



Bundesministerium für
wirtschaftliche Zusammenarbeit
und Entwicklung



Teaching Numeracy in Pre-School and Early Grades in Low-Income Countries

Published by

giz Deutsche Gesellschaft
für Internationale
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Abbreviations



BMZ	German Federal Ministry for Economic Cooperation and Development	L1/L2	First language (mother tongue or language spoken at home)/second language (dominant or foreign language)
BNWS	Backward Number Word Sequences	LIC	Low-Income Country
BRAC	Bangladesh Rural Advancement Committee	LoLT	Language of Learning and Teaching
CK	Content Knowledge	NCERT	National Council of Educational Research and Training - India
DBE	Department of Basic Education – South Africa	NGO	Non-Governmental Organisation
DES	Department of Education and Skills - Ireland	OECD	Organisation for Economic Co-operation and Development
EFA	Education for All	PCK	Pedagogical Content Knowledge
EMU	Extending Mathematical Understanding	PISA	Programme for International Student Assessment
FNWS	Forward Number Word Sequences	PLC	Professional Learning Communities
GIZ	Deutsche Gesellschaft für Internationale Zusammenarbeit GmbH - Germany	PRASHIKA	Prathamik Shiksha Karyakram-India
GPE	Global Partnership for Education	SES	Socio-Economic Status
GPLMS	Gauteng Primary Literacy and Mathematics Strategy – South Africa	SSA	Sarva Shiksha Abhiyan – India
HLE	Home Learning Environment	TAL	Tussendoelen Annex Leerlijnen – India
ICMI	International Commission on Mathematical Instruction	TDMS	Talent Development Middle Schools – USA
IEA	International Association for the Evaluation of Educational Achievement	TEDS-M	Teacher Education and Development Study in Mathematics
INSET	In-Service Teacher Training	TIMSS	Trends in International Mathematics and Science Study
		UNESCO	United Nations Educational, Scientific and Cultural Organization

Executive Summary



This desk study addresses the development of numeracy in the early years of childhood and schooling in the context of low-income countries. We, the authors of this study (Bill Atweh, Arindam Bose, Mellony Graven, Jayasree Subramanian, and Hamsa Venkat), have conducted a wide literature review to determine evidence-based knowledge addressing two key questions:

- What do we know about teaching numeracy in the early years of schooling in low-income countries?
- What do we know about supporting teachers to teach numeracy in the early years in low-income countries?

The study considers different understandings of numeracy based on the traditional and more recent models of learning to present a three-dimensional model for the development of numeracy. Here, numeracy education is understood to consist of a wide variety of ‘contents’, including knowledge, skills and dispositions, needed to function in different ‘contexts’, including inside and outside the school, and for achieving different ‘aims’, including developing the learners’ identity and active participation in the world.

With respect to the first key question, evidence points strongly to the importance of the early years in the development of numeracy and the consequent need to establish quality pre-schools that develop solid foundations in and for numeracy that can narrow the gaps in development with disadvantaged children. This evidence is particularly important given the ongoing links between performance and socio-economic background in low-income countries. Broad access to high-quality pre-school experiences remains limited in these contexts, but where implemented, these studies have shown good results.

Successful programmes to develop numeracy in the early years frequently employ a holistic approach involving a range of government policies and agencies. They include the design of relevant and appropriate

curricula for the early years, and involve whole-school and whole-community approaches. In particular, quality teaching in the classroom makes a difference in the development of numeracy with the most disadvantaged in society. Evidence shows that participatory pedagogies based on problem solving, group work and discussions can be more effective than those based on recitation and rote learning. Effective numeracy learning commences with early numeracy activities linked to the child’s life experiences and contexts. The need for teaching that recognises numeracy aims within life-related situations is important and has been shown to be more effective than for example worksheets and colouring-in activities that lack these links. With this approach, the children’s daily experiences can be used as a step in the progression towards more abstract knowledge and skill in school numeracy.

There is a long history of discussion of the importance of concrete resources for early numeracy teaching. Current evidence provides some limitation about their usefulness. While these resources are considered very important at the introductory stages of learning, the need for a careful ‘fading’ of concrete resources linked to the increasing use of iconic and symbolic representations is advocated. Access to concrete resources remains problematic in low-income countries, although some studies have pointed to a variety of locally made resources that can be used effectively in the early years.

The development of early numeracy programmes cannot be successful if we use the ‘one size fits all’ approach (Meaney et al., 2013). Specific groups of students have their own needs and point to particular types of intervention to meet them. In particular, this study deals with the special issues that arise in the context of teaching numeracy to children from high-poverty backgrounds, children from indigenous backgrounds and children from non-dominant language groups. Likewise, the participation and achievement of girls in education remains a problem in many countries and calls for specific policies and intervention programmes.

Introduction



With respect to the second key question of the study, i.e. supporting teachers to teach numeracy, research and international reports point to the need to attract quality teachers into the profession and to improve their working conditions and salaries. Regular changes in policy, curriculum and assessment that do not involve the teachers and do not provide appropriate professional support for them, not only fail to achieve their aims but also lead to lowering of teachers' morale.

In terms of the level of teachers' knowledge that is necessary for developing numeracy in low-income countries, a wide range of evidence from higher and lower income countries makes clear that while teachers' lack of content knowledge is detrimental to developing numeracy, content knowledge per se is not sufficient. Researchers point out that teachers' knowledge about pedagogy is needed to achieve effective teaching. Pedagogical content knowledge should include the ability to connect teaching to the context of the student and build on their level of performance.

Developing high-quality and longitudinal teacher support is essential to enable early years teachers to become effective numeracy teachers. Evidence reviewed for this study stresses the importance of

linking teacher education to the realities of contexts of poverty rather than preparing them for some 'ideal' classroom context, where learners have the required competence for learning in the grade level they are placed, where class sizes are manageable and where conditions of poverty and high levels of absenteeism are exceptions rather than the rule.

In-service training based on 'cascade' models has been found to be an ineffective means of professional development. They are often associated with watering down of the intention of the professional support and misinterpretation of key concepts. Several studies point to the importance of in-service support and forming long term in-service communities of practice or professional learning communities for enabling continuous numeracy teacher development.

Finally, several studies lay strong emphasis on in-service programmes that support teachers to develop partnerships with families and communities. This is seen as particularly critical for young learners and for minority groups in which school-home-community links have been identified as an important supportive factor.

This study was commissioned by the Deutsche Gesellschaft für Internationale Zusammenarbeit (GIZ) GmbH Education Section on behalf of the German Federal Ministry for Economic Cooperation and Development (BMZ). The views expressed are those of the authors and do not necessarily reflect the position of the GIZ and BMZ.

The German Government, represented by the Federal Ministry for Economic Cooperation and Development (BMZ), supports the Global Partnership for Education (GPE) in its vision of a good quality education for all children everywhere so they fulfil their potential and contribute to their societies. In particular, it supports the implementation of GPE's Strategic Plan 2012-2015. As targeted support to GPE's goal of developing numeracy skills in the pre-school and early grades in low-income countries worldwide (Implementation Plan, strategic objective 3), the BMZ commissioned the Deutsche Gesellschaft für Internationale Zusammenarbeit (GIZ) GmbH with the implementation of a 'Sector Programme Numeracy' for the promotion of numeracy competencies in pre-school and early grades in low-income countries. In line with GPE's strategic plan and implementation plan, the Sector Programme Numeracy aims at supporting the building of knowledge on effective, replicable and implementation-oriented approaches for the sustainable strengthening of numeracy skills. Numeracy skills development in pre-school and early grades forms one of its four focal areas¹.

This desk study addresses the development of numeracy with young children in the context of low-income countries. The authors were commissioned by GIZ Sector Programme Numeracy to conduct a literature review addressing two key questions:

1. What do we know about teaching numeracy in the early years of schooling in low-income countries?

2. What do we know about supporting teachers to teach numeracy in the early years in low-income countries?

The research team consisted of five international academics in mathematics education who share a commitment to improving mathematics education and numeracy² and a commitment to raising the access to mathematics education for the most disadvantaged in society: Bill Atweh, Visiting Professor De La Salle University and Philippines Normal University – Philippines; Arindam Bose, Doctoral Candidate, Tata Institute of Fundamental Research – India; Mellony Graven, Professor, Rhodes University – South Africa; Jayasree Subramanian, Associate Professor, Tata Institute of Social Sciences – India; and Hamsa Venkat, Professor, University of the Witwatersrand – South Africa.

We conducted a wide literature review on the topic addressing one or both of the research questions above, based on a database search guided by our knowledge of the field and supplemented by further readings and comments from our international contacts with expertise in numeracy and primary mathematics education. We also consulted relevant sections from published handbooks of mathematics education. Four educators from Lebanon, Canada and Australia with experience in relevant research areas and staff from GIZ provided critical comments on the first draft of the study. We are grateful for their valuable contribution; however, we assume responsibility for the final content of the study.

¹ Further information on the GIZ Sector Programme Numeracy can be found at www.giz.de/numeracy

² The relationship between mathematics, as a school subject, as is articulated in curriculum documents and textbooks, and numeracy is a complex one. Very generally, in this context we understand numeracy to be the ability to apply mathematics in a variety of contexts inside and outside of school to function and meet the demands on the context. However, such application is not unary and simple as we argue in chapter 1.

SCOPE OF THE LITERATURE REVIEWED

A few words of introduction are necessary to understand the scope of the study and the nature of the recommendations that arise from it. Much of the research we have drawn upon is located in contexts of disadvantage and poverty in middle³ to high-income countries, rather than solely from low-income countries' contexts. This was valuable for two reasons.

Firstly, there is a limited amount of evidence on effective numeracy education directly based on research in low-income countries. Arguably, this lack of research follows from the lack of established structures and resources for academic research in these countries that plays into significant global imbalances in published research (Skovsmose, 2011). This is particularly problematic given that educational research, and particularly mathematics education, has consistently identified poverty as the key hurdle preventing children from participation and achievement in schools. However, this research is all but silent when it comes to problems faced by students and teachers in low-income countries. Given this skew, we have drawn upon numeracy research that has been conducted in contexts of poverty with low socio-economic students in medium and high-income countries, alongside the limited published numeracy research in low-income countries.

Our sense is that this combination of sources can result in useful research-informed policy-level recommendations for numeracy teaching and teacher education in low-income countries. Within our review, we pay specific attention to the contexts where findings arose, and thus point to the need for further research within low-income countries that can start to design and evaluate programmes for supporting numeracy teaching and teacher education. Here, we make a distinction between 'generalising' certain knowledge across contexts and 'transferring' well-informed learning from one context to another in ways that remain tentative until tested in the new context. Our findings should therefore be interpreted as informed recommendations that remain open, and require testing in a new context.

³ In this study we have utilized many sources of research from India and South Africa, both classified as middle income countries. Arguably, their status as middle income countries allows them to conduct research more than their lower-income counterparts. Both countries have significant low-income communities and a significant amount of research on disadvantage and education has been conducted in them.

Secondly, while the conditions of disadvantage may well differ significantly between higher- and lower-income countries, there are some elements of early numeracy practices and competencies that are widely agreed upon in the mathematics education field. Thus, our contention is that there are intervention studies relating to numeracy teaching and teacher education in higher-income contexts that have potential for broadening access to numeracy in low-income countries. In particular, we believe that a particular focus on disadvantaged students within higher-income countries can contribute to the agenda of social justice in terms of access to numeracy in any country.

Two limitations of the scope of the study need to be noted. Firstly, while we understand numeracy as a set of practices that can be developed inside and outside the school, this study limits its focus to school-based numeracy (as illustrated in the key questions above). We retain within this understanding an elaborated notion of numeracy (detailed in chapter 1) that can include the use of numeracy for participation in different out-of-school contexts.

The final limitation relates to the ability of low-income countries to implement recommendations made here with regards to numeracy and its development. While we believe that advancing numeracy (and of course, literacy) in any country should be a national priority (alongside others such as nutrition, health and housing), we acknowledge that with limited public finances, this is a serious challenge for many low-income countries, a challenge that is also noted in other reports (UNESCO, 2013). It is beyond the scope of this study to deal with government policy priorities and issues of funding and resourcing. What this study does is identify the directions that policy and practice can take based on the best available evidence to develop numeracy with early learners. The challenge for low-income countries, non-governmental organisations (NGO) and the international partners working with them is to develop means that allow for the development of policies and programmes that are based on the empirical evidence reported here. In stating this, we emphasise our belief that addressing the need for numeracy in low-income countries is the collective responsibility of global partners, rather than the localized responsibility only of low-income countries.

STRUCTURE OF THE STUDY

The study is divided into three chapters. The first chapter, "Numeracy and its Development: General Considerations" contains theoretical material that guided our thinking and in postulating the recommendations in the study. It discusses a shift in current learning theories from those that are based on acquisition models to those that are based on participatory models. Based on aspects of both models, the chapter presents a multidimensional model to understand numeracy that should guide its development in schools as well as policy and government programmes in low-income countries.

The second chapter, "Teaching to Learn Numeracy" deals with evidence from the international literature on effective practices to develop numeracy in the early years in the context of low-income countries – with a particular focus on disadvantaged contexts. It

discusses the importance of building numeracy on the child's experiences and context. It also discusses learning from research about the needs of specific groups of students, prevalent among early learners in low-income countries, towards making numeracy accessible to all.

The third chapter, "Learning to Teach Numeracy" deals with the needs to support teachers, both in pre- and in-service education, to increase their effectiveness in assisting children in the early years to develop a broad base of early numeracy understandings and skills. It discusses the important issue of the status and condition of teachers that promote productive numeracy teaching and the importance of including parents and the community. Issues related to pre- and in-service development of teachers of numeracy are also discussed.



This desk study considers findings from the literature about numeracy teaching and teacher education in the context of low-income countries. A wide variety of terms is used in the literature and policy documents to refer to numeracy. As Skalicky (2007) noted, related terms are often used as synonyms. These include “quantitative literacy (Steen, 2001), mathematical literacy (Organisation for Economic Co-operation and Development [OECD], 2006), critical numeracy (Johnston, 1994), mathemacy (Skovsmose, 2004), and numeracy (Australian Association of Mathematics Teachers, 1998)” (p. 662), and are based on different theoretical, psychological, social, and/or cultural viewpoints. Joblanka (2003) asserted that it is not possible to have a specific understanding of numeracy without promoting certain social practices relating to policy, teaching and assessment. Thus different understandings of numeracy may be based on different political commitments, and often lead to alternative policies and practices at national and school levels. Singular definitions of numeracy tend not to describe the different contexts in which numeracy is essential. Rather, different types of numeracies are needed to function in different contexts. Instead of referring to different numeracies⁴, in this study we present a multidimensional understanding of the term.

Our discussion of the learning of numeracy and learning to teach numeracy takes into account recent developments in learning theories. Hence, in this chapter, we deal with general considerations that guided this study in these two constructs, namely of “numeracy” and “learning”. The chapter identifies a major shift in learning theories from those that regards learning as an acquisition of knowledge to those that regard learning as a participation in different contexts that give rise to different types of learning. This brief discussion of shift in learning theories, (or as Lerman (2000) calls it, the “social turn”), is used both to construct the model of numeracy as understood here, and is used in the following two chapters as basis for the recommendations on “teaching to learn” and “learning to teach” numeracy.

⁴ ,Numeracies’ is a term parallel to ,literacies’ common in the literature on literacy.

1.1 Learning Theories: From Aquisition to Participation

Sfard (1998) discussed two main metaphors of learning that many learning theories are based upon. The first group of theories falls under, what she calls the acquisition model which “may point to a gradual reception or to an acquisition by development or by construction. [In spite of differences between the different theories in this model], all of them seem to imply gaining ownership over some kind of a self-sustained entity” (p. 5).

Atweh (2007) noted that within this metaphor, the entities that are acquired may vary: knowledge, concepts, notions, misconceptions, meaning, sense, schema, attitude, motivation, and in relation to this study, numeracy. The role of the teacher may vary according to the different theories adopted: delivering, facilitating, explaining, and mediating. However, the overall similarities between these theories lie in a) the existence of an identifiable body of knowledge/skill to be acquired; b) this acquisition ultimately occurs in the individual learner; and c) such acquisition can be measured and assisted by external intervention. Other theories of learning, according to Sfard (1998), follow the participation model. The author noted that one manifestation of a shift in model of learning from acquisition to participation is the change in the language used to talk about learning; for example, from “concepts” and “knowledge” to “knowing” and other doing verbs - along with the wider use of terms such as “practice”, “discourse” and “communication”. This shift implies a shift from thinking of the learner as an accumulator of knowledge to a participant in social activities that involve (and develop) knowledge. In particular, learning is what happens between individuals and groups, and not what happens in/ to the individual. More crucially, a different kind

of knowledge/learning is needed to participate in different contexts. Moreover, in contrast to the context as seen in the acquisition model as secondary, or as a hurdle to be overcome, in the participation model “the ongoing learning activities are never considered separately from the context within which they take place. The context, in its turn, is rich and multifarious, and its importance is pronounced by talk about situatedness, contextuality, cultural embeddedness, and social mediation” (Sfard, 1998, p. 6).

