Designing a Game-based Learning Environment to Foster Geometric Thinking

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***Abstract*—While there has been a steady rise in enrolment in secondary schools in recent years, many of the issues and concerns related to the quality of mathematics education have persisted. The challenges cut across several dimensions – access to resources, a gap between the intended and the implemented curriculum, and pedagogical practices that promote rote rather than reason, are a few important ones. These result in considerable learning gaps and limited opportunities for students to actively engage with the subject. In this paper, we describe the specific challenges of teaching geometry to high school students in India (as part of a project spanning four states), and argue that a carefully designed game-based learning environment, working in tandem with focused classroom discussions, could address some of the key challenges. We describe the design of our game-based learning environment and present the results of preliminary investigations that demonstrate its potential.**

Keywords-Game-based learning, van Hiele levels of geometric thinking, Geometry learning in India.

I. INTRODUCTION

Researchers and teachers have invested considerably in making mathematics more accessible and interesting for high school students. While the problem is global in nature, there are numerous systemic issues and challenges that further affect the quality of learning in the context of state-run secondary schools in India. These include a lack of resources, paucity of teachers, pedagogical practices that promote rote learning (obfuscating the intended curriculum), low student motivation, and student learning levels that are not commensurate with grade. The Connected Learning Initiative (CLIx) project aims to address some of these by designing solutions that factor in the complexity and scale of the challenges, selectively using technology to do so. A component of this project addresses a particularly difficult problem in high school mathematics education: teaching and learning geometry. The core of the CLIx geometry module is a learning game, but other vital components include: classroom discussions that complement the gameplay, and activities (both hands-on and digital) where students construct shapes, reason with them, make and prove conjectures. This paper focuses on the game's design, and explains why and how it fosters a non-threatening, interactive environment that stimulates thinking and peer dialogue, helping students learn geometric concepts and reasoning.

Our experiment is in two stages. Stage 1 consisted of a preliminary *diagnostic study* done in the field with a target group of students, followed by an extended *design phase* where the game was iteratively refined based on feedback, including from *small-scale student trials*. Based on this, the design was finalized for Stage 2 (currently underway): *implementation at scale*. This paper reports findings from Stage 1, focusing on the rationale for design and observations from student trials.

The interactions during the diagnostic study showed that high school teachers considered geometry as one of the most difficult topics to teach in mathematics. While the high school geometry curriculum in India focuses entirely on formal Euclidean Geometry, most students find it difficult to understand or appreciate the need for formal deductive proofs in geometry. Research suggests that this problem is not isolated [1], and several possible ways of addressing the issue have been discussed [2].

After reviewing related work and discussing findings of the diagnostic study (Section II), Section III discusses the rationale and requirements for a game-based intervention. Section IV presents a game designed to meet these requirements, and Section V discusses the extent to which these are met, based on pilot studies. Lastly, we present our conclusions in Section VI.

II. RELATED WORK

Geometry is a foundational topic in mathematics, but TIMSS[1] and PISA[2] studies have indicated the status of geometry as one of the weakest areas of learning [3]. Several researchers have studied students' difficulties with formal reasoning and proofs in mathematics. The van Hiele theory [2], in particular, effectively charts the progress of a learner's geometric thought. According to this, learners progress through several levels of thinking about geometric shapes and their properties before reaching a level where formal deductive proofs and axiomatic systems are accessible to them. Levels 1 to 4, as outlined by Battista and Clements [4], are presented in TABLE I. Level 5, *Rigor,* is rarely encountered amongst high school geometry students, and is therefore not relevant for our discussions. In later work, Battista [2] also details the progression through a level, describing the sublevels within it.

[1] The Trends in International Mathematics and Science Study (TIMSS) is a global series of international assessments of the mathematics and science knowledge of students.

[2] The Programme for International Student Assessment (PISA) is a worldwide study by the Organisation for Economic Co-operation and Development (OECD) in member and non-member nations of 15-year-old school pupils' scholastic performance on mathematics, science, and reading.

