ANALYSING THE ROLE OF LANGUAGE IN STUDENTS' CONCEPTION OF PARALLEL LINES

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The concept of parallel lines is foundational to much of the work that students need to do in high school geometry – especially in reasoning tasks involving parallel lines and transversal and in developing a relational understanding of different quadrilateral classes. There appear to be a few exceptionally tenacious alternate conceptions related to parallel lines amongst students who learn mathematics in Hindi. This is likely to add to the already considerable difficulty that students in state-run secondary schools in India have in learning Euclidean geometry. This paper examines data from a large-scale research project to get a nuanced understanding of students' concept of parallel lines and also explores whether a language-related issue might be challenging concept formation.

INTRODUCTION

The teaching and learning of Euclidean geometry at the secondary level is a complex challenge that several researchers and frameworks have tried to address (Battista, 2007; Shaughnessy & Burger, 1985) over the past few decades. In the context of learners studying in state-run secondary schools in India, formal geometric reasoning and proofs, which constitute over a third of the secondary mathematics curriculum, pose an even greater challenge. Additionally, there are complexities related to the multilingual context of India. A module on Geometric Reasoning (GR) designed for the Connected Learning Initiative (CLIx) project aims to address some of these by designing learning experiences that factor in the complexity of the challenge, making selective use of technology to do so. The GR module attempts to help students progress from lower to higher van Hiele levels of reasoning.

Some diagnostic studies done prior to module development had revealed several learning challenges – some of them expected, but also a few unexpected ones – like some extremely tenacious alternate conceptions in students' understanding of parallel lines (Srinivas, Khanna, Rahaman, & Kumar, 2016). It was important to understand the nature of the challenge faced by learners and address it, since a robust understanding of parallel lines is a foundational concept in geometry. It was also somewhat curious, since no other study that we came across had stressed these specific alternate conceptions previously, even those specifically on parallel lines (Mansfield and Happs, 1992). A recent research had discussed the influence of language on students' relational understanding of squares and rectangles (Bussi & Baccaglini-Frank, 2015). This led us to examine whether something in the language of instruction – Hindi, might be influencing students' concept formation of parallel lines in some

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manner. It seemed liked an interesting and less-explored problem to study and report at the PME.

THEORETICAL ORIENTATION

In the secondary classrooms where this study is situated, students are typically introduced to formal concept definitions from the textbook and expected to use those for reasoning tasks. But contrary to this expectation, it is not the concept definition, but a very personalised mental image of the concept that is evoked in each student when they have to retrieve a concept for a task. This is what some researchers have called "concept image" (Vinner, 1983). Vinner argued that concept definitions "remain inactive or even will be forgotten. In thinking, almost always the concept image will be evoked." While teachers at the secondary expect concept formation to be a one-way process – a precise formal definition leading directly to a clear and accurate concept image, this in fact is not so. The final concept image, which accommodates all features present in the formal concept definition and discards the non-essential features, is formed and refined over time. Watson and Mason (2002) discussed the idea of 'personal example spaces' and propounded 'extending the example spaces' as an important aspect of concept formation. We use the idea of concept images and the extension of students' personal example spaces to reveal students' existing and developing understanding of 'parallelness'.

SAMPLE, TOOLS AND METHODOLOGY

Our research is a sub-study done as part of a large-scale learning outcomes study done in CLIx. The study on geometry learning, which provided us data for our sub-study was undertaken with Grade 9 students in 10 Intervention schools (INT) and 9 non-Intervention schools (non-INT) in the Dhamtari district of Chhattisgarh, a state in east-central India. The official language of learning at the secondary level here is Hindi. The tools used in the larger study from which we have drawn data for our sub-study are mentioned in the table below.