There remains the question as to the relationship between the acquisition models and the participation models with respect to each other. Salomon and Perkins (1998) argued that the two models are complementary. In other words, the authors discuss the two models “not as separate logical categories but as two perspectives on the [one] phenomenon of learning” (p. 2). Similarly, Sfard argued that neither metaphor is sufficient to explain all aspects of learning (in this case, numeracy), and that both perspectives are needed for informing research and practice. In the following sections, we examine what numeracy may look like under the acquisition model, and in particular in terms of its implication to testing. Then we discuss some limitations of such understandings leading to the multidimensional model of numeracy that reflects aspects of both acquisition and participation understandings of learning.

1.2 Numeracy as Aquisition

Arguably, in much academic and public policy, numeracy is taken as something that citizens need to acquire in order to function in school and in their daily life outside the school. Moreover, the level of such acquisition can be facilitated by effective teaching and can be measured by standardised instruments. The practice of testing numeracy is so widespread that in

⁵ We include a reference to Bishop here, not because it fits in the acquisition model of numeracy, but because he identifies a comprehensive list of elements of what we believe are crucial component of numeracy.

the minds of many people, and often teachers, it has become synonymous with numeracy. In other words, what is tested has become an indication of what is worthwhile to teach as well as a measure that it has been learnt.

Numeracy as acquisition has wider or narrower meanings depending on the particular author dealing with it. For example, the early work of Dehaene (1997) understood numeracy as informal number sense that even new born babies have and that infants acquire through interactions with people in their immediate surroundings. However, care must be taken not to limit the understanding of numeracy to simple acquisition of knowledge and skills as they relate to numbers (as the word may imply) and the basic operations on numbers. Montague-Smith (2002) stated that numeracy should include facility with number, counting, patterns (algebra), shape and space and measurement.

In his discussion of the nature of mathematics as a cultural phenomenon, Bishop⁵ (1991) identified 6 categories of mathematics as developed in any culture: counting, locating, measuring, designing, playing, and explaining. Although the author was not discussing numeracy directly, it is clear that if numeracy is a foundation of mathematics and how mathematics is used in any culture, then these categories necessarily involve numeracy – taken to mean the use of mathematics in different contexts.

Similarly, we warn against the stance that takes numeracy to refer to “basic mathematics” (basic operations and knowledge of mathematical terms) often found in the curriculum of primary level of schooling. Some policy statements have taken numeracy to also involve problem solving. French (2013) quoted the Irish Department of Education and Skills (DES) policy stating that

Numeracy encompasses the ability to use mathematical understanding and skills to solve

problems and meet the demands of day-to-day living in complex social settings. To have this ability, a young person needs to be able to think and communicate quantitatively, to make sense of data, to have a spatial awareness, to understand patterns and sequences, and to recognise situations where mathematical reasoning can be applied to solve problems. (DES, 2011, p.8)

In their report, Literacy, Foundation Learning and Assessment in Developing Countries, Nag, Chiat, Torgerson, and Snowling (2014) asserted that “the aims of teaching mathematics in these early years should not be reduced to teaching only arithmetic but should be inclusive of activities that foster a clear and logical approach to problem solving” (p. 19).

Relevant here is the understanding of mathematical literacy in the Programme for International Student Assessment (PISA) tests, widely used around the world, which focus on problem solving in real-world problems beyond typical word problems found in many mathematics textbooks. It includes students’ capacity to “analyse, reason and communicate ideas effectively as they pose, formulate, solve and interpret mathematical problems in a variety of situations” (p. 72). Specifically, PISA (OECD, 2006) defines mathematical literacy as:

an individual’s capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual’s life as a constructive, concerned and reflective citizen. (p. 72).

While this definition assumes knowledge of mathematical terminology, facts, procedures and skills, it involves “the creative combining of these elements in response to the demands imposed by the external situation” (p. 72). The OECD (2006) also acknowledged that while “mathematics-related attitudes and emotions such as self-confidence, curiosity, feelings of interest and relevance, and the desire to do or understand things, are not components of the definition of mathematical literacy but nevertheless are important contributors to it” (p. 73). Therefore, while the PISA definition does not include a disposition component, such a disposition is viewed as a prerequisite to being mathematically literate.

⁶ While certain elements of understanding of numeracy as participation may be more clearly applicable to higher school grades and for adults, here we argue that an effective policy to develop numeracy at early years should be undertaken with a view of a more general national policy on numeracy.

1.3 Numeracy as Participation⁶

Two common assumptions behind acquisition models of school learning of numeracy are that this learning is automatically transferable to real-world performance and that standardised testing is a reliable predictor of readiness to use mathematics in life outside the school.

Studies based on the participation model point to a discontinuity between school mathematics and mathematics as used in everyday life, even in situations that involve “real world” or “word” problems (Dowling, 2001; Lave & Wenger, 1991). In their chapter on Learners in Transition Between Contexts, Meaney and Lange (2008) discussed that for many students the transition between contexts, for example, out of school to school, is problematic – particularly in contexts where “there are differences in what content knowledge is valued, the relationships between participants and how activities are undertaken” (p. 1). For example, the authors illustrated problems that indigenous students face in coping with school learning in terms of transition between a familiar to an unfamiliar context. The authors rejected the acquisition model as sufficient to deal with learning in the context of such transitions. They used Radford’s (2008) assertion that learning is “not just about knowing something but also about becoming someone [else]” (p. 215).

A number of studies have demonstrated that students’ performance on numeracy tasks depends on the context and form of its measurement. Bose and Subramaniam (2011) investigated numeracy abilities of middle grade students from deprived socio-economic conditions living in an economically active low-income settlement in an urban metropolis in India and showed that these students have functional ability to deal with measurement and money in real world contexts. Similarly, West, Denton and Reaney (2000) demonstrated that high poverty kindergarten students have basic knowledge of counting and geometric shapes. In a study of indigenous students in Australia, Grootenboer and Sullivan, (2013) showed that in one-to-one interview situations, students have shown a workable knowledge of measurement by young students – albeit individualistic and perhaps not corresponding to school ways of doing measurement. All authors pointed out that students from these varied backgrounds are often seen in school as low achievers and have low numeracy skills.

Jurdak and Shahin (1999) using Vergnaud’s (1988) notion of a “conceptual field” – which posits that students’ performance on a task is inseparable from the situation in which the task is encountered and the way the task is presented – and not only on the mathematical properties – studied the daily use of mathematical thinking of a group of sellers in Beirut. The authors concluded that “the difficulties [the subjects in their study] showed when using school-taught algorithms were related to the abstract representations of the computational strategies advocated by schools rather than to the underlying mathematical concepts” (pp. 169-170). The argument here is not that a particular means of measuring numeracy is flawed or limited, but that someone can be numerate (i.e. apply mathematics to meet the demands of a particular situation) in one context but not in another.

However, a warning is needed about numeracy development focusing only on useful mathematics outside school at the expense of numeracy needed for formal school subjects. Hoadley (2007) demonstrated differential access to different forms of education for different social classes with a focus on concrete methods with low socio-economic status (SES) students and more formal mathematics for high SES students. She wrote:

Everyday knowledge is often emphasized in the progressive agenda aimed at empowering learners and facilitating their access to school knowledge. However research points out that it is often the marginal groups (lower ability learners, working-class children) who are exposed to local, everyday knowledge, often at the expense of the more specialized knowledge of mathematics... Muller and Taylor (2000, p. 68) comment that “the lower ability student, paradoxically, is left free to be a local individual but a failed mathematics learner”. (p. 682)

Further, we pose the question of kind of participation in society is desirable – a question not often raised in studies that adopt the acquisition understanding of numeracy. Acquisition understanding of numeracy seems to be based on an assumption that all citizens have the same needs in order to have the same opportunity to function in society. These assumptions are problematic in culturally diverse societies where identifiable groups have their own needs and aspirations. There is a potential tension between developing the ‘same’ while respecting and promoting ‘difference’. Talking about Aboriginal students in Australia, Meaney and Evans (2013) warned “that it is a flawed notion to

perceive the gaining of the same achievement levels in national [numeracy] tests as non-Aboriginal Australians by Aboriginal children, in remote communities of Australia, to be the pinnacle of school success” (p. 51). However, this does not mean that low school outcomes should be explained away as due to cultural differences in aspirations and needs. The authors added

Mathematics educators need to take seriously both their role in supporting students to transition into the wider Australian society and also the need for these students to remain strong members of their home community. Without this joint aim, mathematics remains as Bishop (1990) wrote “one of the most powerful weapons in the imposition of western culture” (p. 51).

Similarly, Atweh and Brady (2007) discussed an approach to mathematics education that aims to develop responsible citizenship that are not only able to obtain employment and meet the demands of the economy, but are also able to use their mathematics knowledge to, using Freire’s terminology as adapted by Gutstein (2006), “read” (i.e. understand) the world and “write” (i.e. change) the world. In the literature, this understanding of the role of mathematics, and here we argue numeracy, is often called critical mathematics education (Frankenstein, 1983; Skovsmose, 1994), mathematics education for social justice (Gutstein, 2006), ethnomathematics (Barton, 1996; d’Ambrósio, 2006; Knijnik, 1999), and socially responsible mathematics education (Atweh & Brady, 2009). Gutierrez (2002) argued for an equitable mathematics education in which dominant [school] mathematics and critical mathematics are developed jointly, rather than pitted oppositionally.

1.4 A Multidimensional Model of Numeracy

While an understanding of numeracy as based on acquisition models is useful to identify certain skills and procedures for meeting the demands of different contexts, participation models allow us to recognise that different contexts in which numeracy is demanded generate different types of knowledge. Hence, from this perspective, context is integral to knowledge and not a hindrance to it. Further, numeracy as participation allows us to question the purposes of knowledge as an intrinsic component of the knowledge. This leads to the construction of a multidimensional model of numeracy.

CHAPTER TWO TEACHING TO LEARN NUMERACY



CONTENT OF NUMERACY

- **Knowledge/Skills/Procedures:** Understanding and use of the language, concepts, and skills, as they relate to counting, locating, measuring, designing, playing, and explaining in a variety of contexts. This refers to the use of the objects of mathematics, its operations and their properties to solve problems in different contexts.
- **Dispositions:** The confidence and disposition to choose and use mathematical understandings wherever required. Willingness to take risks and persevere in approaching new mathematics and new contexts.

CONTEXT OF NUMERACY

- **School:** Dealing with formal concepts and procedures of mathematics curriculum and other school subjects required to succeed in school and pursue higher education.

- **Social life:** Ability to select and apply the appropriate mathematical tools for sense-making in a given context and understanding how the context impacts on the mathematics. Contexts related to school, everyday and work life and work life, public and social issues, and an awareness of mathematics connected to history and culture.

AIM OF NUMERACY

- **Understand:** Ability to raise questions about the social world and use the numeracy ability to answer them to increase knowledge about the world.
- **Reform:** Using numeracy to develop the ability to envisage and evaluate alternative means of living in the world at personal and social levels.
- **Cultural Identity:** Identify how different contexts relate in their numeracy demands and processes and relate mathematics to its cultural and historical roots.

Concerns have been raised on the low performance in numeracy for large proportions of learners in primary school mathematics across many countries in Africa, Latin America and parts of East Asia (Pritchett & Beatty, 2012). For example, evidence from South Africa indicates limited moves beyond highly concrete unit counting approaches across the primary years (Schollar, 2008; Ensor et al., 2009). In international comparative assessment, the performance in early numeracy in low-income countries frequently indicates the majority of children falling below low performance thresholds (Mullis, Martin, Foy, & Arora, 2011). Further, some studies point to little change in performance trends in numeracy over time (Uwezo, 2014). This body of work points to general limited progress in early and primary years, rendering access to broader mathematical understandings largely beyond reach.

Lee (2002) noted that while there has been notable success in improvement by middle schools' performance of high-poverty students in the USA, the gaps in achievement still persist. Balfanz and Byrnes (2006) attempted to explain this phenomenon by raising a question on the implementation of some "easy and inexpensive legislative reforms" that focus on high stakes testing and setting standards, at the expense of the "more expensive and difficult reforms, however, which directly impact classroom practice—strong instructional programmes, better supported, trained, and more knowledgeable mathematics teachers, and improved learning climates—have not been implemented successfully on a broad scale in high-poverty, high-minority middle schools" (pp. 144-145).

This chapter identifies central issues in the international literature on effective teaching that develops numeracy. First, we consider evidence that even though numeracy commences its development in the home, for many students from disadvantaged backgrounds participation in a pre-school assists in their

numeracy learning and in bridging the gap between their performance and that of other students. Second, we consider a challenge identified in the literature in teaching numeracy for disadvantaged students of how to take students' background into account without resorting to pedagogies and curricula that lock the child in a cycle of low-level experiences, and hence, prevent the development of high-level performance.

Third, we note that consistent evidence from the literature demonstrates that effective teaching for numeracy is achieved in contexts that employ holistic approaches: it involves stakeholders from within and outside the school including developing appropriate curriculum in the early years, classroom teaching and the whole school and the community. Fourth, while evidence points to prevailing practices in low-income countries of numeracy teaching methods that depend on recitation and memorisations, we discuss alternative pedagogies that are based on the participation learning models discussed in Chapter 1.

Fifth, we consider evidence that points to the efficacy of commencing with teaching numeracy by focusing on the daily-life experiences of the student and their context. Such experiences can be used as a springboard to more generalised knowledge and skills that are important components of numeracy for school success. Sixth, the question on whether resources are helpful or essential to develop numeracy is discussed. Here, we argue that while resources have an important role to play, they are not the universal panacea to the challenge of making numeracy more accessible to all students.

Finally, in keeping with the challenge posed by dealing with diversity in many classrooms around the world, we discuss particular issues that arise in dealing with specific groups of students in relation to poverty, ethnicity, language needs and gender.

2.1 Importance of Early Years for Numeracy Development

The importance of the early years on children's cognitive, emotional, language and social development is well established. In a comprehensive independent report to the United Kingdom Government, The Foundation Years: Preventing Poor Children Becoming Poor Adults, Field (2010) reported on the “overwhelming evidence that children's life-chances are most heavily predicated on their development in the first five years of life” (p. 5). In particular, it is during these early years that the foundations of numeracy are laid through the experiences of children at home with parents and other adults (Barnett & Esposito Lamy, 2006; Dickinson & Tabors, 2002). The quality and quantity of children's interactions with adults through conversations, songs, games, and play are important to develop basic knowledge of numeracy terms and processes. Melhuish et al. (2008) noted that active parenting promotes young children's numeracy and facilitates later academic achievement. Some authors go as far as claiming that “Like it or not, the most important mental and behavioral patterns, once established, are difficult to change once children enter school” (Heckman & Wax, 2004, p. 14).

However, Melhuish et al. (2008) went on to argue that the responsibility for developing numeracy should not be laid on parents alone. Evidence suggests that low income and low education families may be at a disadvantage in exposing their children to experiences that develop numeracy as needed in school. Some researchers (Mercy & Steelman, 1982; Sammons et al.,

2004) found that parents' education level (in particular the mothers') is a strong predictor of later school achievement. Often, in low SES families, all adults and many children are involved in daily work in the house, farm or business to allow for regular and quality interactions with children. Hence, it is also during these years that social differentiation in access to numeracy materialises.

Increasingly, many countries are establishing pre-schools and day-care centres to cater for greater sectors of society. There is significant evidence that children who attended pre-school benefit through developing numeracy skills, thus bridging the gap between less and more affluent groups. Taiwo and Tyolo (2002) reported on two groups of students from Botswana, those who had attended pre-school and those who had not, taken from urban, semi-urban and rural contexts, and found that those who attended pre-school gained better scores in English, Mathematics and Science on the related items on the assessment they used. In addition, they showed greater readiness to commence school among students who attended pre-school. In a wide review of research on effect of pre-schools targeting low SES children, Burger (2010) found that “positive short-term effects and somewhat smaller long-term effects on cognitive development and that in relative terms children from socio-economically disadvantaged families made as much or slightly more progress than their more advantaged peers” (p. 140). Pre-school education, in other words, can bridge the gap between the disadvantaged and the more advantaged. However, the author warned that “early childhood education and care cannot compensate completely for developmental deficits due to unfavourable learning conditions in disadvantaged milieus” (p. 140).