TABLE I. THE FIRST FOUR VAN HIELE LEVELS OF GEOMETRIC THOUGHT [4]

Level	***Name***	***Description***
1	*Visual*	Students reason about geometric shapes on the basis of their appearance and the visual transformations that they perform on images of these shapes. The identify such figures as square and triangles as visual gestalts, often after viewing prototypes. For instance, they might say that a given figure is a rectangle because "it looks like a door".
2	*Descriptive/ analytic*	Students reason experimentally; they establish properties of shapes by observing, measuring, drawing, and making models. They identify shapes not as visual wholes but by their properties. For example, a student might think of a rhombus as a figure with four equal sides.
3	*Abstract/ relational*	Students reason logically. They can form abstract definitions, distinguish between necessary and sufficient conditions for a concept, and understand and sometimes even present logical arguments. They can classify figures hierarchically by analyzing their properties and give informal arguments to justify their classifications (e.g., identifying a square as a rhombus because "it's a rhombus with some extra properties").
4	*Formal deduction*	Students reason formally by logically interpreting geometric statements, such as axioms, definitions, and theorems. They are capable of constructing original proofs by producing a sequence of statements that logically justifies a conclusion as a consequence of givens.

To get a better understanding of students' difficulties with geometric reasoning in the Indian context, we carried out diagnostic case studies in two of the four partner states[3]. This study involved administering a diagnostic assessment tool (based on van Hiele's levels of geometric thought) and detailed small-group interviews. An analysis of the responses showed that most of the high school students assessed were at the very early levels (Level 1 or early stages of Level 2) of geometric thinking [5][6]. The studies indicated that many students of grade 9 identify basic shapes by comparing them to visual prototypes, rather than by their properties. Less than half the students in the highest performing group amongst the two states could identify a tilted square as a 'square', and a similar number could identify triangles correctly amongst a given set of shapes e.g., the triangle *P* in Figure 1, articulating reasons like "it is too thin" or "it does not *look* like a triangle". In a higher level task, students of grades 9 and 10 were shown three shapes: a square (with a horizontal base), a rectangle (with a horizontal base), and a tilted rectangle. When asked to decide which of these shapes were rectangles, most students chose only the rectangle in standard orientation. Hardly any student (including those in grade 10, with one year of formal geometry behind them) understood the idea that the square was a rectangle too. This suggests that the students do not really understand hierarchical shape classification, though their textbooks introduce this idea by grade 8. There were also several basic geometrical misconceptions. For instance, a large number of students thought that parallel lines are necessarily congruent in length.

These observations highlighted the need for a solid foundation in geometry and geometric reasoning. Prior research suggests engaging high school students in tasks related to van Hiele Levels 1 to 3 before moving on to formal reasoning tasks (the 'Spadework Prior to Deduction in Geometry' [1]). Our Geometric Reasoning module attempts to help students with this 'spadework' by developing their thinking from the existing levels (typically Level 1) up to Level 3, which is the threshold of formal reasoning. In particular, the learning game is designed to help students progress through these levels.

[3] The CLIx project is implemented in four partner states in India: Chhattisgarh, Mizoram, Rajasthan and Telangana.

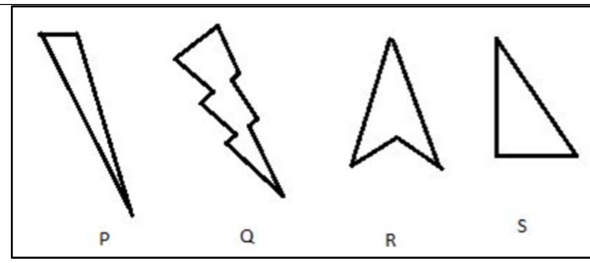

Figure 1. Triangle identification task

III. RATIONALE FOR DESIGN

In this section, we present the pedagogical considerations shaping the design of our Geometric Reasoning module, with emphasis on the game. The discussion is split into three parts – the underlying principles guiding the curricular and pedagogical choices, the rationale for choosing game-based learning as the core of the module, and the need to deploy this game in a digital form.

A. Principles guiding curricular and pedagogical choices

The position paper by the National Focus Group on Teaching of Mathematics [7] has highlighted a pressing need to shift from learning procedures and formulas to the *processes of mathematics*. Reasoning systematically, problem solving, mathematical communication and the use of heuristics are some of the important processes that students need to actively engage in. Our aim is to transform current classroom pedagogy by providing the learner opportunities to be an active participant in these processes. The first change that the module aims at is to *shift the focus to geometric thinking*, from the present practice that focuses primarily on geometric terms and memorization of proofs. At the same time, the module needs to be aligned to the prescribed curricular content, as the intervention is meant to work within the school system. There is also a need to address known conceptual gaps, as they could hamper future learning. Considering this, the module design needs to support the development of reasoning and problem solving skills, while being situated in class-appropriate content. The content found most suitable for the purpose of developing geometric thinking was the topic 'Quadrilaterals' (present in the grade 9 NCERT textbook, as well as in the textbooks recommended by the four partner states).

