EXAMINING KNOWLEDGE DEMANDS FOR TEACHING CLASS INCLUSION OF QUADRILATERALS

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Though teacher knowledge has been assessed using paper-pencil tools, little is known about how teachers use their mathematical knowledge for teaching in in-the-moment challenging instructional situations while attempting to change their practice to encourage student reasoning. This paper illustrates using the case of teachers' discussion on class inclusion, how analysis of these challenging situations serves to not only identify the knowledge demands for teaching geometry but also illustrates the dynamic nature of mathematical knowledge for teaching as it may support use and development of Subject Matter Knowledge (SMK) and Pedagogical Content Knowledge (PCK) to structure the classroom discourse. The need for safe space for teachers to explore their teaching and learn from their mistakes is needed by reflecting on and discussing their teaching with other educators.

INTRODUCTION

Teachers' knowledge of geometry has been studied mostly in the context of measuring prospective or in-service teachers' knowledge through the development of paper-pencil tools and have been found to be dismal (Jones, 2000). These tools have been largely based on two major frameworks prevalent in literature. One of these frameworks is van Hiele's theory (1959) which has been used for over 50 years since it was proposed and has been used to assess both students as well as teachers' knowledge of geometry. Another framework is by Ball, Thames and Phelps (2008) about Mathematical Knowledge for Teaching (MKT). However, these paper-pencil tools have the limitation of not being able to capture the situations in which this knowledge is used or developed "in-situ" in the process of classroom interaction. Further, the use of tests to show deficits in teacher knowledge may lead to teachers' distrust towards researchers. In the Indian context, where we are faced with a situation of lack of highly qualified mathematics teachers, we need to characterise the challenging situations that teachers face in addressing students' learning and the topic-specific knowledge of mathematics for teaching that is required to overcome that challenge while working with the teacher.

Attempts have been made to identify topic specific knowledge of geometry used by teachers in the moment of teaching. A study by Chinnappan, White and Trenholm (2018) identified a symbiotic relationship between the SMK and PCK. In this paper, we illustrate and use a topic specific framework to identify knowledge demands and relation between SMK and PCK, while analysing classroom interactions of teachers making an attempt to change their instructions to encourage student reasoning. The main research question addressed in this paper is

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How can analysis of challenging instructional situations contribute to our understanding of knowledge demands for teaching geometry, particularly class inclusion?

RELATION BETWEEN TEACHERS' CONTENT KNOWLEDGE AND TEACHING OF GEOMETRY

It has been well recognised that both SMK and PCK play a crucial role in determining the quality of instruction (Hill, Blunk, Charalambous, Lewis, Phelps, Sleep, & Ball, 2008). However, the relationship between SMK and PCK has not been explored at depth especially in in-situ situations that illustrate the dynamicity of this knowledge (Takker & Subramaniam, 2017; Chinnappan, White & Trenholm, 2018).

SMK for teaching class inclusion can be identified using van Hiele theory (1959). According to van Hiele theory, the concept of class inclusion develops at level 2 when the learner is able to establish interrelationship between properties within and among figures, create and identify figures as examples and non examples of a class, deduce properties of figure, identify similarities and differences in figures and is able to classify correctly. It also includes giving and evaluating the alternative definitions for their correctness and knowledge of necessary and sufficient properties to arrive at the minimal definition, give figures as examples, non examples and also be able to evaluate non-routine figures for recognising a shape. A common theme that runs across the development of knowledge of geometry is understanding that the process of generalisation is a deductive process, often mistakenly assumed to be an inductive process.

A teachers' knowledge of the teaching of geometry or PCK is built over these foundation blocks, wherein a teacher uses this knowledge to generate and sequence the tasks for teaching or assessment. A teacher uses the knowledge of students' thinking combined with SMK to come up with examples, challenging examples and even counterexamples to make students think and focus on a particular or connected property of a shape. The understanding of correct and minimal definition guides the task implementation as well as evaluation of alternative and partially correct definitions given by students. A teacher may use the knowledge of different representations to depict and elicit statements of class inclusion from students and work on their alternative conceptions of shapes and hierarchical classification. The knowledge of deductive process of generalisation guides the norms of classroom interaction and how the generalisations are stated and validated. Teacher's response to visual and perceptual approaches by students are guided by this knowledge of the nature of mathematics.

METHODOLOGY

The instructional episodes reported in this paper are selected as cases from two teachers (Sunil and Shruti – Pseudonym) from a cohort of 10 teachers associated with Connected Learning Initiative (CLIx)[1] since 2016 to support the implementation of Geometric reasoning module. The selection of the instructional episode case was done based on similar content used in lesson and then within case and cross case patterns were analysed to come up with the theory of role of challenging situations in teacher learning (Eisenhardt, 1989).