RECOMMENDATION

Early years experiences are critical for developing numeracy with young children. In particular, quality pre-school experiences assist the development of numeracy in the early years, lay a good foundation for future development and help narrow the gap between students from low-income backgrounds and the rest of the population. Educational planning for developing numeracy in the context of low-income countries should involve the provision of quality pre-school education particularly for the most disadvantaged students.

2.2 From a Focus on Background to a Focus on Foreground

A significant amount of research has been conducted around the world on background factors that are related to lack of achievement in school mathematics and numeracy. Often such studies are based on country-wide testing or international comparisons such as Trends in International Mathematics and Science Study (TIMSS) and PISA and deal with specific factors in students and family backgrounds that may explain or predict lack of achievement. This literature is not the focus of review in this study primarily because its contribution to the development of numeracy, either at policy or practice levels, has not been demonstrated. However, we identify one possible contribution of this research and raise some concerns about its unintended (negative) implications.

This research has identified groups of students that are not well-served by schools and are left behind in achievement and numeracy development. One of the factors often associated with school disadvantage and lack of numeracy performance is socio-economic background. Other factors studied were gender in certain contexts, racial and ethnic backgrounds, parents' education levels - in particular their numeracy and literacy level- and the living conditions. However, as Valero (2012) noted, research from different contexts has highlighted alternative factors that may explain/predict school performance, such as race in USA, socio-economic factors in Europe, rurality in China and many African and Latin American contexts, health issues in South Africa, thus making international generalisations difficult. What is evident from international research is that many cultures have student populations that are not benefiting from schools to the same extent as the rest of the student population, and their own sets of factors associated with disadvantage.

However, several educators warn that such ‘deficit model’ (Bourdieu, 1997) constructions of school marginalisation are counterproductive. Graven (2014) pointed to side effects of such research relating to teachers adopting a position of seeing the problem as a lack of student “abilities”, the “inevitability” or normality of failure of students from such backgrounds and lack of holding high standards of such students. In discussion of indigenous students, Meany, Trinick and Fairhall (2013) argued that in adopting the deficit model:

the solutions revolve around ‘fixing’ the students so that they become more like their successful peers and, thus, shed their inappropriate cultural practices. Yet, quelling discussions on the relationship between ethnicity and schooling and focusing solely on deficit theories is often unproductive. (p. 236)

In other words, focusing on lack of achievement and on factors related to student background as its causes may lead to seeing certain types of students as inherently deficient.

Research provides consistent evidence that suggests that teachers often adjust their teaching to their perceptions of students' achievement levels. While this may appear to be appropriate, it can restrict the opportunity to learn for low-achieving students. This is of particular concern when it involves groups of students from certain social, cultural or language backgrounds. Sztajn (2003) noted the tendency of using rote teaching for low SES students and problem solving with high SES students. Hoadley (2007), Carnoy et al. (2011) and Hoadley and Ensor (2009) in South Africa report on similar findings. Luke (1999) warned that the “dumbing down” of the curriculum for low-achieving students excludes them from developing high order thinking and intellectual quality work. It also diminishes their opportunity to learn content needed at higher levels of schooling.

Related concerns link to the lack of exposure to opportunities to take risks and become independent learners. Graven et al. (2013) argued based on a large scale survey with Grade 3 and 4 learners in lower achieving schools in South Africa that several learners tend to equate mathematical success with teacher dependence, compliance and careful listening rather than relating it to independent thinking, problem solving or making sense of mathematics.

Similarly, such focus on students' background may be taken as a convenient excuse to divert the ‘blame’ from the failure of the system and place it totally on the individual, their families and context. In other words they may lead into what Ryan (1976) calls in the title of his book ‘Blaming the Victim’. Some research projects have investigated the opportunity to learn rather than background as determining factor in students' performance. For example, the Early Childhood Longitudinal Study-Kindergarten cohort (Wang, 2009) found that

African American kindergartners have differential opportunity to learn mathematics than

their Caucasian peers from low income families. African American students were found to have received, on average, more reported instructional time spent on mathematics, higher use of math manipulatives, worksheets, textbooks and chalkboard work than their Caucasian peers. Moreover, greater opportunity to learn mathematics predicted higher mathematics achievement for both African American and Caucasian students from low income families. (p. 295)

Using the construct of students' disposition, Vithal and Skovsmose (1997) warned against too much focus on students' background at the expense of focusing on the vision and possibilities of what the student has a chance to become – this they call the foreground. They summarised their argument as

Intentions of learning emerge out of dispositions. Dispositions are concerned with "background" as well as "foreground" and are revealed when the learner produces, creates or decides his or her intention. A situation which could raise intentions for learning does not automatically belong to the background of the student having to do with his or her situation and social or cultural heritage. It is just as much to do with the students' possibilities but the possibilities as the student perceives them. The decision of the learner to act or learn therefore has a role to play when conditions for learning are created. The student has to be involved in the learning—should want to learn—if the learning activity is to become learning as action. Furthermore, the learning has to be performed by the learner if it is to include reflections and a critical awareness. (p. 147)

RECOMMENDATION

While it is crucial for effective teaching to take into consideration the background of the child, deficit understanding and blaming the background for low participation and achievement are not helpful. The possibilities of what the child can become should be paramount in designing teaching experiences and in interacting with the child. In particular, high expectations should be the basis for working with children from low-income countries and they should experience teaching that allows them to develop, not only high-level numeracy but their agency and learning independence.

2.3 A Holistic Approach for Numeracy Education

Effective development of numeracy is not to be seen as an isolated endeavour that happens only in the classroom between the teacher and the students. It involves a comprehensive effort on the part of many stakeholders. It should also be a component of other relevant government policies and programmes.

Developing numeracy in the early grades has implications to the design of relevant and appropriate curriculum at a national level. With the increase in standardisation of the curricula and introduction of regimes of high-stake testing in many countries, teachers often face tension between meeting the demands of the curriculum and helping students develop the desired learning outcomes. In a report to the Center for Global Development – a non-profit organization dedicated to reducing global poverty and inequality, Pritchett, Beatty, and Beatty (2012) used data from three recent studies in South Asia and Africa that demonstrated that many students after years of formal schooling show little progress on acquisition of the basic skills. They point to a gap between curriculum expectations and students' deep learning. The authors developed a simulation which illustrates deeper learning results when students' achievement level is matched to teaching rather than to a curriculum that assumes that all students learn the same skills at the same time. The simulation showed that

If the curricular pace – the level and material teachers are expected to teach – moves faster than actual student learning, this alone can generate enormous differences in cumulative learning. Calibrating a baseline model to reproduce typical OECD grade 8 results in internationally comparable assessments (e.g. PISA or TIMSS), we show that all of the observed learning differences between poor performing and OECD countries could be accounted for only by an overly accelerated curriculum in poor countries – even if the countries have exactly the same potential learning.

That is, the observed learning profiles (rates of learning per year of schooling completed) can be flat just because the material being taught to too hard for students as the curriculum has moved ahead, leaving students behind. (pp. 47-48)

Evidence points to many successful programmes to develop numeracy if it involves the whole school.

As an example of a whole school system approach to numeracy development, the Sarva Shiksha Abhiyan (SSA) is a comprehensive programme of the Indian government to achieve its aim for a universalisation of elementary Education. The SSA activities for developing numeracy in the early years include:

- Preparations at national and state levels for improving quality of mathematics education in schools
- Envisioning exercises for better understanding of mathematics education
- Material development for different activities
- Training of trainers and teachers
- Promotion of innovative 3 'r's (reading, 'riting & 'rithmetic) guarantee programme
- Diagnosis and remedial measures for quality mathematics education
- Action research on basic numeracy related issues
- Internal and external learning achievement tests to track children's progress
- Quality monitoring for tracking children's performance on a regular basis

Evidence of a successful whole school approach is developed by Gervasoni et al. (2010) in a low achieving SES school in Australia. The whole school approach consisted of

- The appointment of a school mathematics coordinator to provide curriculum leadership
- Assessment by the classroom teachers of all students at the beginning of each year using the early numeracy interviews and the associated Growth Point framework⁷
- Identification of mathematically vulnerable students
- Professional learning team meetings during which issues associated with learning and teaching mathematics are discussed
- Implementation of the Extending Mathematical Understanding⁸ (EMU) programme for some Year 1 mathematically vulnerable students.

Some successful programmes to develop numeracy, which would be of special interest to low-income countries, involve NGOs. The UNESCO Global Monitoring Report (2013) noted how many NGOs have implemented literacy and numeracy projects aiming at

⁷ See section 3.4 of this study

⁸ Further information on the EMU can be found in http://www2.ceosale.catholic.edu.au/services-to-schools/Extending_Mathematical_Understanding.aspx

supporting teachers in targeting disadvantaged populations. The report calls for governments to “monitor these efforts so that they can learn from, adopt and expand initiatives that provide useful lessons and have the potential to be scaled up” (p. 6). NGOs, for their part, need to consider “whether their projects can be replicated and collaborate with governments to strengthen systems and sustain any gains” (p. 6).

Discussing the particular needs of physically disabled people, Groce and Bakhshi (2011) pointed to the role of NGOs in raising awareness about disadvantaged groups, both locally and internationally. In many countries they have formed networks with poorer communities that can be utilised in numeracy development. The authors called for increased funding and greater collaboration between governments and NGOs towards advancing programmes to alleviate low numeracy in society by ensuring a match between policy, other government programmes and delivery of services.

The following are two examples of wide-ranging NGO programmes.

Shahjamal and Nath (2008) noted that pre-primary education is limited in Bangladesh with only about 13.4% of the children aged 4-5 years receiving pre-primary education. Although the government has no set curriculum at that level of education, many NGOs have developed their own curricula for young children. The Bangladesh Rural Advancement Committee (BRAC) programme operates over 20,000 pre-school centres around the country. In order to increase participation of girls in schools, BRAC has a policy of maintaining a balance between the sexes to be 60:40 in the favour of girls. Similarly, the Penreach (2011) programme (operating in South Africa) is a project that works successfully with a whole-school community approach and shows positive mathematics gains. It utilises multiple interventions from different angles – pre-school centres, girls’ clubs, teacher development.

Developing numeracy does not only involve the schools and the education system but must include parents and the whole community. In her work with indigenous students in Australia, Goos (2004) argued for the importance of forging parental and community involvement

in the development of numeracy and mathematics education where recognition of social and cultural differences and relations of power are brought to the foreground. She called for the need to support administrators and teachers to work with parents and enhance communication between teachers and parents. Thus, effective partnerships are characterized by a “long history of building relationships often involving the whole school, whole family, or whole community” (p.20). She added that the holistic focus “does not necessarily have numeracy as its first [and only] priority” (p. 20).

Hoadley (2013) argued that:

Educational interventions are only part of the solution to low numeracy. Family background remains the most powerful influence on how children will fare in school. Many of South Africa’s children enter formal school with their developmental potential considerably compromised, and with limited attention to their physical and psychological well-being, which affect their ability to learn. It is therefore important to find ways to secure the nutrition, health, safe transport and after-school care of young children in the foundation phase, in addition to improving the quality of teaching and learning. (p. 76).

Lastly, assisting disadvantaged young students in developing their numeracy cannot be totally successful in isolation from the social conditions giving rise to their disadvantage. The UNESCO Global Monitoring Report (2013) asserted that “children cannot benefit fully from school if they live in poverty, are malnourished, suffer from ill health or live in conflict zones” (p. 2). Robinson (1996) went further to state that successful reform in mathematics education cannot succeed without a social transformation that liberates young people from the hunger, poverty and violence that trap so many people in the ghetto of despair. Robinson asserted that the most successful education reform efforts will be those that acknowledge the need to focus on the whole child, the whole family and the whole community – i.e. the whole socio-economic matrix. In other words, attempts to increase numeracy levels should go hand in hand with policies and programmes to alleviate poverty.

RECOMMENDATION

Curriculum design should avoid overloading the early years with content and outcomes, thus allowing children to develop at their own pace. Productive efforts to develop numeracy should include contributions from a whole community, with successful practices based on whole-school and integrated approaches commencing with national policy and down to school policy and practice. At policy and practice levels, these efforts take the whole-child approach catering for all the needs of children, with numeracy development forming one important aspect.

2.4 Pedagogy of Participation

Research from around the world demonstrates that quality teaching matters. Balfanz and Byrnes’ (2006) study compared the performance of students in three schools in the USA that adopted to “implement a comprehensive set of instructional, teacher support, and school climate reforms (embedded in the Talent Development Middle Schools [TDMS] reform model).” In comparison, they “had significantly greater numbers of students close their mathematics achievement gaps than did the other 23 middle schools in the district also serving high-poverty and high-minority student bodies” (p. 143). Further, they demonstrated that differences in catching-up were also a function of the homerooms between these schools indicating the critical role of the teacher within these comprehensive reform schools. Classroom attendance was also an important factor indicating students who consistently attended such classes tended to show greater gains than other students. Also effective was the high expectation of students’ behaviour in those classes.

Ngware, Oketch, Mutisya and Abuja (2009), using the construct of ‘Opportunity to Learn’ and teacher content knowledge, investigated differences between top and bottom performing primary schools in Kenya. The authors found that “teachers in the bottom-ranked schools made some difference in the performance of their pupils in mathematics” (p. 2) while little teacher effect was found in top-ranking schools. Teachers who used more interactive teaching methods assisted students towards higher achievement. The authors concluded that: “In low-performing schools, teachers may require more pedagogical skills-upgrading with a view to enabling them shift their lessons to more learner-centered approaches” (p. 3).

We turn to the question of how to develop “good” teaching. Atweh (2007) stated that several frameworks, based on different theoretical underpinnings, exist in the literature that may be useful to assist in the development of a variety of teaching techniques that cater for a wide diversity of students. Perhaps widely known such frameworks are those of Bloom’s Taxonomy (Bloom, 1956), Gardner’s Multiple Intelligences (Gardner, 2000), de Bono’s Thinking Hats (de Bono, 1985) and Myer-Briggs Personality Types (Briggs & Myer, 1995). These frameworks have few characteristics in common. None of them contain prescriptive teaching tools for the teacher. Rather, they provide tools for reflection by the teacher in their critique of their own pedagogy and in designing

alternative pedagogies. Similarly, none of these schemes are content-based. That is, they can be used in a variety of subject areas and at different levels of teaching. Without dismissing their value for teachers, Atweh (2007) noted that the educational research base on which they are based is perhaps limited in that they focus more on higher order thinking and intelligence constructed under the individualistic and acquisition models of learning as discussed above. While some of them might acknowledge individual differences in thinking style and preference to learning, they do not account for the effects of student background and their social context.

A promising and comprehensive framework developed recently in the state of Queensland in Australia, called Productive Pedagogy⁹ is an example of an attempt to integrate research findings on effective teaching from a variety of areas of research within education itself. The framework was based on the previous work of Newman and his colleagues (Newman & Associates, 1996) at the University of Wisconsin on Authentic Pedagogy and based on a longitudinal study conducted in that state (Queensland School Reform Longitudinal Study, 1999). Luke (1999) explained that

the Productive Pedagogy model does not provide ready-made techniques for teaching. Rather, it is an approach to creating a place, space and vocabulary for teachers to get talking about classroom instruction again. The approach does not offer a magic formula (e.g., just teach this way and it will solve all the kids problems), but rather it is a framework that provides a vocabulary for staffroom, in-service, pre-service training, and for teachers to describe the various things that can be done in classrooms – the various options in our teaching ‘repertoire’ that we have – and how we can adjust these ... to get different outcomes. (pp. 5-6)

The Productive Pedagogy framework identifies four main essential characteristics of ‘good’ teaching:

- Intellectual Quality
- Connectedness
- Supportive Classroom Environment, and
- Recognition of Difference

We note that the available evidence about teaching numeracy in low-income countries point to a limited variety of pedagogies that are utilised in the classroom. In their final report on Literacy, Foundation Learning and Assessment in Developing Countries and, in

⁹ Further information about Productive Pedagogy is available from the website of the Queensland Department of Education and the Arts at <http://education.qld.gov.au/corporate/newbasics/>.

particular, with respect to numeracy, Nag et al. (2014) concluded that

The review provides a fairly consistent picture of the teaching of numeracy in the countries that have been studied. The emphasis is on relaying number facts, with considerable reliance on recitation and rote learning. There is comparative neglect of the teaching of number concepts and arithmetic strategies and very little attention is paid to embedding problem solving in familiar contexts. A major issue is the language of the classroom. This can be abstract and removed from everyday experience. (p. 2)

Similar observation was made by Akyeampong et al. (2013) in discussing numeracy teaching in African countries investigated. In the education literature, these pedagogies are often referred to as 'traditional' and 'teacher-centred'. Further, these teaching practices seem to be based on the acquisition theory of learning as discussed in chapter 1.