Tool	Description and Purpose	Sample
Pre-test and Post-test	Written assessment tools based on van-Hiele levels with 8 MCQs + constructed response items	INT: 466 (91.9% of the cohort, paired) non-INT: 499 (88.3% of the cohort, paired)
Student Interviews	Interviews done with pairs of students immediately after the Pre, and then again after the Post-test	4 pairs from each school – 2 pairs each of high-performing and average-performing, ensuring equal representation of boys and girls
Observation Freewrite	At least 2 classroom observations in each school	(All students present in class on observation days)

Table 1: List of tools used in the sub-study on parallel lines

OBSERVATIONS AND ANALYSIS

According to the data from the MCQ items of the written pre and post-tests, the INT group (Gain score 0.93) showed significantly higher ($p<0.001$) overall learning gains than their peers in the non-INT group (Gain score 0.19). This was true even on individual items, including those on difficult concepts like understanding hierarchical class relationships amongst quadrilaterals. However, both groups had difficulty on the item on identification of parallel lines even in the post test. In the following sections, we discuss in detail the students' concept images of parallel lines that were encountered and explore a few probable explanations.

Data from the Pre-test and Post-test

In this sub-section, we discuss the MCQ items on parallel lines in the written tests.

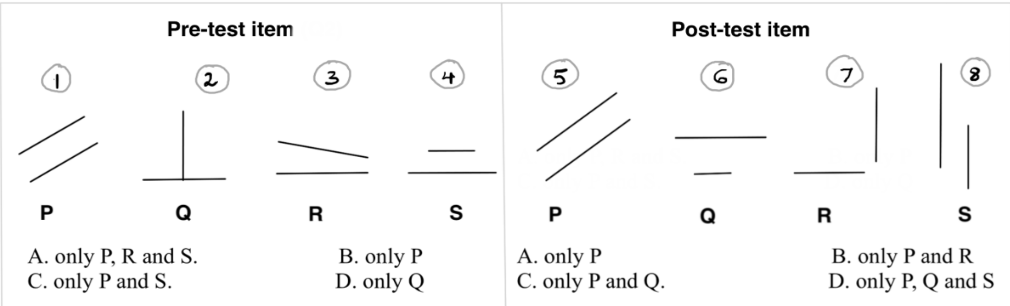

Figure 1: Test items on parallel lines: 'In which of these figures are the lines parallel?' (Translated to English, figures numbered here for easy referencing. The figures will be referred to as Img.1 to Img.8 in the text)

	Pre-test				Post-test			
	A	B	C	D	A	B	C	D
INT	13.3	48.3	21.9	7.3	51.1	12.0	7.8	23.8
non-INT	11.2	52.3	24.7	7.4	53.7	12.6	6.2	25.3

Table 2: Performance data on the Pre-test and Post-test items on parallel lines

In the pre-test, only 21.9% of the students in the INT group, and 24.7% in the non-INT selected the correct option. Just over 10% in each group selected option A, perhaps intuitively, based on overall appearance. Nearly half the students in each group selected option B -indicating, perhaps, that they couldn't identify unequal parallel line segments (Img.4) as a valid example. In the interviews that followed the pre-test, students' concept images were explored through further probing.

In a bid to understand whether they had a concept definition in place, students were asked to explain what they meant by lines being 'parallel'. Many students could retrieve some form of mathematically acceptable definition of parallel lines. Most students who tried to give a formal definition used the ‘lines that never intersect’ construct in their definition rather than the ‘maintaining a constant distance’ construct. This was especially interesting for two reasons:

- In Hindi, the language of learning and teaching in these schools, the term for ‘parallel’ is *samaantar* (or *samaanantar*)– which is a conjugation of *samaan*[equal] + *antar* [distance], a direct statement of the ‘maintaining a constant distance’ construct, and
- their grade 8 textbook defines parallel lines using the ‘constant distance’ construct and in the context of state-run schools in India, the textbook is the primary (and in most cases, the only) resource and considered sacrosanct.