All teachers belonged to and worked in a rural setting in the state of Chattisgarh, India. The teachers were associated with the initiative since 2016 and had undergone around 9 days of face-to-face time in workshops prior to study. Based on reports of challenges faced in managing classroom discussion, follow up classroom support of around 4 weeks was provide to teachers by research team after a two-day workshop discussing the module in detail. During the follow up, researchers did classroom observations and follow-up discussions to discuss the activities, lesson plan, content related doubts and analysis of students' understanding. The data from the professional development workshop and classroom observations was collected by writing detailed notes by the researchers through participant observation and in some cases, audio was recorded after teachers' permission.

Sunil was 39 years old high school teacher (Male), had 8 years of experience of teaching high school maths and has the qualification of M.Sc in Physics and diploma in IT, though no formal qualification in the education field. Shruti was a 42 years old high school teacher (Female) with 11 years of experience and had M.Sc maths and B. Ed as a formal qualification. Both the teachers were motivated to implement the modules and engaged heartily with the researcher team on the discussions of content, pedagogy and student learning. However, Sunil had a very relaxed and conversational approach to teaching mathematics making students comfortable in expressing their thoughts and trying to engage all learners in classroom discussion. Shruti was a bit authoritative in classroom transaction. She felt that mathematics is too difficult for some students though she understood the importance of engaging students in discussion and supporting reasoning and expressing their thoughts.

FINDINGS

In this section, we report challenging episodes that made gaps in teachers' knowledge explicit and provided opportunities to us as teacher educators or teachers' themselves to reflect on their knowledge. These challenging events arose in situations when teachers attempted to support students' engagement in reasoning about shapes and came across contingent situations in practice.

The normal instruction for teaching geometry in these rural schools was that of a teacher explaining concept of shapes through examples of stereotypical figures from the textbook, listing its properties and definition and expecting students to memorise them. The teaching involves explanations of concept or definition followed by questions, most of which are replied in chorus or in few words. The attempt in the CLIx initiative was to encourage teachers to focus on developing property based reasoning through challenging and non-routine shapes for recognition and classification of shapes thereby developing the meaning of the definitions which may have been learned by rote by the students in earlier grades. The pedagogical pillars to provide safe space to students to learn through their mistakes and learning through collaborative work was emphasised in the workshops.

The transcripts presented from both the lessons are one week after the intervention started. Teachers were discussing tasks of class inclusion using several challenging figures, for example, if the particular figure is a square or not, identify the shape if it was not a square and give reasons based on properties of shape.

Sunil's discussion on class inclusion

The lesson from which the episode is discussed, Sunil was discussing several figures from the task described above and one of them was a rectangle. He started the discussion on rectangle as indicated in the transcript below. In the transcript, T indicates the teacher, Sn indicates the different students speaking and Sch indicates chorus response of students.

36 T: What is a rectangle?

37 S1: One which has opposite sides equal.

38 Teacher draws a parallelogram in figure 1 and labels the opposite sides as 2 and 7 cm respectively.

40 T: Would you call this a rectangle?

41 S *chorus*: Yes

42 T: *Arre? (what!!)*

45 S *chorus* : No

46 S2: Because all its angles are not 90 degree

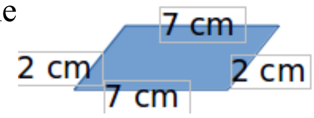

Figure 1: Parallelogram

In the above episode, one needs to note that instead of responding to the student's incomplete definition of rectangle with an evaluative statement or explanation of the correct definition, the teacher responded by making a figure which satisfies the property given by the student but is not a rectangle. It is thus a counterexample given by the teacher which is contingent on the response given by the student. Knowledge of geometry is involved in both the pedagogical tasks, but giving a counterexample based on student response requires in-the-moment use of both SMK and PCK to respond in mathematically appropriate way to students' definition of rectangle. The teachers made the figure having the property of opposite sides but varied the angle using the knowledge of similar and different properties of parallelogram and rectangle. However, the decision to respond by drawing a counterexample figure rather than the correct definition involves knowledge of PCK of how to draw students' attention towards the missing necessary property of the ninety degrees angles of the rectangle. He expected that students would experience conflict through the figure, identifying that the property is same but the figure looks different from rectangle, but is surprised when student agrees. It made student realise the mistake and change the answer. However, in continuation of the pattern of giving reason and justification for their response in previous task, the student S2 justifies their response by identifying that angles are not ninety degrees which is the necessary property of the rectangle. Thus, by using the counterexample, the teacher was able to highlight the need of the necessary property of angles being ninety degrees and help students take the step towards arriving at the minimal definition. One can still doubt, however, that whether the students have understood the concept of opposite sides being parallel too since this property can be deduced from opposite sides being equal. Identifying that this point needs to be discussed, again involves both SMK of properties and PCK of selecting the points to be discussed an assessed for understanding. The discussion returns to this point in the following transcript when the teacher asked the students to recognise a rotated parallelogram (not in horizontal or vertical orientation).

67 T: What is this?

68 S *chorus*: Rectangle

69 S1: In rectangle opposite sides should be equal and ninety degree angle should be there. (In response teacher points to the figure drawn earlier of the rectangle and the parallelogram.)