Current literature, as well as the participatory learning theories discussed in chapter 1, point to the need to create learning environments that are more active and engaging for the students. For example, French (2013), argues that a Vygotskian¹⁰ approach "with its emphasis on the role of the adult and/or more knowledgeable peers in social interaction in learning and development, supports professional practice in numeracy in the early years" (p. 39). He advocated for small group interactions mediated by a trained key person. The role of adults is to sustain motivation, provide resources, aid children in seeing numeracy through modelling, using the language of numbers, measures, patterns and shape and critically capitalising on interactions to enhance children's experiences.

¹⁰ Lev Vygotsky, a Russian psychologist known for his contribution on the role of language and sociocultural factors in cognitive development, believed that community plays a strong role in the process of 'meaning making' and placed a strong emphasis on cooperative, collaborative social interaction in the development of cognition. According to him, working with a more knowledgeable other- teacher or parent or peer- who is in a position to give the most appropriate instruction or guidance required for the child achieves cognitive development.

RECOMMENDATION

Classroom numeracy teaching should be based on more participatory practices in which the students are active in discussions, explorations and problem solving in small groups. Classroom activities should provide students with opportunities to develop their ability to discuss and explain their thoughts, which develop their numeracy.

Similarly, Young and Richardson (2007) in their International Association for the Evaluation of Educational Achievement (IEA) pre-primary study which looked at 2000 children across 15 countries identified some characteristics of effective pre-school numeracy programmes that include:

- Having free choice in participatory learning activities
- Engaging in few whole-group activities. (p.76)

Here, we add that such activities should involve high order thinking and problem solving appropriate to the level of experiences of early learners according to the 'intellectual quality' stipulated by the Productive Pedagogy framework discussed above.

In this context, we note that such pedagogies call for an alternative role, and knowledge, of the teacher to traditional classroom. Chapter 3 of this study discusses teachers' knowledge and its development. Here, we point to one such summary of teachers' knowledge that specifically deals with pedagogies of participation. French (2013) focused on the need for teachers to develop knowledge of what specific children already understand through observation and knowledge of where we should aim to help children to get to as a result of classroom interactions. Such knowledge includes:

- Content (what we want children to understand – one-to-one correspondence, cardinal numbers, distinguishing between a square or a rectangle).
- Orientation (how we would like children to tackle the experiences – dispositions to engage numerically, for example, finding a way of remembering where they started counting, organising into sets, aiming at accuracy when measuring, learning to record, observing and knowing patterns).
- Knowledge of how we pace our support (for example, while playing a card game). (p. 42)

2.5 Building on Children's Life Experiences and Context

In the following section (2.6) we discuss research evidence on the use of resources in teaching numeracy skills. In this section, we refer to research findings on the importance of using, arguably the most effective and readily available 'resource', namely the child's own life experiences and context. This is in line with the 'connectedness' principle of Productive Pedagogies discussed above which stipulates that productive teaching must make connections to the world outside the school.

Building on findings from other researchers, French (2013) argued for teaching numeracy by bringing in contexts that make sense to children, building on their own first-hand experiences of numeracy. He stipulated that "Early childhood educators need to connect and build on the variety and range of children's everyday experiences of numeracy in the home and early childhood settings" (p. 42). Moyles (2001) noted that the sole use of worksheets and colouring-in activities fail to gauge the level of development of an individual child or even to motivate them to develop their disposition for the use of numeracy skills in their daily life, that the use of the child's real world context may provide. French (2013) provided some examples of using the context of the child to develop numeracy including:

children's exposure to the concrete experience of dividing and naming things in halves and quarters (sharing fruit or playdough "half for you...") supports the understanding of fractions. Children learn what is "big" and what is "little" when choosing what size spoon to eat with, or when hearing the story of Goldilocks and the three bears. Later they will use centimeters, kilograms and degrees to measure and compare. Children need many of these experiences. (pp. 42-43)

Rampal et al. (1998), in their book Numeracy Counts!, stressed the importance of using 'folk mathematics', riddles and stories, patterns that they see in their vicinity, tiling activities, and body measures for measurement. Mathematical potential in stories and games needs to be exploited for strengthening numeracy. Use of stories, songs, contexts that relate to the children, games and activities are stressed in most of the literature. It is known from the studies of several mathematics educators (see Bose & Kantha, 2014; Wright, 2013) who have worked with first-generation learners from poor homes who work outside home to supplement the family income that many of these children are very good at oral mathematical competence but they cannot read the symbols and operate on them. The challenge

for teachers is to use these out-of-school experiences to develop school numeracy.

Many curricular documents lay emphasis on acknowledging children's language, culture and resources. For example, India's major curricular document for school education mentions that connecting with the child's environment also has a role to play in creating an educational culture that is equitable. "Our children need to feel that each one of them, their homes, communities, languages and cultures, are valuable as resources for experience to be analysed and enquired into at school; that their diverse capabilities are accepted" (National Council of Educational Research and Training [NCERT], 2005, p. 14).

Perhaps, as an example of progression from a concrete contextual problem to abstract thinking about numeracy can be illustrated by the Tussendoelen Annex Leerlijnen (TAL) (van den Heuvel-Panhuizen, 2001) from the Netherlands. The TAL team observed that there are individual differences between children in attaining these levels and that the teacher should provide each child with the opportunity and time required to move from one level to the next.

Context-Bound Counting and Calculating (level 1) takes place in meaningful problem related situations in which "how many" and comparison questions can be put in a meaningful form, even though children at this level may not be able to do pure counting required in word problems such as "If there are seven sweets in a tin and I take one out how many remain?". For example, in finding out the number of candles on a birthday cake that corresponds to the age of the child or in games such as snakes and ladders or ludo, played with rolling dies and counting, children employ counting correctly as the purpose of counting makes sense to them. Operations such as ordering, comparing, estimating, add, takeaway, more, less and as many as can also be introduced in contexts that children find relevant for them.

Object-Bound Counting and Calculating (level 2) occurs in problem situations that are focused directly on the quantitative aspect pushing the context to the background. It makes sense to answer "how many candles are there?" without the supporting context of one's birthday. At the level of object-bound counting, the skilful organization of counting plays a central role. The children have to lay out the objects in a neat pattern to get a better grip of counting.

Pure Counting and Calculating via Symbolization (level 3) occurs at the next level where children can answer questions such as "what is seven take away three?", without reference to context or the support of objects. They use fingers to represent the quantity and perform calculations.

While the TAL project does not address low-income contexts directly, it provides an interesting interpretation of what we mean by ‘everyday life context’ of the child. Using their argument, here we understand the child’s experiences not only to refer to the material and concrete living conditions and experiences; what is ‘real’ for children’s need not necessarily be something tangible or concrete. Children have imagination and, in their minds, can make up all kinds of ‘beings’ and objects that have no real existence. It is important that early numeracy curriculum takes advantage of this.

The TAL team in Netherlands refers to this approach as ‘realistic’, where emphasis is on making something real in one’s mind (van den Heuvel-Panhuizen (2001). In other words, context could be imaginary or refer to popular culture and may constitute something that is common and shared among the learners. This approach is particularly useful when a classroom has children coming from socio-culturally diverse backgrounds that they cannot easily make sense of the practices that are not part of their own cultural background.

RECOMMENDATION

For developing numeracy in the early years, children should be assisted to identify numeracy thinking within their experienced or imagined world, including counting things in their environment, retelling of stories, songs, riddles and games and identifying patterns in the environment.

2.6 Access to and Use of Resources

While the resource situation in contexts of low-income countries may be improving, access to resources remains at low levels, even in terms of ‘low-end’ resources like textbooks and workbooks (see Valverde & Näslund-Hadley, 2010, for a Latin-American overview). Further, moves towards broadening access have actually led to increases in class sizes, and hence limitation in resources per child, in the early years in many low-income countries.

Resource provision is being targeted as a priority area in several contexts of poverty, following multi-context evidence that improved individual textbook/workbook availability in particular, was related to increases in learner performance (Baker, 2002). South Africa, for example, launched a national programme of learner workbooks linked to their national curriculum, with resources for early number learning such as structured bead strings and abaci having been distributed as part of the preceding ‘Foundations for Learning’ policy (South Africa Department of Education, 2008).

In this section we focus on one type of resources that in the literature are often referred to as manipulatives, concrete materials or teaching aids. Moyer (2001) described resources as: “objects designed to represent explicitly and concretely mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences” (p.176). The rationale for using resources varies in the literature. For example, in India, Rampal et al. (1998) supported the use of resources in ways that align with curricular goals, while Srivastav (undated) argued for their use for promotion of active learning. Drews (2007) identified a comprehensive list of justifications for their use in educational literature and research, including:

- motivate children;
- provide variety to teaching and learning experiences;
- connect ‘classroom mathematics’ with application to the real world;
- act as a visual aid to allow children to build up a store of mental images;
- enable teachers and children to model mathematical processes involved in specific number operations or calculations;
- encourage mathematical communication to take place;
- support teacher assessment of children’s knowledge and understanding of aspects of mathematics; and

- support the understanding of mathematical ideas through allowing children to make connections between, what for them may be, disconnected aspects of mathematical learning. (pp. 25-26)

Here, we identify three themes that are relevant to the context of this study, namely: the importance of use of resources for developing numeracy, the need for attention to supporting transitions from concrete manipulatives to abstract mathematical ideas and access to such resources for teachers in contexts of low-income countries.

With respect to the importance of the use of resources in teaching early learners, based on a review of the literature, Drews (2007) asserted that “while acknowledging that the teaching and learning of mathematics does benefit from effective use of visual and practical aids, recent research has questioned whether such use is always needed, or helpful, to children’s mathematical understanding” (p. 19). She goes on to identify that “crucial to the debate is the rationale which teachers use to support the planned use of mathematical resources within their lessons (Moyer, 2001), teacher beliefs about how best to teach mathematics to assist children’s learning (Askew et al., 1997), and assumptions which teachers may make regarding children’s interpretations of the use of mathematical resources (Cobb et al., 1992)” (p. 19).

A longitudinal study conducted by Guarino, Dieterle, Bargagliotti and Manson (2013) including a sample of 22,000 young students in the USA examined the effect of certain teachers’ characteristics and practices (including the use of resources) in kindergarten and grade 1. The authors found that the use of resources was effective in kindergarten but not in grade 1. Even though statistical significance was obtained for the lower grade, the effect of resources on learning was small. The authors posited the hypothesis that the level of development of children may contribute to such findings. More verbal and problem-solving pedagogies were more effective in grade 1. Hence, it seems that students may outgrow the need to depend on resources to develop their understanding and skills in numeracy.

With respect to the proper use of the resources, in the South African context, Venkat and Askew (2012) pointed to improving access to resources in some urban settings, but noted the need for in-service teacher development in the use of what they described as ‘structured’ resources (resources inlaid with aspects of mathematical structure – e.g. 1-20 bead strings structured in 5s or 1-100 abacus structured in 10s). Their evidence points to dangers of structured resources being used in unstructured ways (i.e. through unit counting strategies) by teachers who have not had access to use of these resources in their own learning and prior training and teaching experiences.

Wang (2009), in the US context, found that use of mathematical manipulatives was a significant negative predictor of mathematics achievement, with detrimental effects for African-American children, pointing again to the need for attention not just to resource provision, but to supporting constructive resource use.

Lastly, with respect to the availability of resources in the contexts of low-income countries, we note that numeracy programmes that seek sustainability in the context of poor and low-income background also need to generate teaching learning materials that are low cost or no cost (such as leaves, twigs, seeds and measurement units) rather than employing commercially available teaching learning materials or poor imitations thereof. This requires fresh imagination and the participation of the community so that one can source appropriate materials that will be freely available and turn them into tools for the classroom.

In this regards, Drews (2007) identified a variety of locally made resources that can be used effectively in the early years including:

[everyday resources] can be brought into the classroom and used successfully as resources to support and develop children's understand-

ing of some of the purposes of mathematics in real-life contexts. The examples of materials which help relate 'school' mathematics to everyday applications are endless, but could include packaging materials, patterned fabric or paper, timetables, receipts, catalogues, scaled plans, photographs of shape/number in the environment and any form of container or measuring device. Such types of resources have use in whole-class teaching, small group activities, displays and cross-curricular role-play situations. In addition to these real-world artefacts, many resources not specifically designed for mathematical learning can be exploited to assist with early learning in particular. Toys, stories, environmental or malleable materials such as sand, water and play-dough can be used to support early concepts in aspects of number, shape and measurement. The advantages here are that they are tactile and more likely to connect with children's home/prior/real-world experiences. For Edwards (1998: p. 8), the value here lies in the fact that "handling of familiar 'everyday' objects enables children to learn about their properties and components". (p. 25)

RECOMMENDATION

The availability of key resources for early numeracy is helpful especially in kindergarten years. However, it is crucial that teachers using such material incorporate a focus on the ideas and the progressions that these resources are intended to develop. A variety of everyday resources not only reduce the cost of making resources available in low-income countries' classrooms but also support establishing connections between school numeracy and the everyday life of the student.

2.7 Teaching Specific Groups of Students

In keeping with the challenge posed by Meaney et al. (2013) whether 'one size fits all', here we argue how engaging with the development of early numeracy programmes cannot be successful if it fails to take into consideration a range of mediating factors which add complexity to the situation and demand a re-vision of the programme in the context in which it is to be implemented. In particular, in the following sections we consider specific needs of three student background factors that the research has paid significant attention to in the development of numeracy and that are prevalent in low-income countries. These include:

- high poverty backgrounds
- indigenous backgrounds, and
- non-dominant language backgrounds.

In addition, we discuss special policies that are needed to increase the participation and achievement of girls. In doing this, we stress that multiple factors often overlap in the same student population. For example, many indigenous students use a different language at home than in school and come from very low-income families. Clearly, dealing with such complex contexts calls for context-specific approaches. In this section, we point out specific types of interventions that may be needed in addition to the general considerations argued for in the rest of the study.

Having identified the needs of these particular groups, however, opens the possibility for critique on the silence about the needs of other student groups - for example, rural students - also prevalent in low-income countries. We note here that numeracy in rural areas is an emergent issue in some high-income countries such as Australia (Goos, Dole, & Geirger, 2011; Tracey, 2012), Europe (Aubrey & Godfrey, 2003) and to a lesser degree in low- and medium-income countries, for example, Uganda (Wamala, 2013). However, such research reinforces the general principles argued for in this study. Hence, due to space limitations, it will not be dealt with here in detail. Other chapters of this study deal with other issues related to rural teachers and the support they need.

Another group of students who often have low participation in numeracy and school achievement are students with mental or physical disability. While beyond the scope of this study, the needs of this group are crucial to identify for success in developing numeracy for all. Groce, and Bakhshi (2011) considered the needs of these populations. Although the project dealt with adult literacy, their argument can also be relevant to

young children's numeracy. The authors argued that the needs of children with disabilities can be met if these interventions are designed in ways to cater for their inclusion. However, the authors go on to argue that this may not be sufficient

At the same time, persons with certain types of disabilities will need specialised programmes - for example, targeted initiatives for teaching Braille to blind adults or having instructors fluent in sign language to teach deaf adults. Literacy for individuals with intellectual impairments needs to be done at a pace which those receiving instruction can benefit from. However, it is important to emphasise that the needs of subgroups within the larger disabled population are predictable and can be anticipated, planned and budgeted for. Even if such targeted literacy programmes reach only certain subgroups of the overall number of illiterate disabled adults in a community, the effects will be cumulative. Over the course of time, literacy levels within the general disabled population will rise, and in so doing, literacy among disabled adults will become the norm and not a striking exception to the rule. (p. 1163)

We urge for more directed research on the needs of students from rural areas as well as mental and physical disabilities in the context of low-income countries.