The idea of lines that ‘never intersect’ was articulated with varying degree of sophistication by the students – ranging from the very mathematical “*kabhi pratichhed nahi karti* [never intersect]”, to the informal “*ek doosre ko kaat ti nahi hai*” [don’t cut each other] or “*ek doosre se takrayengi nahi*” [won’t collide with each other]. One common colloquial phrase used by some students for parallel lines was ‘*sojh*’ [straight] or ‘*sojh-sojh’* indicating lines that move ‘straight’ and don’t intersect. A few students produced incorrect concept definitions – for instance, one boy who used the term *sojh* interpreted it as ‘horizontal’ and mentioned in the interview that he would choose only Img.4 (pre-test) as parallel if that had been an option.

Interestingly, not even half the students who could produce mathematically acceptable definitions in the pre-test could identify Img.4 as parallel. This seemed to indicate a gap between the concept definition and the evoked concept image. Data from the pre-test and interviews revealed that a concept image that students frequently evoked was that ‘parallel lines are necessarily equal’. While we foresaw this happening at the pre-test stage, a considerable change was expected post teaching - especially in the INT group, as the GR module had several tasks specifically designed to reveal students’ temporary concept images, and ‘extend their example spaces’. Despite this, in the post-test item too over 50% of students in the INT group chose only Img.5 (Option A), the equal parallel lines. This led us to examine whether the Hindi terms being used was somehow informing this concept image. Examples from the student interview data and the class interactions provided some insights about how the terms used were influencing student thinking about these concepts and are discussed in the next sub-section.

A classroom snapshot: *The story of sam, samaan, samaantar and smaanaantar*

This sub-section reports an excerpt from a classroom discussion between the teacher (T) and a group of girls (G1 to G3) that happened in INT-10, one of the INT schools.

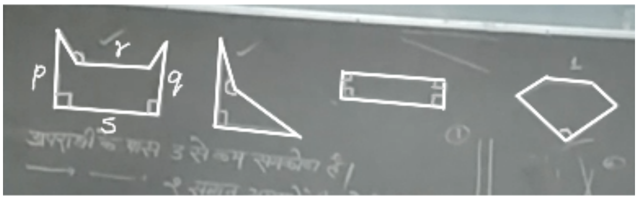

Figure 2: A snapshot of the blackboard during the classroom discussion on '*samaan*' [equal] vs '*samaantar*' [parallel]

The class is discussing properties of shapes and T asks whether the first shape (see Figure 2) has equal sides. The students reply 'yes' in chorus. T points to sides *p* and *q* and asks the class if those were the sides that are *samaan*[equal]. A few students say 'yes', while others, including student G1, disagree. She indicates sides *r* and *s* and says those are equal. T asks whether they appear to be equal. Some students say 'no', but G1 and some others say 'yes'. T ignores those who said yes and confirms that *r* and *s* are not equal and asks students to justify. A discussion follows:

G1: Par sir, yahan *baraabar* nahi bol rahe, *samaan* bol rahe hain!/ [But sir, they aren't saying *baraabar* here, they are saying *samaan*!] (*'Baraabar'* is the colloquial word for 'equal' in Hindi, while '*samaan*' is the formal one.)

T: *Samaa*n aur *baraabar* mein kya antar hai? [What's the difference between *samaan* and *baraabar*?]

G1: (indicating sides *r* and *s*) *'Baraabar*' matlab, jaise yeh 2 cm hai toh yeh bhi 2 cm Aur *samaan* mein na sir, usko aage bhi badha sakte hain… (trails off, appearing unsure)/ ['B*araabar*' means if this one is 2 cm then this one is 2 cm too. And sir, in *samaan,* they can be extended further…]

T then asks G1 and another girl G2 to come up to the board and there is a discussion with an isosceles trapezoid drawn on board where G2 uses the construct of 'lines that never intersect' to explain 'parallelness'. G1 repeatedly points at the pairs of parallel lines during her explanations, using the word *samaan* for them. At this point T specifically asks her to define what '*samaan*' means.