The intervention also aims to change the *learning environment in the classroom*. The position paper on Teaching of Mathematics talks about offering a learning environment in our classrooms that will "invite participation, engage children, and offer a sense of success" [7]. We believe that such learning environments help break out of the dominant pedagogy where the teacher is in full control, and instead encourage and motivate the learner to drive their own learning.

Accordingly, the main expectations from the module (that the design is required to take care of) are:

1. *Targeting foundational geometric concepts and vocabulary.* The content of the module has to specifically address known areas of difficulty, especially those identified in the baseline studies.
2. *Providing opportunities to engage in reasoning and problem solving*. The sequence of learning activities should start at the learner's current level and take them to progressively higher levels of thinking, throwing up challenges appropriate to the level of thinking, and providing insights into the processes and core ideas of the domain.
3. *Providing opportunities to engage in peer discussion and dialogue*. The tasks and learning activities should be designed to prompt students to share and respond to (each other's) mathematical ideas. The design should also create opportunities for presenting and debating different viewpoints.
4. *Creating a safe space for students to make mistakes and learn*. The design needs to create an environment where students have opportunities to make mistakes, struggle, fail and rectify their errors without anyone explicitly telling them to do so. This is especially important in our context, where students often reach high school without having mastered earlier grade content. These students, in particular, need an environment that helps them revisit foundational concepts and learn without fear of reprimand.

In Section V, we discuss the findings related to each of these expectations from our initial trials.

B. Why game-based learning?

Why did we choose game based learning as the centerpiece of the module? There have been several prior experiments where technology has been used to enhance geometric thinking, including Logo-programming based tools [2] and dynamic geometry environments (DGEs) [2][8]. What does game-based learning have to offer that makes it better suited to our purpose and context? The first reason, is that game-based learning can work without demanding any additional time or effort from the teacher. Unfortunately, most technology-based tools require the teacher to become an expert in that tool, then teach the students, and finally, assess the work students produce. With existing student-to-teacher ratios, it would be impractical to expect teachers to take on such a workload. A well-designed digital game on the other hand, helps the students learn independently, and also carries out formative assessment on a continuous basis. This frees the teacher up for more meaningful interactions with the students (triggering discussions, or giving personalized feedback, for instance).

The other question that needs to be addressed here is 'why design another game – aren't there enough out there already?' We had two main constraints to consider before determining our central resource – it had to align (even if loosely) to the prescribed curriculum, and it had to be open source. To the best of our knowledge, no resource meeting both these criteria exists. Most of the geometry games available addressed spatial reasoning or transformations which are not part of the curriculum in the target schools. Well-designed games like DragonBox Elements [9] that are interesting and target Euclidean Geometry concepts are not open source. Also, this game places greater emphasis on geometric concepts than on reasoning. Another issue faced was that most available games were specifically designed for younger, predominantly western audiences. One game, said to be based on the van_Hiele levels-based approach, is Smart City Learning [10]. It focuses on contextualizing geometry learning with the use of mobile devices and maps. Such resources are infeasible for the current project, with its severe infrastructure and internet availability constraints. Hence, a new game-based environment that we could customize to expressly target some of the expectations listed in Section III was deemed necessary. The game is played by students in very small groups (ideally pairs), providing the necessary 'safe space' for students (Expectation 4), and students can learn from failures via discussions and debates (Expectation 3) with their peers [11]. These ideas are elucidated further in Section V, based on observations from field trials.

C. The need for a digital implementation

Any technology-based intervention at scale is risky because equipment can easily fail in the challenging environment of schools (intermittent power, dust, rough use, infrequent maintenance, etc.). It is necessary for us to justify why we have chosen to implement our games digitally, despite these challenges. We have two key reasons for our choice: the need to manage game complexity, and the (eventual) need to record the behaviour of students. Both these points are elaborated in subsection IV.B below.

IV. DESIGN OF THE GAME

Mizuko Ito [12] categorized learning games into three genres: (1) Educational, which are tied to a curriculum and target the honing of specific procedural skills, (2) Entertainment, which are "exploratory, narrative-based" [13], and (3) Construction, which are simulation based. In India, games of genre (1) – the drill-and-practice "solve a mathematics problem before you blast something" type – are common. Many games available as Android apps fall in this category. The various versions of Geometry Dash [14][15] are among these – where students burst bubbles or race in between answering objective type questions related to geometry and measurement. Such games have not been shown to have any special educational value, and rarely help students learn anything new or meaningful. In fact, as Klopfer and Osterweil argue: "If your spaceship requires you to answer a math problem before you can use your blasters, chances are that you'll hate the game and the math" [13]. On the other hand, totally open-ended and exploratory games can be extremely time consuming, and difficult to implement in our current school system.