2.7.1 Children from High Poverty Backgrounds

Even though children from middle and low-income countries constitute a significant percentage of children in the world, poverty still gets marked as a special condition in educational research. In other words, educational initiatives often start from the context of children coming from developed countries and from upper and middle income homes and then various measures are sought to extend the same to children from low-income background.

First, we note that poverty is an all-encompassing condition that affects the whole life of people and not only their educational attainment. According to the Children's Workforce Development Council report (2011)

Poverty is not simply about being on a low income and going without. It is also about having poor health, education and housing, impacting on basic self-esteem and the ability to participate in social activities. Poverty can have a profound impact on children, their families and the rest of society. It can set in

motion a deepening spiral of social exclusion and create problems in education, employment, mental and physical health and social interaction. Poverty makes people's lives shorter and more difficult than they need to be. (p. 3)

In particular in the context of this study, we refer to studies that point to a lag in the cognitive development and emotional well-being of children living in poverty. Melhuish, et al. (2008) argued, “poverty is linked to poorer child outcomes as well as poorer parenting” and “children in persistent poverty have greater cognitive and behavioral deficits at age five than those exposed to transient poverty, who in turn have more deficits than children in non-poor families [Korenman, Miller, & Sjaastad, 1995]. Some deficits can be attributed to health problems associated with poverty, but the greatest part can be explained by reduced emotional support and less cognitive stimulation from parents” (p.10). Fleisch (2008) pointed to similar combined effects of multiple problems affecting primary mathematics learning and schooling in South Africa.

While the construction of lags in development in deficit terms illustrates the wide understanding of differences in performances as “deficits”, we use the term cognitive skills to refer to those that are required or expected for success in school. Arguably, children reared in a context of poverty develop other cognitive skills that are needed for survival in the harsh reality of poverty more than their better-off counterparts. Often these skills may not be recognised in the education system.

In terms of numeracy, based on a study, Sood and Jitendra (2011) reported that number competencies

are highly sensitive to socioeconomic status, suggesting the importance of early input and instruction’ (Jordan et al., 2010, p. 82). Specifically, children from low-income households do not perform as well as their middle-income peers on verbal number combinations and story problems involving addition and subtraction (Jordan et al., 1994). Furthermore, their use of counting strategies is less adaptive in that they do not avail of their fingers to count on from addends (Jordan et al., 2008). (pp. 328-29).

A particular issue related to numeracy development in contexts of poverty is the amount of support from home. In a study that explores early numeracy skills and growth in this cognitive domain over two years of pre-school (age 3–5 years), Anders et al. (2012) concluded that “pre-school education may only be an effective means of promoting the development of cognitive abilities” and hence “emphasize that it is important to make high-quality pre-school education accessible

for all children” (p. 242). Findings on child and family background factors revealed that gender, parental native language status, maternal education, and SES were associated with initial numeracy levels as well as with cognitive growth. Both SES and parental native language status independently predicted numeracy skills at age 3. The achievement gap between higher and lower SES children widened over the pre-school years. “Children with a medium- or high-quality HLE [Home Learning Environment] seem able to take advantage of a high-quality pre-school, whereas children with a low-quality HLE do not seem able to benefit from two years of high-quality stimulation at pre-school.” The study by de Coulon, Meschi and Vignoles, (2011) also provided clear support for the notion that identifying parents with poor literacy and numeracy skills can help us predict which children are most at risk of having poor skills themselves.

Second, we note that in many education systems, students from all backgrounds are subjected to the same curriculum, taught in the same way and the performance measured in standardised instruments. Here we argue that a programme that takes the existing conditions as given and evolves standards that can be met in a realistic way has a better chance of succeeding and building on its strengths. For communities placed outside the dominant socio-economic and cultural space, the shared experiences of those within this space, which define the normative practices in school, are not available.

Such effective programmes must include a knowledge by teachers of the special context of poverty, the experiences that their students have outside school and what students can do in terms of numeracy, rather than what they cannot do.

In the context of early numeracy, working with children from poor backgrounds need to be based on the awareness that they may not have the same opportunities as those coming from better off backgrounds. Children from higher SES backgrounds are more likely to have active parental involvement in shaping their number knowledge and spatial understandings that are in line with what is expected in school and have access to educational toys and games which further enhance their ease with numbers and spatial skills. While, as Sood and Jitendra (2011) argued,

Research indicates that providing these students, who are less likely to be supported in learning mathematics both in the home and school environments compared to their peers from middle- and high-income families, with systematic and purposeful activities, can alleviate their lack of experiences and lead to improved mathematics achievement (e.g., Griffin & Case, 1997; Klein & Starkey, 2004). (p. 29)

A similar if not a much stronger recommendation is made by Balfanz and Byrnes (2006), calling for strong student-teacher bonds. According to them, “high-poverty schools need strong instructional programmes and sustained and intensive teacher support to provide students with the opportunity to attend a high-gain classrooms every year. They need organizational reforms to create stronger student-teacher bonds and more caring and daring classroom environments that promote student effort and improve attendance” (p. 155).

RECOMMENDATION

Children from high-poverty backgrounds often show a lag in their cognitive development and limited experiences needed to build numeracy in school. Such students need special understanding, empathy and support in early years teaching to bridge the gap in their numeracy development. Programmes developed to deal with children living in poverty conditions should be based on teachers' knowledge of the out-of-school experiences of these students and provide experiences that are necessary to develop cognitive and numeracy skills expected of all students.

2.7.2 Children from Indigenous Backgrounds

Knowledge transmission is culturally coded. Trying to impose on children from indigenous backgrounds forms of knowledge that are not supported by their own cultural contexts results in alienating the children. Perhaps issues related to cultural relevance and numeracy are best illustrated in the research on indigenous students.

Zevenbergen (2001) identified two approaches taken by different studies that deal with indigenous students and numeracy. Some of the studies focusing on children belonging to indigenous traditions seek to understand pedagogical approaches that will eventually bring the indigenous children on a par with the non-indigenous children in mainstream schools. Other studies seek to figure out approaches to mathematics curriculum designing and pedagogy that will contribute to strengthening the existing cultural practices of the community.

The Productive Pedagogy framework discussed in chapter 1 stresses the principle of ‘recognition of difference’ that includes the development of cultural identity as an essential component of productive teaching.

The second approach identified by Zevenbergen, Meaney and Evans (2013) argued

that mathematics education must take seriously its responsibility to support Indigenous students to gain school mathematics and also to help maintain the use of traditional

Arguably, these considerations are more difficult to implement in context of low-income countries as they may be in context of poverty in medium- to high-income countries. They do call for a greater commitment to education and public spending at a local level and collaboration with international organisations and governments, towards achieving more social equality and the aim of universal primary education.

mathematical ideas. If this does not occur, mathematics educators will contribute, intentionally or unintentionally to the loss of Indigenous knowledge that present and future generations of Indigenous people will hold them responsible for. (p. 481; bold added here for emphasis

In the context of engaging with quantifying for children from indigenous backgrounds, they suggested a three-pronged approach:

The first is to discover what reckoning strategies continue to be used or have been used in the past. The second prong is to determine how reckoning practices could be connected to Western calculating and the third is to develop strategies for maintaining traditional mathematical practices. All of these research approaches need to be controlled by Indigenous people, with the support of non-Indigenous people when needed. (p. 491).

In their seminal study in the USA, Boaler and Staples (2008) found that the mathematical performance of disadvantaged students improved markedly when the pedagogies matched their expectations and backgrounds.

In their study of New Zealand indigenous populations, Meaney, Trinick and Fairhall (2013) argued that “Equity of outcomes involves ‘at risk’ students being provided with extra support, so they are more likely to achieve the same results as other students” (p 236).

Contesting the standard practice of accounting for low performance of indigenous students on their background using deficit theories or on the teachers non-indigenous background, they quoted the work of Brenner (1998) showing “how students’ learning could be improved by combining their understanding of everyday activities with knowledge of cognitive processes” and (Lipka, 1998) calling for a “focus on bringing existing community practices and ways of interacting into classrooms that had used Western-focussed curricula and teaching pedagogies” (p. 236-237).

The need for early years pedagogies that work inclusively with the understandings that learners bring from their home and community environments has been highlighted as a feature that promotes numeracy learning with indigenous students. Frigo et al. (2004), looking at schools with reputations for supporting numeracy learning of indigenous learners in Australia, suggested that “inclusive” practices at classroom level and “the provision of contextually and culturally relevant pedagogies, and explicit teaching strategies which recognised and valued home languages and dialects other than standard Australian English” (p xii) – were related to successful working.

Similarly, Warren and de Vries (2009) who also focused on indigenous learners in Australia found several ‘bridging’ pedagogies to be helpful. The teaching actions that supported indigenous students’ numeracy learning are:

- Use of oral language, using mathematical vocabulary and transition to standard English
- Starting from numberless context before moving to numbered contexts
- Mapping oral language and representations onto number contexts.

Building numeracy language is highlighted as a key aspect of inclusive numeracy practice in this work. Their paper concluded that indigenous students begin school at a disadvantage because Western notions of quantity

are not only irrelevant to their world but contrary to it. The approach adopted results in the research to bridge the gap enabling students to begin compulsory schooling on equal footing (Barton, 1996; d’Ambrosio, 2006; Knijnik, 1999; Vithal & Valero, 2003).

Grootenboer and Sullivan (2013) in their study to understand and build on the Australian indigenous measurement schemes found that these students had some clear understandings of the measurement concepts involved, and a “task-based one-on-one interview gave better insights into students’ knowledge than the written form of the National Assessment Programme–Literacy and Numeracy assessment.” In this study we highlight our believe that this finding is not a statement that one form of assessment is more accurate than another, but that different numeracies are demonstrated through different contexts of assessment.

The authors went on to identify some of the features of effective numeracy education in schools that had a significant proportion of indigenous students as “using real-life contexts, having an explicit focus on the language of mathematics, and building on students’ existing knowledge, understandings and skills” (p. 183). They emphasised the role of context in problem solving and said:

For some of the questions, the context seemed to all but prohibit the participants from displaying their mathematical abilities and understandings because the contexts were foreign. However, when the same mathematical ideas were assessed in a familiar context, many of the students were able to show that they did have some comprehension of the mathematics involved.

A similar experience was pointed out by the Prathamik Shiksha Karyakram (PRASHIKA) team in relation to their work with first generation learners in Central India (Agnihotri et al., 1994).

RECOMMENDATION

Educational programmes involving indigenous students must be sensitive to their context and aspirations and should aim to develop both the cultural and mathematical identities of the children and their ability to participate mathematically in society. This can only be done in collaboration with the local communities.

2.7.3 Children from Non-Dominant Language Backgrounds

In many countries, a single classroom may consist of children with different languages/dialects spoken at home or life outside of school. For these students, school instruction is in the dominant language of the country. For other students in low-income countries, the language of instruction in school is in a language (often English) other than the home language of the majority of students. Language policies differ across countries, with some adopting an ‘English from the outset’ policy in formal schooling such as Kenya (Mazrui, 2000); others adopting a transition model, wherein early school years numeracy teaching occurs in home language(s) and a transition to English (or other dominant language) occurs later, while others still maintain home language teaching across pre-school and the whole of the primary age range (Barwell, Barton & Setati, 2007; Mendes, 2007).

While some research offers language as a factor associated with low achievement or as a hurdle that needs to be overcome for effective numeracy development, increasingly, researchers are seeing language as a useful resource that can contribute to students learning and relate it to their everyday context. Setati, Molefe, and Lange (2008) urged that multilingualism should be reconceptualised as a resource rather than a disadvantage, thus shifting the deficit discourse around multilingualism and mathematics performance towards a proficiency discourse (see also Setati, Chitera & Essien, 2009; Phakeng, Moschkovitch, 2013). They argue that

most language ‘factors’ in large-scale studies correlate with low mathematics performance but this should not be interpreted as causal. According to them, English is one of the eleven official languages, but is the language of teaching and learning in all schools; however, is not the language spoken at home for the majority of teachers and students. Research shows that teachers and learners in these classrooms prefer that English be used as the Language of Learning and Teaching (LoLT).

For these authors, the critical feature relating to language use is ‘transparency’: what language is used is not the central issue, rather the key issue is that the language used within teaching and learning has ‘transparency’ in terms of allowing learners to access meanings. Setati (1998) and Setati and Adler (2000) also found that code switching between languages was a useful resource for teachers to use to aid learner access to meaning.

Based on their study of multicultural classrooms in Botswana, Kasule and Mapolelo (2005) reported the following strategies to be used by teachers to overcome the difficulties imposed by their classroom situation. They pointed to code switching being seen as important and effective by teachers yet discouraged in policy. An example of the typologies used in teaching mathematics in L2 ¹¹ (adopted from Kasule & Mapolelo, 2005, p.612) is:

¹¹ L1 refers to home language or ‘mother tongue’ while L2 refers to a second language (either the country’s dominant language or a foreign language other than the home language of the majority of the students – such as English).

Linguistic strategies	Games and other strategies	Organizational strategies
1. Code switching from English to Setswana	1. Using concrete objects	1. Group work based on ability so that the weak ones can be helped
2. Inviting questions from pupils	2. Using flash cards and playing cards	2. Providing variety in teaching method and class activities
3. Teaching the language of mathematics	3. Jigsaw puzzles	3. Forming a Maths Club
4. Translating the L1 textbook (called “Dipalo”) into English	4. Snakes and ladders	4. One child solving a problem as others listen
5. Using a simpler English word whenever possible	5. Using multiple tables	5. Using the solution to check the meaning of a problem
6. Doing “oral” maths	6. Role play in a mini shop	6. Involving the pupils when working out a problem on the board
7. Using discussion	7. Pupils converting number to story problems	7. Providing different ways of getting to the answer
8. Encouraging pupils to speak English	8. Participation in maths quizzes, fairs, and other competitions weekly	
9. Relating a new to an old concept		

RECOMMENDATION

Teachers need to use language as a resource for enabling access to mathematical engagement and ideas. Teachers need to maximize this resource in their local context. In particular, code switching may assist in the developing of understanding by young learners and should be allowed by government policy.

2.7.4 Special Policies on Education of Girls

The participation of girls and women in education and their educational outcomes remain a problem in many countries around the world. A Fact Sheet on Girl's Education published by Global Partnership for Education (GPE) (2014) stated the following:

- Women represent nearly two thirds of the world's 775 million illiterate
- 53% (31 million girls) are still out of school around the world
- Too many girls in developing countries are still shut out of school, because they have to work, are married early, or have to care for younger siblings, denying them their fundamental right to education
- Participation of girls in school decreases as they progress through the education system
- Even when girls are in school, they are often treated differently from boys, and discouraged from taking leadership roles. Lower expectations from families and school communities hinder their performance. (p. 1)

Research on the numeracy outcomes of girls in schools provides a complex picture that requires careful analysis. Some authors (e.g. Hanna, 2000/1) investigated the patterns of gender differences between the first and third IEA studies concluding that the "clear message from the IEA cross-national studies is that gender differences in mathematics decreased considerably over the thirty years or so which these studies covered and indeed are on the way to disappearing" (p. 16). Similar results were reported by Else-Quest et al. (2010) in their meta-analysis of both TIMSS and PISA results. However, in considering variations between countries, Else-Quest et al. went on to point out that the patterns of gender equity shows huge variations with some showing advantage in favour of boys with others in favour of girls. Ma (2007) explained this as "gender differences in favour of girls balanced off gender differences in favour of boys, resulting in the lack of [overall] gender differences at the 4th grade in TIMSS 2003" (p. 35).

In a background paper prepared for the Education for All Global Monitoring Report 2008, Ma (2007) studied gender differences in different school subjects and countries as reflected by international and regional cross country comparisons. Ma (2007) conducted a comparison between PISA 2000 and 2003 results and showed "that the male advantage in mathematics literacy was on the rise in proportion (from 47% of the participating countries in 2000 to 70% of the participating countries in 2003)" (p. 32). These results were subject to country development status. "Participating developing countries were the majority in the category of countries with the largest gender differences in both 2000 and 2003. In general, the percentage of the participating developing countries with gender differences increased from 67% to 70%, and that of the participating developed countries increased from 46% to 69% between 2000 and 2003" (p. 33).