70 S2: Opposite sides are equal but …

71 S3: First one is a rectangle because opposite sides are equal. In parallelogram, it is not necessary to have a ninety degrees angle.

73 S4: Rectangle can be called as parallelogram because… but we cannot call a parallelogram as a rectangle.

75 Teacher repeats the assertion by S4 followed class repeating it in chorus.

In the above episode, all the students were able to successfully identify a rotated rectangle as a rectangle, thus were able to understand that orientation of the figure does not constitutes as the property of the figure itself. The students then gave their observations which represents the modified definition including the necessary property and statements of generalisations about parallelogram by S3 and relation between rectangle and parallelogram by S4. The generalisation by S3 is correct while that of S4 is partially incorrect. Teacher not evaluating and repeating the S4's assertion in line 73 indicates gap in SMK of not being able to correctly evaluate a generalisation which is behind the gap in PCK of not identifying students' misconception and ways to address students' misconception. In line 73, student is arguing visually considering the static figure of a parallelogram always having two acute angles and two obtuse angles and thus excluding the rectangle from the category of the parallelogram. It is confirmed that the teacher agrees with this line of thinking, since in the next task of analysis of square and rectangle, the teacher concludes that square can be a rectangle since it fulfils all the necessary properties of the rectangle (correct) but incorrectly concludes that rectangle cannot be made into a square and thus is not a square. This is also reasoning based on empirical nature of drawing where the argument is based on physical properties of drawing rather than mathematical properties. There is also a gap in the use of mathematical language as the teachers is not careful in use of qualifiers or terms that indicate generality like "All squares are rectangle" is a different statement from "A rectangle *can become* a square" constraining thinking in generality of shapes. Thus although the norms for giving reasons and articulating thinking have been established, the knowledge demands during teaching indicate gaps in teachers' knowledge about the process and nature of class inclusion especially the process of generalisation and therefore students' understanding fall through the cracks.

Shruti's discussion on class Inclusion

We now discuss an episode from Shruti's class on 30 July 2018 wherein a similar discussion took place about the relationship between square and the parallelogram. Here the teacher was discussing a challenging example from the module (rhombus) and was asking students to evaluate if it was a square or not. Students identified that it is not a square and some identified it as a rhombus while others as a parallelogram. When students were not able to articulate what a parallelogram is, the teacher asked the students to make different figures of parallelogram and herself identified the property of a parallelogram as "one who has opposite sides equal and parallel". What happened afterward is given in transcript below.

96 S4: 2 acute angles and 2 obtuse angles

97 T: Very good! What is the main property?

98 S *chorus*: All sides equal

99 T: It is not necessary. (Draws a parallelogram with one pair of sides longer in horizontal orientation) Is this not a parallelogram?

100 S *chorus*: Opposite sides are parallel…

101 T: (Draws a rectangular candy and explains) Opposite sides will not ever meet, the angles of the parallelogram cannot be ninety degrees…. It is always 2 acute angle and 2 obtuse angle…. Or in some case, it may be possible…. In some cases, rectangle can also be a parallelogram.

In the above episode, in line 96 the student gave the property of 2 acute angles and 2 obtuse angles for a parallelogram based on a visual stereotypical image of it. Though wrong, teachers praised the student indicating it to be correct but tried to make student think about the necessary property as the "main property" and also gave a correct counterexample when students responded as "all sides equal" based on the visual figure of rhombus in front of them. Just like Sunil, she was also using her knowledge of SMK of necessary properties to generate PCK through the use of counterexample in the moment of teaching. However, the most interesting part in this episode is when teacher realised her own mistake of giving the property on the basis of visual diagram of the parallelogram rather than based on its necessary properties in line 101. While discussing the properties of the parallelogram, she at first said that angles of a parallelogram cannot be ninety degrees but in the next instance after a long pause and looking at diagram, she realised that a rectangle is a parallelogram and in that case the angles would be ninety degrees. Here we see an instance of teacher deepening her own SMK while responding to student's incorrect assertion. It is possible that this reflection may not have occurred during the classroom, would have occurred later or not at all. However, the potential of development of SMK as well as PCK while responding to and evaluating students' assertions cannot be denied.

THE WAY FORWARD

The analysis of challenging episodes related to class inclusion from the teaching of two teachers struggling to change their practice to encourage students' reasoning indicates how this type of analysis can shed light on the knowledge demands for teaching class inclusion in terms of both SMK and PCK and their interdepenencies. The main knowledge demand is knowing and supporting the process of generalisation among students and to be able to consistently operate at the level of analysing properties of figures mathematically rather than perceptually in contingent situations. The episodes also illustrate the dynamic nature of mathematical knowledge for teaching. The classroom interaction not only gave opportunity for teachers to develop their PCK contingent to student responses to develop their thinking through use of counterexamples and to identify necessary properties of the shape but also allows opportunities to deepen their own SMK through reflection and student engagement. However, to support teachers' exploration and learning from practice one need to create safe space for teachers to explore their teaching and learn from their mistakes perhaps by reflecting on their teaching with other educators.

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