In designing a game that addresses the specific issues we have highlighted, we have attempted to blend elements from different genres. Our main goal is to ensure that *the mathematics in the game* – the challenge it offers – is engaging in itself. Also, our pedagogical approach is a blended one – the game is used to "hook" students and trigger thinking about a particular concept or idea, which is then taken up in the classroom for further (and more formally structured) discussions that crystallize these ideas. Studies have advocated such blended approaches, where the respective strengths of learning games and traditional instructional approaches are used in tandem for more effective learning. Arena and Schwartz [16] have argued that "Well-designed digital games can deliver powerful experiences that are difficult to provide through traditional instruction, while traditional instruction can deliver formal explanations that are not a natural fit for gameplay. Combined, they can accomplish more than either can alone." In the next section, we describe our game, *Police Quad*, and how we envisage learning happening through it.

A. Overview of our game: Police Quad

Our game has been designed keeping in mind the need to focus on geometric thinking and learning processes, even as it is loosely mapped to textbook content. In the game, the player's role is that of a police detective tasked with solving cases. All cases require the use of deductive reasoning in the context of shapes – particularly those in the textbook chapter on Quadrilaterals. Hence the name: *Police Quad*. The context of 'police' and 'eliminating suspects using clues' was chosen to provide an authentic real-life scenario where deductive reasoning is used.

The game has a narrative, which establishes the player in the role of an alien, Geo, who has migrated with fellow aliens from their home planet to Earth. Geo is a police officer who solves crimes (related to theft of water) using his/her rational thinking skills, but all the while driven by a strong sense of justice to make Earth a safer and better place for both humans and aliens. We spent considerable effort in selecting a theme for the game that would feel authentic to students from four different states (hence our choice of police and crimes like 'water robberies'), but not so familiar as to trigger unwanted stereotypes (hence our choice of aliens).

The player needs to use reasoning to solve different types of cases, presented as four different 'Missions'. While playing the game, the player develops an understanding of the properties of shapes, shape classes, and properties and relationships of special quadrilaterals. The game is designed with the belief that the key to student motivation lies in the opportunities that the game provides to think, and engage in reasoning – rather than blasting or popping anything.

Within the module, the game begins after a foundational unit on the concept of shape, where the students (many of whom are likely to be at the *visual* level, van Hiele Level 1), develop the initial notion that a shape is defined by its properties, and not by its appearance. Missions 1 and 2 of the game focus on analyzing and describing shapes based on their properties, hence scaffolding the students' understanding of basic properties, and facilitating the progress through the sub levels of the *descriptive/analytic* level (van Hiele level 2) of thought. At the end of these two missions, the aim is for students to "have made a decided shift from visually dominated reasoning" and "explicitly and exclusively use formal geometric concepts and language to describe and conceptualize shapes" [2]. Mission 3 builds on this understanding to move on from properties of individual shapes to defining classes of shapes. This also lays the foundations for an understanding of the complex idea of hierarchical classification of shapes, which is the key idea in the last mission of the game. This mission – Mission 4, corresponds to van Hiele level 3 – *abstract/relational* – and aims to engage students in property-based inferential reasoning.

B. A closer look: Mission 2

To illustrate the complexity of our game and its adherence to our stated design principles, let us take a closer look at Mission 2. Missions 1 and 2 have similar gameplay: several aliens are suspected of a crime, but all are innocent except one – the culprit. Each alien has a unique shape marker on their chest, and the suspect grid displays these shape markers. The game consists of a series of progressively harder levels, and the number of shapes and clues increases with levels. Each shape, chosen from a pool of about 70, has several associated geometrical properties (number of straight sides, number of right angles, number of pairs of parallel sides, etc.). Please refer to Figure 2.