G1: *Samaan* woh hotein hain 'jo ek doosre ko kabhi nahi kaat ti./ [*Samaan* means those (lines) which never cut each other.]

T: Usko *samaan* bolte hain?/ [Are those called '*samaan*'?]

G1: *Samaantar* bolte hain sir…/ [They are called *samaantar*(parallel), sir…]

T: Toh phir…?/ [Then…?]

G1: *Samaantar* matlab *samaan* hota hai. / [*Samaantar* means *samaan*]

T: Kaun bolta hai?/ [Who says so?]

At this point, T comes to the board and unpacks the terms used by explaining their conjugation: *samaanaantar* as *samaan* + *antar*, and *samaantar* as *sam* (also meaning equal) + *antar* (distance*),* explaining how this actually defines the notion of parallelness. The discussion continues and by now many students are seen discussing this in their own groups. After a while when T asks if the distinction between *samaan* and *samaantar* was now clear, about half the class says ‘yes’. When T is about to move on, G1 and G3 stop him and ask him something (inaudible), and again there is a discussion on *samaan* and *samaantar*. This happens twice. T explains the derivation of the terms again and finally asks students to classify a few pairs as examples and non-examples.

Post-test interview responses

In the interview following the post test, G1 was paired with student G3, and an attempt made to reveal their ‘personal example spaces’ and the extent of concept formation through some extension tasks that had not been part of the interviews done after the pre-test.

Draw (if possible), a pair of lines that are:

Task 1: parallel but not equal
Task 2: equal but not parallel
Task 3: neither parallel nor equal

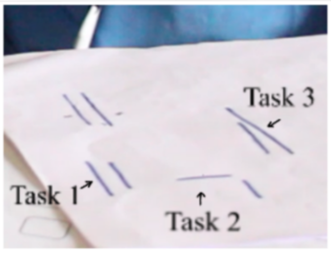

Figure 3: Extension tasks 1-3 of the Student Interviews and G1’s responses

On Task 1, both G1 and G3 could individually produce correct examples. However, when presented with Task 2, G1 looked puzzled, and her immediate response was “*...jo samaan hai ussey samaantar bol sakte hain hum*” [lines which are equal ‘can be called parallel’]. G3 interrupted her to counter this by pointing out Img.7 of the post-test as an example that fits the task. G1 pondered over this, and subsequently, both students produced correct examples that resembled Img.7, with G1 agreeing that it is indeed possible. Further on, in Task 3, G1 could produce an example, and justify why her example fitted the task. But her initial response to Task 2 showed that her concept images of ‘*samaan*’ vs ‘*samaantar*’ were still not quite robust, and she was not able to isolate one concept from the other with consistency.

DISCUSSION

Based on class observation data and the student interactions, it appeared that a few students (like G1, at the initial stages of the reported classroom discussion), and possibly those who chose the lines in Img.7 too as ‘parallel’, were unable to discern the difference between the two terms and often used them interchangeably. In both pre and post intervention interviews, many such students expressly stated that there is no difference between ‘*samaan*’ and ‘*samaantar*’. However, a much higher percentage of students (51.1% in INT group) had a different issue – while they understood the idea of ‘parallelness’ as lines or line segments that would never intersect even if extended, they *assumed equality of length* as an essential feature of the concept. This was a notion that they found (as the data showed) extremely difficult to discard. This seemed to be the case with G1 during the post-test interviews.

Several possibilities were explored while trying to figure out the reason why for so many students the evoked concept image of parallel lines was of *those that would never intersect, but were also necessarily equal in length*, and why this particular concept image was so unshakeable. The examples that students have had prior exposure to, was deemed to be a possible influence. However, it did not seem like the most compelling one, especially because their previous class (Grade 8) textbook had roughly 20 examples each of equal and unequal parallel lines (or line segments).