These variations between countries and between years of testing imply important lessons for the reasons behind such gender differences. Rather than explaining these results in terms of gender differential in terms of ability, Else-Quest et al. (2010) noted that "the gender stratification hypothesis maintains that such gender differences are closely related to cultural variations in opportunity structures for girls and women" (p. 103). A United Kingdom report (Brock & Cammish, 1993) studied the factors affecting the participation of females in education in seven developing countries and has identified the following factors with various roles in each of the countries studied: Geographical, socio-cultural, health, economic, religious, legal, political/administrative, and educational. Else-Quest et al. (2010) added: "Gender equity in school enrolment, women's share of research jobs, and women's parliamentary representation were the most powerful predictors of cross-national variability in gender gaps in math" (p. 103).

Ma (2007), commenting on the changing expectations of girls and women in different countries and the changes in gender differences patterns, concluded

[The] distinction in gender stereotypes between developed and developing countries may in fact explain why most female breakthroughs

in mathematics and science where girls begin to outperform boys come from the developing world (see TIMSS 2003 in Tables 11 and 15). Philippines reports female breakthroughs in both mathematics and science at both 4th and 8th grades. Bahrain, Jordan, and Singapore show

female breakthroughs in mathematics at the 8th grade. Palestinian A. T. and Saudi Arabia shows female breakthroughs in science at the 8th grade. In contrast, very few female breakthroughs are observed in mathematics and science from the developed world. (pp. 91-92)

RECOMMENDATION

Government and local school policy towards developing early years' numeracy should include provisions to monitor girls' access to quality educational opportunity and outcomes counteracting any stereotypes of careers and subjects as essentially preferred or needed by either sex. Working with families and communities is essential towards raising expectations of girls of developing numeracy.



Broad advocacy of the need for ‘universal primary education’ has led to the need for many more well-qualified early years and primary level teachers. Concerns have been raised about the prevalence of short duration training in low-income countries (Abadzi, 2002), although significant variations exist between countries. Further, many current teacher education courses depend on generalist rather than subject-specific approaches with marked separations between general theory and mathematics-related components (Opolot-Okurut, 2005). This tends to mean that mathematics is taught in the form of traditional content courses, rather than in the profession-related ‘mathematical knowledge for teaching’ orientations that are widely described as useful in the literature, with little if any link to the theories of learning that are encountered in general education courses.

While widespread evidence of gaps in teacher knowledge continues in low-income countries, concerns have been raised about lack of capacity and appropriate experience within early years teacher education, and lack of openings to develop ‘mathematics knowledge for teaching’ (UNESCO, 2013). Given the attention to language issues within early numeracy learning, it is worth pointing out that capacity issues in multilingual countries like South Africa extend to insufficient numbers of teachers qualified to teach in the early years of compulsory schooling with mother tongue language proficiency (Green, 2011).

In the introduction to this study, we noted the limited research that addresses numeracy and its development in low-income countries. Here, we similarly note that leaders in the field of researching teacher education internationally (such as Tatto, Navotna and Lerman) acknowledged in their report from the International Commission on Mathematical Instruction (ICMI-15) study, *The Professional Education and Development of Teachers of Mathematics*, that:

We know little about the organization of the opportunities to learn mathematics and mathematics pedagogy offered to prospective and practicing teachers across the world and their relative effectiveness (Tatto, Lerman & Novotna, 2010, p. 313).

While comparative studies such as the Teacher Education and Development Study in Mathematics (TEDS-M) have recently begun, we must note that such research is in its infancy and there are as yet no clear recommendations as to which strategies are more effective and the reasons for this. Indeed, such inter-

national studies combine first world and developing world contexts and it would be naïve to think that a ‘one size fits all’ could emerge from comparative studies as the proposed ‘best practice’ for all contexts. Such comparative studies tend to raise the presence of ‘unresolved questions’ rather than solutions. Tatto, Lerman and Novotna (2010) did, however, point clearly to trends in the field that foreground increased periods of teacher preparation in schools and the importance of mathematical knowledge for teaching, which points to problems with more generalist teacher education orientations in early primary years:

The mathematics-specific information presented here provides indicators of larger and important trends in the field. For instance the examination of the system for the mathematics preparation of teachers indicates that although a larger proportion of pre-service programmes are located in the universities, a global market economy seems to be pushing teacher preparation out of universities and into the schools where future teachers may not find the expertise to further their mathematics knowledge.... Although the research literature seems to increasingly point to the importance of mastering mathematics among those who teach it, our preliminary inquiry makes evident a worrisome trend especially among primary teachers who find themselves in programmes that prepare them as “generalists” and where the curricular emphasis in mathematics and mathematics pedagogy is seen as “low” (p. 323).

Thus, while here we provide an international review of literature, we focus on studies relating to addressing numeracy teacher education of particular relevance to low-income contexts. In particular, this chapter considers the issue of the low status of teachers in low-income countries and its implications to their effectiveness to assist young learners in developing their numeracy skills. Likewise, we discuss literature on teachers’ knowledge, both of content and pedagogy, necessary for effective numeracy development. Of particular importance is the teachers’ knowledge of numeracy progression. The following two sections deal with the research findings on issues relating to pre-service and in-service development of teachers respectively. The chapter concludes with a discussion of the need for equipping teachers with skills to work with parents and other community members towards provision of numeracy development in young children.

3.1 The Crucial Role of the Teacher

Atweh (2007) noted that in a study commissioned by the US congress, Coleman, Campbell, Hobson, McPartland, Mood, Winefeld, and York, (1996) reviewed the long term effect of many interventions to alleviate economic disadvantage through education and concluded that schools do not reduce social inequality. Rather, research consistently shows that the family socioeconomic wealth is the best predictor of educational success. Similarly, the increasing gap between the rich and poor in many western countries (and between countries) does not support this utopian view of education. Perhaps Basil Bernstein (1971) was correct in his conclusion that schools do not compensate for society.

However, there is some good news. Coleman and his colleagues demonstrated that under school reform the most disadvantaged students benefited the most. In other words, although good teaching benefits all students, under certain conditions it also closes the gap between the least disadvantaged and the rest of the students. Further, out of all the school factors that effected students’ achievement was the teacher. Hence good teaching “can make a difference, but not all the difference” (Hayes, Mills, Christie & Lingard, 2006, p. 178). The research shows that higher-level achievement, in particular, with student populations often referred to as disadvantaged is related to quality teachers.

What qualities of teachers affect student achievement is a complex issue and, at times, the results are not intuitive. Using data from several USA wide surveys and case studies, Darling-Hammond (1999) conducted a study to examine the effect of teacher qualifications on student achievement. Interesting findings of this study include:

- Teacher quality characteristics such as certification status and degree in the field to be taught are very significantly and positively correlated with student outcomes.

- School resources, such as pupil-teacher ratios, class sizes, and the proportion of all school staff who are teachers, show very weak and rarely significant relationships to student achievement when they are aggregated to the state level (p. 29).

Of particular interest in this comprehensive review of literature is the observation that content knowledge in the subject was less effective than knowledge of teaching and that “when aggregated at the state level, teacher quality variables appear to be more strongly related to student achievement than class sizes, overall spending levels, teacher salaries (at least when unadjusted for cost of living differentials), or such factors as the statewide proportion of staff who are teachers” (p. 38).

The crucial role that teachers play in children’s education is also highlighted by the UNESCO (2013) Policy Paper 7 on Addressing the Crisis in Early Grade Teaching. The policy paper pointed out that in many countries, teachers at lower grades are not well prepared to teach leading to hundreds of millions of students completing primary schools without having learnt the basics. The policy paper named the example of East Asian countries which show that teacher knowledge and the support they receive in pre-service and in-service is reflected in their students’ achievement. The report went on to state that: “This achievement shows what is possible, although lack of resources and institutional capacity makes it difficult to replicate in poorer countries” (p. 3).

Similarly, the UNESCO Global Monitoring Report (2013/4) called for policies that “include attracting and retaining the best teachers, improving teacher education, deploying teachers more fairly, and providing incentives in the form of better salaries and attractive career paths” (p. 218) while realizing that finding the means “to end the learning crisis requires a delicate juggling act on the part of policy-makers” (p. 18).

RECOMMENDATION

The most effective means to develop numeracy with young children are quality teachers. An education system cannot exceed the quality of its teachers. While quality teaching benefits all students, evidence shows that it benefits low-achieving students more, thus it contributes to narrowing the gap between different groups of students. Attracting and retaining well-qualified teachers should be a high priority for developing numeracy in the early years. Teachers should be supported to adopt effective pedagogies to assist young learners in developing their numeracy.

3.2 Teachers' Conditions, Status and Morale

There is evidence that teachers' conditions and pay are related to the quality of teaching in schools. The UNESCO Global Monitoring Report (2013/4) pointed out that many countries that experienced a rapid expansion of their primary education sector, to meet the demands of universal primary education, have experienced a shortage of well-trained and supported teachers. The report gave examples of some countries that have been able to increase the quality and qualification of their teachers by mandating higher entry requirements to the profession and increased pay to attract more able candidates. However, the report acknowledged that measures such as these are difficult for some low-income countries. In some countries (Ghana, for example), students would train as teachers in order to obtain higher status and better paid jobs outside the education system.

Tatto et al. (2012) reported on an IEA study on variations on teacher preparation programmes across 17 countries focusing on the relationships between teacher education policies and readiness of teachers to teach mathematics. The study found that "countries where teaching conditions are relatively favourable can readily attract the required number of talented, highly motivated teachers. In those countries where conditions are unfavourable, recruiting teachers tends to be difficult" (p.38). In particular the reports pointed two examples:

In Botswana, for example, the challenges include heavy workloads, shortages of teaching and learning resources, large class sizes in some areas, an insufficient number of classrooms, and considerable diversity in student abilities and home languages (p. 39).

The salary situation in the Philippines is so bad that finding a solution is proving difficult. At the time the Philippines submitted their TEDS-M country report, salaries were close to the poverty threshold, with new teachers receiving a salary of US\$194 per month compared to the poverty threshold of US\$156 (p. 39).

Teachers' conditions and pay are not the only factors that are related to teachers' performance and morale (OECD, 2008). Bennell (2004), in his report on teacher motivation and incentives in low-income countries in Sub-Saharan Africa and Asia, argued that the status of teachers in most countries (developed and developing) has declined over the past decades, and that the de-professionalisation of teachers is more pronounced

in low-income countries. He reiterated that pay on its own does not increase motivation (p. iii). He also pointed to the poor and declining public education in many of these countries (especially low-income countries in Asia) leading to a mass exodus from public schools. In Southern Africa, there has been a related rapid rise of 'low fee' private schools where many teachers in the public sector send their children (Bernstein & Schirmer, 2010). Bennell (2004) continued, in relation to Sub-Saharan African and Asian low-income countries, to argue that:

Increasing hours of work, larger class sizes, more subjects, and constantly changing curricula are cited as major de-motivators in many countries. What is expected from teachers (the 'social contract') is not pitched at a realistic level in many countries given material rewards, workloads, and work and living environments. In many countries, teachers are being asked to take on more responsibilities, including HIV/AIDS education, counselling, and community development.

The work and living environments for many teachers are poor, which tends to lower self-esteem and is generally de-motivating. Housing is a major issue for nearly all teachers. The 'struggling teacher' is an all too common sight, especially in primary schools. High proportions of teachers remain untrained in many LICs [low-income countries], which adversely affects 'can-do' motivation. Too often, teachers are 'thrown in at the deep end' with little or no induction. Multi-grade teaching is common in LICs, but most teachers are not adequately prepared for the special demands of this type of teaching. (p. iv).

The issue of constantly changing curricula has been particularly felt in many countries. For example, in South Africa the remuneration packages of teachers are on par with many other countries but teacher morale continues to be low. Teachers, especially in low-income countries, are regularly tasked with implementing new curricula, that they have had little say over, and in contexts that are extremely challenging, such as overcrowded classrooms, high levels of poverty and absenteeism, few resources, little in-service training and support. Graven (2012) argued that the focus on teacher change is particularly problematic in contexts where dichotomies are set up between 'old' and 'new' practices and reference to 'old' practice as bad and 'new' practice as good. Calls for radical teacher change, where old practice is completely replaced by new practice and where once this has happened the learning process is assumed complete, are problematic.

Such a view of teacher change is clearly disempowering for teachers, especially experienced teachers, and furthermore is not educationally productive. Related to this idea of change from 'bad' to 'good' practices is a 'fix-it' approach to in-service teacher training (INSET). Breen (1999) asserted that the manifestation of INSET culture seems to have the following principle:

There is something wrong with mathematics teaching world-wide, and that we, as mathematics educators, must fix it. Many mathematics teachers have bought into this culture. Such teachers seem to be seeking new ways to fix their practices ... Mathematics teachers need someone to fix them, and mathematics educators need someone to fix ... This culture is based on judging what is right and wrong, paying little attention to what mathematics teachers are actually doing (since it is wrong anyway) in their classrooms, and looking outside themselves for the 'right' way, the newest 'fix' (p. 42).

Despite the negative effects of such approaches on teacher morale - and thus learning - there seems to be continued acceptance among several stakeholders that such training approaches are appropriate, or inevitable, in large scale nationally organized teacher development. Even while some teachers have been critical of the nature and effectiveness of the training they have received (see Chisholm et al., 2000; OECD, 2008), teachers have not actively challenged and rejected identifications that assume their existing experiences and practices are bad and in need of radical change. This is not surprising as across Southern Africa:

Teaching as a career has low public status. The image of teaching as portrayed in the media also tends to be negative... Many teachers have internalized a negative image of their work, experiencing a lack of confidence and low morale (OECD, 2008, p. 299).

Graven (2012) argued rather that teacher education, pre- and in-service, should embrace teaching as a lifelong learning profession and should work on content knowledge (CK) for teaching and pedagogical content knowledge (PCK) as well as developing strong identities of professional participation. Professional learning communities, professional associations and in-service communities of practice are argued to be excellent models for enabling teachers to re-author the negative identifications provided to them by the press and public.

Thus the challenge of current teacher education is that while there is increasing international literature on teacher identity (e.g. Carrim, 2002; Jansen, 2002;

Lasky, 2005; Mattson & Harley, 2002; Moletsane, 2002), teacher emotions (Reio, 2005; Sutton & Wheatley, 2003; Zembylas, 2005) and teaching as a learning profession (e.g. Day, 2000; Darling-Hammond & Sykes, 1999; Kwo & Intrator, 2004) teacher education models remain largely unchanged since colonial times and thus the influence of this literature on teacher education is slow.

In respect to teacher identities, and the status of teaching, Graven (2012) argued that stories of teachers as deficient shut down the space for learning. Denying teachers the opportunity to draw on their existing experiences alienates them from the learning process. Deficiency stories tend to regard teacher experiences as irrelevant at best and at worst as wrong. Thus the rhetoric of deficiency focuses on what teachers don't know and negates the value of what they do know. Through denying the value of teachers' life histories and experiences as a fundamental resource base of the learning process, and the resource which enables the appropriate reinterpretation of curricula change, deficiency identities tend to result in a self-fulfilling prophecy.

This 'storying' of teachers thus shuts down the space for meaningful ongoing learning in the classroom and instead supports a limited learning cycle which involves mastery of 'mimicry' of outward forms of proposed pedagogy (Mattson & Harley, 2002) and adoption of the rhetoric of policy without substantive change in the practice of teaching. That is, it leads to "ritualized learning" rather than "substantial learning" (Sfard & Prusak, 2005, p. 19).

However, Graven (2012) warned that this is not to deny that various kinds of knowledge are essential to effective teaching and should be a focus of teacher development. Shulman's (1986) seminal work on Pedagogical Content Knowledge (PCK) (a "strictly cognitive and individual construct" [Shulman & Shulman, 2004, p. 258]) is an example of crucial knowledge relating to subject specificity which is central to effective teaching. But this is only one piece of the puzzle and thus shifts developing teachers as 'a person-in-the-know' to lifelong learner. This opens up the space for in-service learning where teachers can readily admit to their limitations, which evidence suggests is particularly important for early years generalist trained teachers involved in teaching numeracy and mathematics. So, for example two teachers in Graven's study (2004) in a township context noted the centrality of developing confidence in teachers as part of teacher development programmes as it is this confidence that sustains learning beyond the in-service programme. The quote of a teacher after two years of participating in a mathematics in-service community of practice illustrates this point:

I can expose myself to what I know, I mean to other people and I am willing to say Okay fine, show me wrong, prove me wrong. What is your idea then? What I say is I am open let's learn. That is what that self-confidence is (Karl) (p. 203).