In Mission 1, the player's task is to eliminate suspects and identify the culprit by evaluating clues provided by the *system*. In Mission 2, the *player* has to proactively query a witness (a robot, as per the story), and identify the culprit based on its responses. The player must frame questions regarding geometric properties of the culprit, and the robot witness is constrained to respond in one of three ways: "Yes", "No" or (for an ill-formed query) "I don't understand". The player can maximize rewards by identifying the culprit using the fewest number of queries. As an example, if one suspect's shape marker is a quadrilateral with exactly one acute angle and the player asks whether the culprit has "at least two acute angles", a "Yes" answer would allow this suspect to be freed (thereby shrinking the pool of suspects), whereas the alien would remain a suspect if the answer was "No".

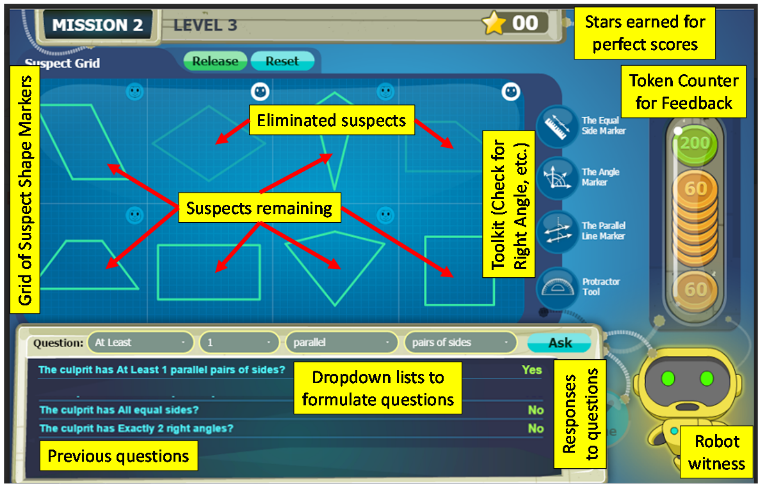

Figure 2. Screenshot from Mission 2, Police Quad

Alternately, the player could query whether the culprit has "exactly 4 straight sides" – here, a "No" answer would allow the suspect to be freed. Thus, the game requires players to genuinely understand concepts, and use appropriate vocabulary such as "acute angle", "at least", "exactly", etc.

During gameplay, feedback is given to the students through a carefully designed system of reward points (in the form of tokens displayed on screen). Every error that the student makes costs him/her a token, but no other comment is made about the error. This is done with a deliberate purpose that is explained in Section V.

The game includes a small number of tools to assist players, each carefully chosen to target specific geometric concepts and the challenges that students face in understanding them. For illustration, we describe the right-angle testing tool in more detail. Originally, this tool was intended to help students measure angles precisely so that they could check whether a specific angle was acute, obtuse or right-angled. During our initial tests, when this tool was not yet available in a digital form, we noticed some students holding up L-shaped corners of paper sheets (which form a 90° angle) against their screens to determine whether an angle was greater than it (obtuse) or smaller (acute). We recognized that such a tool, which provides a *relative* rather than an *absolute* measure of angles, was in fact more suitable for understanding the concepts of acute and obtuse angles. Thus, this is *exactly* how our digital tool is designed: a simple L-shaped interactive that can be moved and rotated. In a similar spirit, there is support provided for the player to revisit some terms that they may be uncertain about, by hyperlinking keywords (such as "acute" and "parallel") to a glossary that provides a brief visual explanation of terms. This support ensures that players can focus on concepts and reasoning, rather than merely memorizing definitions.

Each choice made by the player is also recorded by the game for offline analysis (and also to recover the state of play in case the power fails). It is infeasible for any human to monitor each and every step performed by players, but digital records can be analyzed to determine whether students are finding certain concepts difficult. For instance, we have anecdotally observed students struggle with the concept of "at least", whereas they appear to be more comfortable with concepts such as "exactly" or "more than".

C. *Challenges for digital design*

Developing requisite geometric vocabulary was an important aspect in the game. Considering this and the scale of implementation, it was important for the game itself (and not just the instructions) to be in multiple languages. Our digital implementation allows us to cope well with issues such as multiple languages, but also poses several infrastructure-related challenges.

1) *Multiple languages*

The digital implementation makes it easy to translate *terms* (words, phrases, etc.) appearing in instructions and clues from the base language (English) into the student's language of choice[4]. The translation module maps each base language term to an equivalent term in all supported languages, and this mapping can be easily extended to support new languages. For example, the term "less than 4 straight sides" is represented as "straightSides < 4". This representation is mapped to the English phrase "less than 4 straight sides" and separately to the Hindi phrase "4 से कम भुजाएं". This design allows us to cater to the varied grammatical rules of languages: in English, the quantity 4 appears after the translation of "<" (less than), whereas in Hindi the 4 appears before the translation of "<" (से कम).