A more plausible explanation seems to be the linguistic challenge related to the Hindi words for the two concepts – '*samaan*' and '*samaantar*'. This seems plausible in the light of this particular notion not having been reported as being widely prevalent, or persistent, in hitherto reported studies on students' concept of parallel lines (Mansfield and Happs, 1992). We discuss a few possible ways in which the terms used could be impeding students' concept formation.

The words *samaan* and *samaantar* are close to each other in sight and sound and might be causing students to substitute the usage of one for the other. This is compounded by the fact that there is a third word in Hindi, called '*sam*', also meaning equal (among other things), which is also used in mathematics, especially in the context of geometry. So *sam* and *samaan* both mean the same thing (equal), while *samaantar* (*sam* + *antar*) and *samaanantar* (*samaan* + *antar*) both mean the same thing too (parallel). In this context, we expect students to understand that *samaan* and *samaantar* represent two different concepts. One more compounding factor here could be that the everyday word used by the students for 'equal' is not *samaan*, but *barabar*.

Another challenge could be that the word '*samaan*' is nested in the word '*samaantar*' in its entirety. It is possible that in the students' mental schema, this results in the concept of *samaan* (equal) being subsumed within the concept of *samaantar*(parallel). It's important to note that both *samaan* and *samaantar* are attributes applicable to the entities in question – line segments. This might offer a plausible explanation for students' extreme difficulty in dissociating the equality feature from the concept image of parallel lines.

In their study, Bussi & Baccaglini-Frank (2015) had suggested that the *inclusive* sequence of ideograms in the representation of 'squares' and 'rectangles' in Chinese is perhaps more effective than the separate, everyday names 'square' and 'rectangle' learnt by English speaking students for developing an understanding of their inclusive class relationship in later grades. In our context, the inclusion of one unrelated concept (*samaan*) within another in the Hindi term for parallel (*samaantar*) might perhaps be impeding concept formation, making it more difficult for students to separate the essential and non-essential attributes of the concept of parallel lines. This is something that needs to be studied further. Also, it is not to say that this is the only challenge to students' understanding of parallel lines – there are many others, a few of which have been discussed here. It is important for teachers to understand these challenges and take up tasks that help in revealing students' concept images, and weave a discourse around it to reduce the gap between the concept image and the concept definition, in order for them to be used effectively in reasoning tasks.

Additional information

Study done with Connected Learning Initiative (CLIx). Seeded by Tata Trusts, Mumbai; Founding Partners: Tata Institute of Social Sciences, Mumbai and Massachusetts Institute of Technology.

References

Battista, M. T. (2007). The development of geometric and spatial thinking. In F. K. Lester Jr. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 843-908). Charlotte, NC: Information Age

Bussi, M. G. B., & Baccaglini-Frank, A. (2015). Geometry in early years: sowing seeds for a mathematical definition of squares and rectangles. *ZDM, 47*(3), 391-405

Mansfield, H. M., & Happs, J. C. (1992). Using grade eight students' existing knowledge to teach about parallel lines. *School Science and Mathematics, 92*(8), 450-454

Shaughnessy, J. M., & Burger, W. F. (1985). Spadework Prior to Deduction in Geometry. *Mathematics Teacher, 78* (September 1985), 419-428

Srinivas, S., Khanna, S., Rahaman, J., & Kumar, V. (2016, December). *Designing a Game-Based Learning Environment to Foster Geometric Thinking.* Paper presented at the *2016* IEEE Eighth International Conference on Technology for Education (T4E)*, Mumbai (pp. 72-79). doi: 10.1109/T4E.2016.023*

Watson, A., & Mason, J. (2002). Extending example spaces as a learning/teaching strategy in mathematics. In A. Cockburn, & E. Nardi (Eds.), *Proceedings of the 26th Conference of the International Group for the Psychology of Mathematics Education* (Volume 4, pp. 377–385). Norwich, UK: University of East Anglia

Vinner, S. (1983). Concept definition, concept image and the notion of function. *International Journal of Mathematical Education in Science and Technology*, *14*(3), 293-305