Furthermore, the recruitment and retention of teachers in LICs is particularly complex and distribution of teachers to poorer rural areas can be particularly difficult. Bennell (2004), in his study across low-income countries in sub-Saharan Africa and Asia, raised some crucial points that are worth quoting at length:

The low proportion of qualified teachers working in rural schools is one of the most serious problems preventing the attainment of EFA [Education for All] with reasonable learning outcomes in most LICs. Women teachers at government-funded schools are also disproportionately employed in urban schools because it is generally accepted that they should not be separated from their husbands/partners and there are pervasive cultural concerns about posting single female teachers away from their family homes.

The extent to which the recruitment process is centralised is a key factor in shaping

deployment outcomes. Teacher recruitment is school-based in some countries, especially where missions and other faith-based education agencies own and manage sizeable proportions of schools. At the other extreme are highly centralised teacher recruitment systems where teachers are appointed by the Ministry of Education and then posted to schools.

The failure to provide attractive additional incentives to work in remoter rural schools is a key factor. Relatively very large incentives may be necessary to attract teachers to hard-to-staff schools, but these are not likely to be affordable in most LICs.

Despite the widespread recognition of what amounts to a teacher deployment crisis in many LICs, efforts to tackle the most serious deployment problems have been quite limited and invariably unsuccessful. Very few countries have ... systems that are sophisticated enough to be used for detailed recruitment and deployment purposes (p. v).

In this respect, he argued that it is critically important to provide incentives for teachers to work in these less attractive and more challenging locations.

RECOMMENDATION

Research positions teaching and teacher education within broader socio-cultural contexts. Low status and morale of teachers in contexts of low-income countries lead to difficulties with recruitment and retention. In such contexts, attention to models of teacher education that address the development of teacher identities attuned to ongoing mathematical learning, and supporting the improvement of teacher morale and commitment to the profession are noted as particularly important. Incentives should be provided to encourage teachers to take up posts in remote and particularly challenging contexts in order to ensure that sufficient numbers of well-qualified teachers are deployed to these areas. Additionally supporting low-income countries to develop effective systems for deployment of teachers to reach all areas is needed.

3.3 Teacher Knowledge in/for Numeracy

Further issues that are frequently raised in discussions of teaching in contexts of poverty relate to teacher knowledge and teaching quality. The international literature provides a range of models for thinking about the knowledge base required for high-quality primary mathematics teaching (Ball, Thames & Phelps, 2008; Rowland et al., 2009). A key distinction that has been highlighted as particularly relevant within primary mathematics teaching is the need for both content knowledge and pedagogic content knowledge (Shulman, 1987). Venkat (2013), looking at episodes of early years' numeracy teaching in South Africa, noted that the evidence points to gaps in the way mathematics is understood, rather than simplistically at the level of content knowledge. Several African critiques point to the dangers of over-emphasis on content knowledge and under-emphasis on pedagogic content knowledge within teacher education. Pryor et al. (2012) noted that in sub-Saharan Africa a focus on infrastructure and teacher supply has been at the expense of promoting teacher competencies. Several additional studies point to the need for stronger teacher pedagogical content knowledge in teacher preparation- (i.e. how to teach the foundations of numeracy versus learning the content only). For example, Ngware et al. (2010) and Akyeampong et al. (2013) both pointed to too much focus on content at the expense of PCK. Broadening to pedagogic skills more generally, Ngware et al. (2010) in the Kenyan context proposed that: "In low-performing schools, teachers may require more pedagogical skills-upgrading with a view to enabling them shift their lessons to more learner-centred approaches" (p.4).

While content knowledge is argued to be critically important (Ma, 1999) and the basis for good teaching (UNESCO, 2013), there is acknowledgement that it is not of itself enough. Siraj-Blatchford et al. (2002) argued that content knowledge (CK) and PCK are critical and that qualification levels were important for encouraging children within early years teaching to engage in high cognitive demand and 'sustained shared thinking' in the English context. Askew et al.'s (1997) large-scale study of teacher practices and learner gains, also in the English context, suggested that 'connectionist' teaching was linked to highest learner level gains in primary mathematics. This coheres with Ma's (1999) comparative study that investigated the mathematics knowledge and pedagogical knowledge of Chinese and USA elementary mathematics teachers. Ma (1999) stipulated four properties for developing mathematical understanding: basic ideas, connectedness, multiple representations and longitudinal coherence. She found that a key reason for China's comparatively better performance on

international tests was due to the much more connected subject knowledge of Chinese teachers, which were linked to more connectionist approaches to teaching mathematics. This links with Pryor et al.'s (2012) comments that teachers need to understand what conceptual understanding in numeracy means as distinct from following a set of steps.

Connected to issues of teacher knowledge, there is also evidence of disparities of distribution of content knowledge and teaching quality at primary level within countries by socio-economic locale (Spaull, 2013 in South Africa; Haycock, 2002-3 in the USA). While there is acknowledgement in the research that findings on whether teacher qualifications and teacher quality are significantly related are mixed (e.g. Borman & Kimball, 2005; Carnoy et al., 2011; Carnoy, Chisholm and Chilisa, 2012), all these studies (in USA, South African, and Botswana contexts) found that 'better teachers' were matched with better performing schools and learners as they sought out schools with better conditions and resources. Studies drawn from disadvantaged contexts in both low- and high-income countries point to greater evidence of fragmented, rote and highly localized mathematics teaching than in more advantaged contexts. (See Haberman's (2010) account of a 'pedagogy of poverty').

Related to the above, Askew et al.'s (1997) study on effective teachers of numeracy has some implications for what teaching approaches could be foregrounded in teacher development programmes:

They [highly effective teachers of numeracy] used corresponding teaching approaches that:

- connected different areas of mathematics and different ideas in the same area of mathematics using a variety of words, symbols and diagrams
- used pupils' descriptions of their methods and their reasoning to help establish and emphasise connections and address misconceptions
- emphasised the importance of using mental, written, part-written or electronic methods of calculation that are the most efficient for the problem in hand
- particularly emphasised the development of mental skills (p. 3).

Jorgensen et al. (2010) pointed to the need for exemplars of practice that embody 'connectedness' within pre- and in-service teacher education in contexts of poverty. They found that while teachers realised that they needed to implement changed practices with increased connectedness (and believed in these practices) they had little knowledge of what such practices may look like. This points to the need for in-service support and more explicit inclusion of such practices in

pre-service teacher education. This also highlights the need for development of locally relevant exemplars of connected numeracy teaching within and for teacher development.

Many argue that ways must be found to attract the best into teaching (e.g. UNESCO, 2013) and Brazil has been given as an example of where teacher recruitment has

become more selective. The UNESCO (2013) report however also notes that this is very difficult in countries where there are few candidates with strong subject backgrounds, which makes attracting these candidates to teaching (with low pay and status) difficult.

RECOMMENDATION

Supporting the development of the content knowledge base of primary teachers remains an urgent priority according to the literature. In the context of early numeracy, it is important to note that findings suggest that it is not simply 'more' or 'higher level' content that is required. Numeracy teacher education must provide opportunities for teachers to develop connected understandings of early numeracy content and problem solving, knowledge of the progression of numeracy learning development, syntactic knowledge related to problem solving, and pedagogic content knowledge and skills related to early numeracy teaching.

3.4 Understanding Numeracy Progression

Research evidence from medium- to low-income countries points to concerns relating to the prevalence of 'flat' learning profiles across primary grades in numeracy in contexts of poverty (Pritchett & Beatty, 2012). While there is broad agreement on this phenomenon, ways to respond to it remain somewhat discordant, and particularly so in relation to questions of curriculum coverage/pacing and standardisation. Partially at least, this discord is linked to research studies that point to the effectiveness of teaching approaches foregrounding learner-centredness (Lambert & McCombs, 1998), co-operative learning (Moss and Beatty, 2006) and the teaching of mathematics in exploratory and connected ways (Askew et al., 1997; Ma, 2000) rather than in transmissive ways. In contrast to this view, Pritchett and Beatty (2012) noted this kind of response in radical curriculum change and warn of dangers of "overambitious curricula" in low- and medium-income countries, where teaching can outpace learning with a "coverage orientation" (p.1) rather than a learning orientation. They wrote:

If the official school curriculum covers too much, goes too fast and is too hard compared to the initial skill of the students and the ability of the schools to teach this can produce disastrous results. An overambitious curriculum causes more and more students [to be] left behind early and stay behind forever (p. 13).

Similarly, there has been critique of the ways in which learner-centredness has been taken up in international development cooperation projects. For example Tabulawu (2003) writes:

Recent pronouncements by international aid agencies on their interest in and preference for a learner-centred pedagogy so far appear not to have attracted much scholarly attention. This paper attempts to explain this interest. It argues that although the efficacy of the pedagogy is often couched in cognitive/educational terms, in essence, its efficacy lies in its political and ideological nature (p. 7).

Furthermore Brodie, Lelliot and Davis (2002) found that teachers who had been involved in a longitudinal in-service programme aimed at supporting the take-up of the new mathematics and science curriculum introduced in South Africa in 1997, took up the form rather than the substance of learner-centred practices, thus providing little opportunity for individual learner sense-making.

Several large scale and national intervention studies are moving towards more 'scripted' and standardised approaches to coverage and pacing. An example is the South African provincial level Gauteng Primary Literacy and Mathematics Strategy (GPLMS) which provides scripted lesson plans to over 800 primary schools (Fleisch & Schöer, 2014, forthcoming).

Such initiatives follow critiques that implementation of curricula, that have moved away from more traditional pedagogies and curricula in developing country contexts, failed to take into account the needs of teacher development for enabling implementation and, thus, exacerbated inequality between the high- and low-SES students (see Chisholm et al., 2000). More recently, Carnoy et al. (2011) pointed to low expectations of coverage and learning in South Africa, reflecting earlier studies pointing to poor coverage and pacing in primary mathematics (Reeves & Muller, 2005) and Hoadley's (2006; 2007) studies on learners of different socio-economic classes being provided differential access to mathematics learning. Sztajn (2003), in the USA, found similarly that teachers adapt reform rhetoric and cautions that when 'based on student needs' teachers provide differential access to aspects of curriculum and different opportunities to learn for different SES groups of learners. Lower SES learners mostly receive rote teaching while upper experience problem solving. Close adherence to lesson plans is viewed as a necessity in GPLMS with coaches helping teachers to get the most out of these plans whilst paying attention to the particular context of each class. Language and mathematics coaches are employed within the strategy with one coach overseeing both learning areas in the Foundation Phase (grades 1-3) and specialists working with schools in either language or mathematics in the InterSen (Intermediate and Senior Phase (grades 4-7). Initial analyses of literacy performance indicate some success in the improvement of learner performance of schools in poorer settings (Fleisch and Schöer, 2014, forthcoming).

Numeracy teaching for remediation in the early years of schooling tends to support this view of highly structured intervention, while working with much more individually diagnostic approaches (e.g. see Wright et al.'s, (2006) Maths Recovery approach). Central to the pedagogic approaches advocated here is the development of teacher understandings of progression in early numeracy. This is done alongside the provision of teaching and assessment activities and resources that are geared towards achieving the early number progression that is widely regarded as necessary to achieve access to mathematics in subsequent years of schooling. Wright et al.'s (2006) early numeracy recovery programme is being used by teachers to remediate numeracy understanding of learners who

are performing way below their grade level in several countries. Such programmes are highly structured based on thorough research of the developmental levels learners' need to progress through in order to develop numeracy foundations. While this recovery programme is implemented individually with learners, and thus would need to be adapted for poorer contexts, where individual recovery is unlikely to be possible, it does point to the need for carefully planned and pre-structured activities for remediation that progressively develop learner understanding from where the learner is at.

While much of the 'serial innovation' in developing country contexts points to disappointing results (Reddy, 2006), it is worth noting that Sztajn (2003) also points to absence of support for teachers in addressing reform across contexts. This concurs with similar concerns in the South African context (e.g. Reddy, 2006) as reason for poor national performance in TIMSS. A recurring lesson to be learnt from these failures is the need for curric-

ulum change to be accompanied by intensive teacher development programmes for pre- and in-service teachers that are strongly aligned with curricular goals.

The above studies point to the importance of a structured developmentally progressive framework as the basis of both teacher development and teaching for numeracy learning and numeracy remediation of foundational concepts. While a range of models is available, they all have similar levels of developmental progress that need to be carefully considered in assessment and teaching of learners and thus need to be fully understood by teachers. In this respect a model of developmental progress must be a focus of both pre- and in-service teacher development.

Based on multiple books authored by Wright with others and drawing on earlier work, the following learning and assessment framework is provided by Wright (2013, p. 31):

Table 1: The Learning Framework in Number¹²

Stages:	Levels: Forward number word sequences (FNWS) and Number word after
Early arithmetical learning	
0. Emergent counting	0 Emergent FNWS
1. Perceptual counting	1. Initial FNWS up to ten
2. Figurative counting	2. Intermediate FNWS up to ten
3. Initial number sequence	3. Facile with FNWS up to ten
4. Intermediate number sequence	4. Facile with FNWS up to thirty
5. Facile number sequence	5. Facile with FNWS up to one hundred
Levels: Numeral identification	Levels: Backward number word sequences (BNWS) and Number word before
0. Emergent numeral identification	0 Emergent BNWS
1. Numerals to ten	1. Initial BNWS up to ten
2. Numerals to twenty	2. Intermediate BNWS up to ten
3. Numerals to one hundred	3. Facile with BNWS up to ten
4. Numerals to one thousand	4. Facile with BNWS up to thirty
	5. Facile with BNWS up to one hundred

This framework points to the critical importance of focusing on progression and use of conceptual resources to assist learner 'recovery'. Wright (2013) further

summarises the emerging approaches to early number development drawing on his earlier work (p. 24):

¹² Table used with permission from Copyright owner

Table 2: Contrasting traditional and emerging approaches to early number instruction¹³

Traditional approaches	Emerging approaches
Study of the 'pre-number' topics provides a basis for learning about numbers and should occur before learning about numbers.	Pre-number topics can enhance development of logical and number knowledge but are not necessarily an essential prerequisite for early number knowledge.
Children should study numbers in the range 1 to 10 for an extended period before focussing on numbers beyond 10. Similarly, then study numbers in the range 11 to 20.	Teachers should develop children's verbal (in the sense of spoken and heard rather than written) knowledge of number words and their knowledge of numerals, extending beyond 20 and beyond 100 as soon as possible.
Children should study each number in turn to learn about its cardinality, its numeral, and number combinations involving the number.	Teachers should take a flexible and open-ended approach to learning about number words and numerals.
It is important for children to work with spatial patterns and count the dots in spatial patterns to learn about cardinality in the range 1 to 10.	Instructional activities involving flashing spatial patterns can help children learn to combine and partition numbers in the range 1 to 10 without counting by ones.
Teaching cardinality and ordinality of numbers in the range 1 to 10 is important.	Teachers should de-emphasize the teaching of ordinality and cardinality.
Children should be encouraged to use materials to solve early number problems for as long as they seem to need or rely on the materials.	Teachers should use instructional strategies as soon as possible that help to advance children to levels where they do not rely on seeing materials.
When children first learn about numbers in the range 11 to 20 it is important to teach the associated ideas of place value. Similarly for numbers in the range 20 to 100.	Children should learn about the number words and numerals beyond ten, long before they learn about 2-digit place value.
Children should learn about place value before they learn about addition and subtraction involving numbers beyond 10.	Children can learn about addition and subtraction involving numbers beyond 10, before they learn about place value.
Place value should be formally taught using base-ten materials, before children learn addition and subtraction involving multi-digit numbers.	Place value knowledge should arise from children's developing strategies for addition and subtraction involving 2- and 3-digit numbers.

Additionally, fluency is considered essential for numeracy development. So, for example Askew (2012) discussed fluency with respect to 'elements of fluency', which includes basic facts and knowing basic methods: "The point of being fluent in them [these facts] is to free up working memory when tackling a more interesting and engaging piece of mathematics" (p. 54). He argued that a lack of fluency in basic facts can impede conceptual understanding because certain processes take up too much working memory and

attention is diverted from thinking about the bigger mathematical picture. Basic number facts include, for example, bonds to ten; adding and subtracting 10; adding and subtracting 100 and doubling numbers. Stott and Graven (2013) found that focusing on these basic facts and fluency can greatly improve learning and remediation of basic number sense of learners in their research of after-school clubs in disadvantaged South African communities.