2) *Infrastructure-related challenges*

The key infrastructure-related challenges facing digital game implementations in our target schools are intermittent power, lack of reliable network facilities, and

[4] This may differ from the primary language of instruction in a state for some students (e.g., children of migrant labourers). At present, we support Hindi and Telugu.

limited numbers of computers (in relation to the number of students). Our implementation is therefore designed as a stand-alone, offline game with the ability (in future) to synchronize data gathered during game-play with a server whenever internet access is available. Also, since we cannot be assured of a functional local area network (LAN) within the computer lab, we cannot support any in-game collaboration with players on other terminals. Our implementation captures the current state of each game locally so that, in the event of a power loss, students can pick up from where they left off once power is restored.

V. Observations and Analysis

During the design phase, the *Police Quad* game was trialed iteratively with students of grades eight, nine, and ten in two different contexts – Mumbai and Mizoram. The samples were representative in terms of the expected game audience (grade 9 government school students), the diversity of learning levels, and gender. As detailed in Section III, we have identified key expectations from the module that we believe are necessary in order to bring about a change in the way mathematics learning is approached. This section discusses some experiences from the trials that are indicative of whether these expectations are achievable.

1. *Targeting foundational geometric concepts and vocabulary:* The baseline study that we had conducted prior to module design had helped us get an understanding of the conceptual gaps that needed to be addressed. Most of these were woven into the design of the game. For instance, to address the misconception that a pair of parallel lines must be equal in length, our parallel line tool was specifically designed to have two lines of unequal length. Also, we deliberately included many shapes with parallel sides of unequal length in the pool of shapes from which suspects would be drawn.

   In our trials, we found many students struggling repeatedly whenever these shapes appeared. Ultimately they either asked the facilitator to explain, or (in much fewer cases) figured it out themselves. However, in some cases, the concept still seemed to be shaky: it was observed that on coming back to the game after a gap, they again made the same mistake and needed scaffolding. However, students picked up the concept noticeably faster the second time around. In terms of developing the requisite vocabulary, the progression from Mission 1 to Mission 2 seemed to work well. In Mission 1, students gained familiarity with terms and geometric properties through clues generated by the system. In Mission 2, they were required to use their understanding of these properties proactively in order to formulate questions that would rapidly eliminate innocent shapes. During actual implementation in schools, both acquisition of specific concepts, and the progression of geometric reasoning can be tracked through data streams generated by the game's logging facility.

2. *Providing opportunities to engage in reasoning and problem solving:* Making students aware of specific terms and properties was only one objective of the game, certainly not the main one.

   Did the game succeed in making students reason? In the first mission, most students had a tendency to select the culprit randomly, rather than release the innocents gradually, on the basis of clues. Some of them would even say things like, "I feel this one is the culprit". But as the game progressed, they would realize feeling-based selection was not getting them anywhere, and subsequently, they started reasoning systematically to eliminate innocents as per the clue given in a step. It was a small step towards deductive reasoning.

   Did the game also help the students develop better problem solving strategies? Often, students began by cursorily connecting the clues to the shapes, but started adopting better and more consistent strategies later. We also observed cases where innovative strategies were developed to tackle the cases. For instance, in Mission 2 we observed students going through the options available in the drop down (property) lists in order to zero in on a suitable question. Our game's reward system (discussed in more detail below) is designed to make students mull over the possible causes of their errors, and to try and avoid similar errors the next time around.

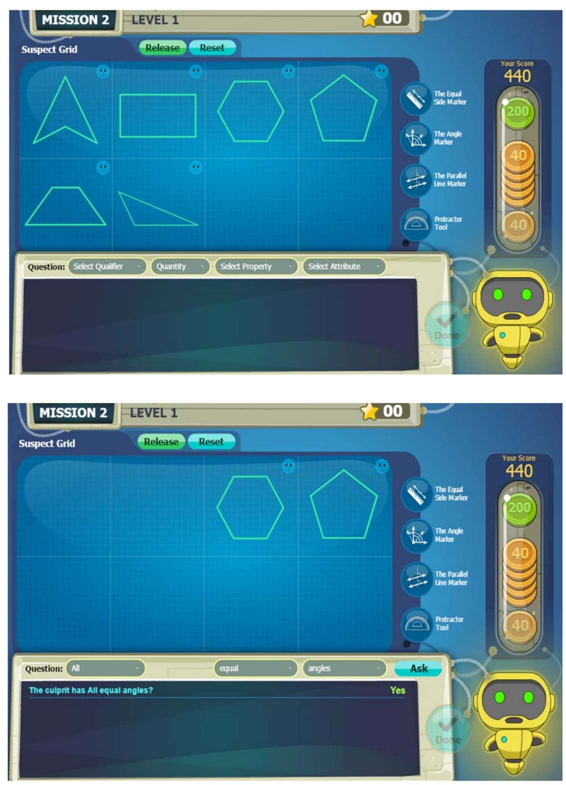

Figure 3. A sample case: Generating peer discussion

3. *Providing opportunities to engage in peer discussion and dialogue*: Consider this sample case from Mission 2 (Figure 3), where the group playing the game has to first formulate a question that can be answered as "Yes" or "No", and then eliminate the innocent shapes on the basis of the system's response. One group asks, "Does the culprit have all equal angles", and the system returns an answer "Yes". While eliminating the innocents in the following step, the opinion of two group members is divided on whether to eliminate the rectangle or not. The first student feels that it should be eliminated (assuming that equal angles implies equal sides), leaving behind just the hexagon and the pentagon. His partner disagrees. The first student has the mouse, and decides to make the move, and the system removes one token and gives them the feedback that their choice was incorrect. This generates an animated debate between the two on why their move was incorrect. They are unable to resolve it themselves, and the facilitator is called in, and an important question gets discussed: does 'equal angles' necessarily imply 'equal sides'?

   While *Police Quad* is essentially a one-player game, infrastructural constraints at participating schools compel it to be played with at least two students per terminal. However, examples such as the one above from the trials (which were carried out with a student-computer ratio that was similar to the anticipated field situation) showed us that the technology and infrastructure challenges were actually *working to the advantage* of the learner in our game-based learning scenario. With every new clue (in Mission 1 or 2), the group working at one terminal would go into an animated discussion and debate over which shapes to keep and which to eliminate. Thus, the game has enabled a model where 2 to 3 students collaborate in the real world by physically being present at the same screen, and brainstorming over the problems they face. One observation was that the students seemed to engage most meaningfully in discussions when they were working on the game in pairs. In larger groups, at least one student seemed to be somewhat passive, engaging less than the others.

   One point to be mentioned here is that the teacher's role is critical for facilitating student dialogue in certain cases. It was observed that there were several kinds of group dynamics during game play – some groups argued well, each member defending their solution strongly, whereas in other groups, one member was content to be more passive, offering suggestions only occasionally, and/or hesitating to defend their solutions too strongly. In such cases, the teacher's role would be to encourage a more balanced dialogue amongst the peers. In some cases, the teacher might even need to set up some guidelines about sharing control of the mouse to ensure equal participation during gameplay. But all in all, this model of two players collaborating to play the game seemed to greatly enhance mathematical communication – one of the process skills stressed by the NCERT position paper [7].

4. *Creating a safe space for students to make mistakes and learn*: The biggest change expected from the game is to create '*a safe space for students to make mistakes and learn*'. We therefore paid special attention to this during the initial trials, and recount specific observations from the trials that highlight this aspect. From the word go, the students explored and discovered the gameplay on their own, having complete freedom to fail, and try again, with minimal inputs from the teacher. At the start of the game, most students had no strategy to speak of, and made numerous errors. However, the system *never* specifies diagnoses errors precisely (even at the start) – it merely indicates their presence (by deducting tokens). Although it is technically simple to give students explicit corrective feedback, this goes against our design principles. Instead, the subtler feedback indicated by lost tokens appeared to work well – we observed students thinking about their errors and correcting themselves (either on their own, or after discussions with peers), without fear of criticism. This 'safe' learning environment also enabled the students to churn their trials and errors and formulate them into reusable strategies. For instance, while querying the robot in Mission 2, students would initially stick to exactly questions, but later realize the inefficiency of this strategy, and start asking more questions leading with 'More than', 'At least' etc.

   In the game, the teacher plays the role of the facilitator, watching students play, and stepping in only on occasions when necessary. It was observed that when students were not sure about a particular concept, they would first keep trying it on their own, and then discuss amongst the group. Only when they were completely stuck would they call the teacher in. For instance, on one occasion a group just could not discern whether to look at the exterior or interior angle in the case of shapes with a reflex angle – they struggled with this repeatedly before calling the teacher for help. The students showed signs of becoming more independent learners, more confident and in control of their own learning, and not relying on the teacher for everything – this was a marked shift from the existing teacher-controlled system.