RECOMMENDATION

Developmentally progressive frameworks are useful for developing numeracy and should be a component of teacher development in low-income countries. This represents an area of common ground between the more learner-centred responses and the more prescriptive coverage-oriented responses as both are responding to challenges of poor learner progress in early numeracy.

¹³ Table used with permission from Copyright owner

3.5 Pre-Service Numeracy Preparation

The need for teacher education linked to the realities of contexts of poverty is noted, with some critiques pointing to teacher development that tends to prepare teachers for some 'ideal' classroom context. In such contexts learners are assumed to have the required competence for learning in the grade level they are placed, where class sizes are manageable and where conditions of poverty and high levels of absenteeism do not exist (Pryor et al., 2012; Tatto et al., 2012; articles based on the international TEDS-M studies' findings). While a concern for teacher education in all countries, this is particularly problematic for low-income countries where contexts of poverty and school dysfunctionality are frequently the norm rather than the exception (Bloch, 2009). Across a wide range of literature and across countries, a key problem seems to be that teacher training assumes a certain well-resourced semi-ideal context of teaching and in so doing does not provide preparation for the diverse contexts in which teachers will teach (Blömeke, 2012; Tatto et al., 2012; Ray, Bowman and Robbins, 2006) and the difficult conditions that many teachers face, especially those in contexts of poverty. Such contexts often involve overcrowded classrooms, learners from poverty backgrounds, learners from diverse linguistic backgrounds that are different from the language of instruction and learners with an enormous backlog of basic skills that are several levels below the grade the teacher is expected to teach according to the curriculum. Furthermore, such contexts are often under-resourced in terms of basics such as infrastructure, texts, writing resources, and so forth.

It is noted that one reason for the absence of this kind of training across contexts is that it is difficult to prepare teachers to face these contexts in colleges and universities – such preparation and support is

most likely needed on site where teachers are struggling to teach based on the specific challenges of their school context. Furthermore, some studies point to what they call a washed-out effect (Lewin & Stuart, 2003) of teacher pre-service training and argue that the socialisation processes of teachers in schools take precedent over the effects of their training (Pryor et al., 2012). In this respect much of the literature reviewed points to the need for localised in-service support of teachers on site in their schools and classrooms which is a theme discussed below (e.g. Tatto et al., 2012; Pryor et al., 2012). A series of studies in Uganda have drawn attention to the kinds of pedagogies that can support moves to more active learning in early years' mathematics in the context of large classes, and pointed to the need for inclusion of attention to these practices within pre-service teacher preparation (e.g. O'Sullivan, 2006).

However, it must be remembered, that while teachers might be supported to find ways to teach optimally in such contexts of poverty, success in teaching and learning requires simultaneous addressing of the poverty challenges that affect student learning (Hoadley, 2013). Hoadley argued that educational interventions are only part of the solution. They cannot replace the need to provide nutrition, health, safe transport and after-school care.

This points to finding ways to support pre- and in-service teachers in understanding the systems available in their local context that can be drawn on in order to support these basic needs. For example, providing teachers with knowledge of what the department of education and district can be held accountable for, such as local feeding schemes and bringing mobile health services to school. Understanding systems for mobilizing governmental systems to function as required could be important aspects of pre- and in-service teacher education, as these are factors that greatly affect learning within the classroom context.

RECOMMENDATION

There is a need for pre-service teacher education to build awareness of the diverse realities of contexts of disadvantage and provide practical information and resources for working substantively for numeracy development. While in school, this means developing pedagogies that are pragmatic for the realities of large classes and low resource levels; it also means developing networks of access to food, healthcare and well-being agencies in the field.

3.6 In-Service Numeracy Teacher Development and Support

Almost all research in teacher education reviewed here points to the critical need for in-service support and teacher development for early years' numeracy teachers, especially in the context of low-income countries (Balfanz and Byrnes, 2006; Borman and Kimball, 2005; Pryor et al., 2012; Tatto, et al., 2012; UNESCO, 2013). Reasons across these include the fast tracking of teachers in various poverty contexts, which has led to the under-preparation of teachers. Teachers are seldom prepared for teaching in conditions of poverty and courses sometimes focus on the mathematical knowledge required rather than pairing this with how to teach this knowledge in contexts of low-income countries which include high numbers of students and weak language and literacy skills.

Also as Tatto et al. (2012) pointed out in their Teacher Education and Development Study in Mathematics (TEDS-M), higher-income countries can afford to be selective and attract the 'best' and most motivated teachers, while in lower-income countries teaching has low status and teachers often have low morale (OECD, 2008). Tatto et al. (2012) pointed to examples of Botswana and the Philippines as contexts where conditions of teaching are much less favourable. Again, this points to the need for ongoing in-service teacher support in order to support the retention of teachers and to help teachers to cope with teaching under conditions of few resources and large class sizes. Graven's (2004; 2005b, 2012) study of teachers working in disadvantaged contexts in South Africa showed that mathematics teachers who had planned to exit the profession chose to stay when provided the opportunities to learn within a supportive teaching community and to participate in a range of activities offered by professional associations, such as teacher conferences and other educational events.

While NGOs are acknowledged as playing a critically important role in supporting education and for doing excellent in-service work, a key problem is that such work is not easily expanded to scale. For example, Graven (2005a) noted several dilemmas one confronts in the design of mathematics in-service programmes with 'disadvantaged' communities. One key dilemma of

NGO in-service work with teachers is that in order to develop a well-functioning and supportive community, one needs to limit the numbers participating, resulting in a few receiving high-quality knowledge and other resources. Kahn (2000) captured this tension, "dispersed low unit cost intervention may not work, but concentrated high cost intervention may succeed. How then to compare costs?" (p.18). This points to the high cost of intensive regular school-based in-service support that most governments in low-income countries are unlikely to afford. In this respect, the UNESCO Global Monitoring Report (2013) argued that we must find ways for partnerships between governments and NGOs to expand and bring support to scale. While there is agreement on the need for intensive and longitudinal in-service support, suggestions on the form and content of the support to be offered for developing numeracy teaching remain relatively diverse. Some studies point towards more structured instructional programmes accompanied by sustained intensive support (for example, in USA contexts of poverty see Balfanz & Byrnes [2006]). Alternative approaches cluster around the need to develop supportive in-service teacher networks and communities that are argued to be more capable of becoming self-sustaining beyond the timelines of more formal intervention.

Several studies point to the importance of forming long-term in-service communities of practice or professional learning communities for enabling mathematics teacher learning that addresses and shifts knowledge, practices and identities (Brodie, 2014; Graven, 2004; Pausigere & Graven, 2013). These models cite problems with the more prevalent 'cascade' type models of training that have been widely acknowledged to be problematic and result in the watering down and misinterpretation of key concepts and curriculum aspects (e.g. Chisholm et al., 2000). In the South African context for example, the establishment of professional learning communities (PLCs) is strongly promoted in the Integrated Strategic Planning Framework for Teacher Education and Development in South Africa by the Department of Basic Education (DBE; 2011) Pausigere and Graven (2013) argued that we need to find ways to increase the size of in-service professional communities while maintaining the opportunities for teachers to develop strong supportive relationships and opportunities of full participation. The optimal size for balancing the tension between small and large-scale professional communities is yet to be found and will likely depend on the nature of the teacher interventions and in-service support being offered.

RECOMMENDATION

In-service development needs to provide longitudinal rather than once-off or piecemeal support for teachers involved in development of numeracy. Longitudinal models, whether these take the more prescriptive 'top-down' or the more 'bottom-up' organic forms, are viewed as more self-sustaining for long-term improvements in learning outcomes.

3.7 Partnerships with Parents and Communities

Several studies point to the critical importance of parents and families for developing numeracy and literacy skills in the home (e.g. Goos, 2004). The argument emanating from these findings is that in-service programmes must support teachers in developing these partnerships with families and communities, and especially for young learners. This is particularly the case for supporting numeracy learning of minority groups. For example school-home-community links were identified as an important factor in supporting indigenous learner numeracy performance (Frigo et al., 2004). In the Kenyan context Ngware et al. (2010) proposed that: “more parental and teacher/head teacher interactions should be encouraged, including parental involvement in what happens in the classroom” (p. 4). In the publication South African Child Gauge, Hoadley (2013) argued that since family backgrounds remain the most powerful influence on how well children do in school it is essential that they are supported in ways that provide nutrition, safe school transport and after-school care. Similarly, Graven and Stott (2014) stated that working with families is a critical element in addressing the challenges of numeracy education and getting children to learn more

numeracy in their home contexts through simple games (such as dice and cards), so that homes become second sites of learning that support the consolidation of basic numeracy concepts. Teacher education must therefore support teachers to partner with parents in order to address the challenges of poverty that affect what is possible in schools.

Goos (2004) highlighted the following critical issues in relation to educating teachers for parental involvement: The need to forge parental and community involvement in mathematics education; recognize social and cultural differences and relations of power; respond to cultural diversity in numeracy practices; support administrators and teachers to work with parents; enhance communication between teachers and parents; and, connect home school support. She noted that

effective partnerships were characterized by: a long history of building relationships often involving the whole school, whole family, or whole community...a holistic focus that does not necessarily have numeracy as its first priority; a specific focus on localized needs and contexts; leadership and liaison by one or two 'champions' (p.20).

RECOMMENDATION

Teacher education needs to prepare teachers for models and modes of working inclusively with parents and communities for numeracy development to be effectively built and sustained. The literature points to the building of relationships that support the learning of the child through supporting the participation and learning of the family and community of which the child is a part.

LIST OF RECOMMENDATIONS



RECOMMENDATIONS ON TEACHING TO LEARN NUMERACY

IMPORTANCE OF EARLY YEARS FOR NUMERACY DEVELOPMENT

Early years experiences are critical for developing numeracy with young children. In particular, quality pre-school experiences assist the development of numeracy in the early years, lay good foundation for future development and help narrow the gap between students from low-income backgrounds and the rest of the population. Educational planning for developing numeracy in the context of low-income countries should involve the provision of quality pre-school education particularly for the most disadvantaged students.

FROM A FOCUS ON BACKGROUND TO A FOCUS ON FOREGROUND

While it is crucial for effective teaching to take into consideration the background of the child, deficit understanding and blaming the background for low participation and achievement are not helpful. The possibilities of what the child can become should be paramount in designing teaching experiences and in interacting with the child. In particular, high expectations should be the basis for working with children from low-income countries and they should experience teaching that allows them to develop, not only high-level numeracy but their agency and learning independence.

A HOLISTIC APPROACH FOR NUMERACY EDUCATION

Curriculum design should avoid overloading the early years with content and outcomes, thus allowing children to develop at their own pace. Productive efforts to develop numeracy should include contributions from a whole community, with successful practices based on whole-school and integrated approaches commencing with national policy and down to school policy and practice. At policy and practice levels, these efforts take the whole-child approach catering for all

the needs of children, with numeracy development forming one important aspect.

PEDAGOGY OF PARTICIPATION

Classroom numeracy teaching should be based on more participatory practices in which the students are active in discussions, explorations and problem solving in small groups. Classroom activities should provide students with opportunities to develop their ability to discuss and explain their thoughts, which develop their numeracy.

BUILDING ON CHILDREN'S LIFE EXPERIENCES AND CONTEXT

For developing numeracy in the early years, children should be assisted to identify numeracy thinking within their experienced or imagined world, including counting things in their environment, retelling of stories, songs, riddles and games and identifying patterns in the environment.

ACCESS TO AND USE OF RESOURCES

The availability of key resources for early numeracy is helpful especially in kindergarten years. However, it is crucial that teachers using such material incorporate a focus on the ideas and the progressions that these resources are intended to develop. A variety of everyday resources not only reduce the cost of making resources available in low-income countries' classrooms but also support establishing connections between school numeracy and the everyday life of the student.

TEACHING CHILDREN FROM HIGH POVERTY BACKGROUNDS

Children from high-poverty backgrounds often show a lag in their cognitive development and limited experiences needed to build numeracy in school. Such students need special understanding, empathy and support in early years teaching to bridge the gap in their numeracy development. Programmes developed to deal with children living in poverty conditions should be based on teachers' knowledge of the out-

of-school experiences of these students and provide experiences that are necessary to develop cognitive and numeracy skills expected of all students.

TEACHING CHILDREN FROM INDIGENOUS BACKGROUNDS

Educational programmes involving indigenous students must be sensitive to their context and aspirations and should aim to develop both the cultural and mathematical identities of the children and their ability to participate mathematically in society. This can only be done in collaboration with the local communities.

TEACHING CHILDREN FROM NON-DOMINANT LANGUAGE BACKGROUNDS

Teachers need to use language as a resource for enabling access to mathematical engagement and ideas. Teachers need to maximize this resource in their local context. In particular, code switching may assist in the developing of understanding by young learners and should be allowed by government policy.

SPECIAL POLICIES ON EDUCATION OF GIRLS

Government and local school policy towards developing early years' numeracy should include provisions to monitor girls' access to quality educational opportunity and outcomes counteracting any stereotypes of careers and subjects as essentially preferred or needed by either sex. Working with families and communities is essential towards raising expectations of girls of developing numeracy.

RECOMMENDATIONS ON LEARNING TO TEACH NUMERACY

THE CRUCIAL ROLE OF THE TEACHER

The most effective means to develop numeracy with young children are quality teachers. An education system cannot exceed the quality of its teachers. While quality teaching benefits all students, evidence shows that it benefits low-achieving students more, thus it contributes to narrowing the gap between different groups of students. Attracting and retaining well-qualified teachers should be a high priority for developing numeracy in the early years. Teachers should be supported to adopt effective pedagogies to assist young learners in developing their numeracy.

TEACHERS' CONDITIONS, STATUS AND MORALE

Research positions teaching and teacher education within broader socio-cultural contexts. Low status and morale of teachers in contexts of low-income countries lead to difficulties with recruitment and reten-

tion. In such contexts, attention to models of teacher education that address the development of teacher identities attuned to ongoing mathematical learning, and supporting the improvement of teacher morale and commitment to the profession are noted as particularly important. Incentives should be provided to encourage teachers to take up posts in remote and particularly challenging contexts in order to ensure that sufficient numbers of well-qualified teachers are deployed to these areas. Additionally supporting low-income countries to develop effective systems for deployment of teachers to reach all areas is needed.

TEACHER KNOWLEDGE IN/FOR NUMERACY

Supporting the development of the content knowledge base of primary teachers remains an urgent priority according to the literature. In the context of early numeracy, it is important to note that findings suggest that it is not simply 'more' or 'higher level' content that is required. Numeracy teacher education must provide opportunities for teachers to develop connected understandings of early numeracy content and problem solving, knowledge of the progression of numeracy learning development, syntactic knowledge related to problem solving, and pedagogic content knowledge and skills related to early numeracy teaching.

UNDERSTANDING NUMERACY PROGRESSION

Developmentally progressive frameworks are useful for developing numeracy and should be a component of teacher development in low-income countries. This represents an area of common ground between the more learner-centred responses and the more prescriptive coverage-oriented responses as both are responding to challenges of poor learner progress in early numeracy.

PRE-SERVICE NUMERACY PREPARATION

There is a need for pre-service teacher education to build awareness of the diverse realities of contexts of disadvantage and provide practical information and resources for working substantively for numeracy development. While in school, this means developing pedagogies that are pragmatic for the realities of large classes and low resource levels; it also means developing networks of access to food, healthcare and well-being agencies in the field.

IN-SERVICE NUMERACY

TEACHER DEVELOPMENT AND SUPPORT

In-service development needs to provide longitudinal rather than once-off or piecemeal support for teachers involved in development of numeracy. Longitudinal models, whether these take the more

prescriptive 'top-down' or the more 'bottom-up' organic forms, are viewed as more self-sustaining for long-term improvements in learning outcomes.

PARTNERSHIPS WITH PARENTS AND COMMUNITIES

Teacher education needs to prepare teachers for models and modes of working inclusively with

parents and communities for numeracy development to be effectively built and sustained. The literature points to the building of relationships that support the learning of the child through supporting the participation and learning of the family and community of which the child is a part.

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Published by
Deutsche Gesellschaft für
Internationale Zusammenarbeit (GIZ) GmbH

Registered offices
Bonn and Eschborn,
Germany

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Design and Layout
Diamond media GmbH, Neunkirchen-Seelscheid

Printed by
druckriegel GmbH, Frankfurt/Main, Germany
Printed on FSC-certified paper

Photo credits
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As at
November 2014

GIZ is responsible for the content of this publication.

On behalf of
Federal Ministry for Economic Cooperation and Development (BMZ)
Division of Education and the Digital World

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