VI. CONCLUSIONS

Overall, we saw the game-based learning approach establish a learning environment that actively encourages reasoning and strategizing. This environment made students comfortable about exploring geometric ideas and trying different solution strategies, unafraid of being chided for their errors, and motivated by being rewarded for effort. The students functioned as independent learners

– learning concepts and solving problems through discussions with peers, and asking for support only when needed. It was a very different situation from the current teacher-dominated classroom set-up with its singular focus on one correct approach or answer.

Research suggests three factors as key contributors to the success of technology-based intervention for learning mathematics [17]. First, the design should be based on "pedagogical and didactical considerations rather than being guided by the technology's limitations or properties." Second, the "teacher has to orchestrate learning, for example by synthesizing the results of technology-rich activities." The third factor is the educational context, which "includes attention for important aspects such as student motivation and engagement" [17]. We have argued how each of these factors has been considered and integrated into the design of the *Police Quad* game that is at the core of our Geometric Reasoning module. The initial trials of the game (albeit carried out in controlled settings) are promising. They seem to confirm that a blended game-based learning approach, where students play a learning game which is then followed up by focused teacher-facilitated discussions, can help students learn geometric concepts, and move to progressively higher levels of reasoning. The game creates the desired learning environment, and the follow up discussions help cement the key ideas more formally. Going into the next phase of the project, we plan to continue this study to get more conclusive evidence about the working of this model of game-based learning, when the module is implemented at scale.

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REFERENCES

[1] Shaughnessy, J. M., & Burger, W. F. (1985). Spadework Prior to Deduction in Geometry. *Mathematics Teacher 78 (September 1985)*, 419-428.

[2] Battista, M. T. (2007). The development of geometric and spatial thinking. *Second handbook of research on mathematics teaching and learning*, *2*, 843-908.

[3] Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. Routledge.

[4] Battista, M. T., & Clements, D. H. (1995). Geometry and proof. *Mathematics Teacher*, 88(1), 48-54.

[5] CLIx (2016). Analysing Geometry Learning Part 1: A Case Study of Two Schools in Rajasthan. Unpublished manuscript.

[6] CLIx (2016). Analysing Geometry Learning Part 2: A Case Study of Two Schools in Mizoram. Unpublished manuscript.

[7] NCERT (2006). Position Paper by the National Focus Group on Teaching of Mathematics.

[8] Laborde, C. (2000). Dynamic geometry environments as a source of rich learning contexts for the complex activity of proving. *Educational Studies in Mathematics*, 44(1-2), 151-161.

[9] DragonBox Elements. Available at: https://play.google.com/store/apps/details?id=com.wewanttoknow.Euclid&hl=en (last accessed on September 25, 2016).

[10] Rehm, M., Stan, C., Wøldike, N. P., & Vasilarou, D. (2015, September). Towards Smart City Learning: Contextualizing Geometry Learning with a Van Hiele Inspired Location-Aware Game. In International Conference on Entertainment Computing (pp. 399-406). Springer International Publishing.

[11] Gellert, U. (2000). Mathematics instruction in safe space: Prospective elementary teachers' views of mathematics education. *Journal of mathematics teacher education*, 3(3), 251-270.

[12] Ito, M. (2008). Education vs. entertainment: A cultural history of children's software. *The ecology of games: Connecting youth, games, and learning*, 89-116.

[13] Klopfer, E., and Osterweil, S. (2013). "The Boom and Bust and Boom of Educational Games." Transactions on Edutainment IX, 290–296.

[14] Geometry Dash Lite. Available at: https://play.google.com/store/apps/details?id=com.robtopx.geometryjumplite&hl=en (last accessed on September 25, 2016).

[15] Geo Dash - Geometry Games. Available at: https://play.google.com/store/apps/details?id=air.com.callystro.games.GeoDashGeometryGames&hl=en (last accessed on September 25, 2016).

[16] Arena, D. A., & Schwartz, D. L. (2014). Experience and explanation: Using videogames to prepare students for formal instruction in statistics. *Journal of Science Education and Technology*, 23(4), 538-548.

[17] Drijvers, P. (2015). Digital technology in mathematics education: why it works (or doesn't). In *Selected Regular Lectures from the 12th International Congress on Mathematical Education* (pp. 135-151). Springer International Publishing.