"Archaeology" of Measurement Knowledge: Implications for School Mathematics Learning

Arindam Bose and K. Subramaniam
Homi Bhabha Centre for Science Education, TIFR, Mumbai, INDIA

This paper explores measurement knowledge that middle-graders from low-income families gain from out-of-school contexts and the implications of such knowledge for classroom learning. Work and other out-of-school contexts entail rich and diverse "funds of knowledge" about measurement. Such knowledge includes conceptual elements which may be fragmented or hidden, but if unpacked (archaeology) can support classroom learning. The out-of-school measurement-related experiences have been analyzed to show the underlying conceptual constructions and their diversity in terms of measures, systems of units, and measurement tools. The paper discusses possible connections between classroom learning and specific aspects of out-of-school measurement knowledge using a characterization that marks such connection.

In the diverse contexts that comprise everyday living, the notion of measurement occurs frequently. It is used in diverse ways in workplaces, in economic exchange, and in homes. The topic of measurement is also a compulsory part of the school curriculum. Much of the literature on out-of-school measurement knowledge has explored the contours of measurement knowledge—diverse measurement tools, modes and units, ways such knowledge is acquired, and its difference from school mathematics. While such studies communicate a promise of reshaping school maths education based on what was known about out-of-school knowledge, there is still a lack of clarity about the implications of such studies for school learning. This paper unpacks the diversity of measurement knowledge embedded in work and other out-of-school contexts and also possessed by school children from an economically active urban low-income neighbourhood, dotted with a micro-enterprise economy in a South Asian developing world context. Many students in our study either participate in, or are aware of, the work-contexts where measurement plays a role. Our purpose is to inquire into the implications of such knowledge for school learning.

Archaeology of Embedded Mathematics

We have argued that many measurement tools used in everyday contexts, such as measuring scales and templates, have embedded in them mathematical ideas and elements which remain hidden even from those who frequently use such objects. Surprisingly, the mathematics textbooks or curricula do not require students to explicitly uncover the hidden or embedded mathematics from such objects. Such uncovering or unpacking of the underlying conceptual constructions is what we refer to as "archaeology" of embedded mathematics (Subramaniam, 2012). To begin with, unpacking of the embedded mathematics in measuring scales can be the starting point of an "archaeological" exploration, that can lead to learning about length measurement and its uses such as the notions of construction of new or sub-units, chunking, equi-partitioning, iterative covering, etc. Such archaeology can have an important role in supporting the mathematical learning of students who gather, as evidenced from our study, fragmented and obscure mathematical knowledge from their work-contexts. We discuss these connexions below.

Funds of Knowledge

Children in low-income conglomerations are often bound in social relationships and work practices from an early age and the broad features of their learning develop in their homes as well as in their surroundings. Households and surroundings contain resources of knowledge and cultural insights that anthropologists have termed as *funds of knowledge* (González, Moll, & Amanti, 2005). The "funds of knowledge" perspective brings to mathematics education research insights that are related to, but different from, the perspectives embedded in studies of "culture and mathematics". In contrast to restrictive and sometimes reified notions of "culture", the concept of "funds of knowledge" emphasizes the hybridity of cultures and the notion of "practice" as "what people do and what they say about what they do" (*ibid*, p. 40). *Funds of knowledge* are acknowledged to be broad and diverse, and embedded in networks of relationship. When they are not readily available within households, then they are drawn from community networks. This concept emphasises social inter-dependence. From this perspective, children are active participants, not passive by-standers.

We have used "funds of knowledge" as a guiding notion in analysing the work contexts that students are exposed to, and in illuminating the nature and extent of everyday mathematical knowledge available within the community of the classroom. We look at "funds of knowledge" as a resource pool that emerges from people's life experiences and is available to the members of the group, which could be households, communities, or neighbourhoods. In a situation where people frequently change jobs and look for better wages and possibilities, members of the household need to possess a wide range of complex knowledge and skills to cope and adapt with the changing circumstances and work contexts. Such a knowledge base becomes necessary to avoid reliance and dependence on experts or specialists, particularly in jobs that require maintenance of machines and equipment.

Children's participation in work, either within the household or in the neighbourhood, allows a closer integration with the social networks that generate funds of knowledge, and makes this knowledge present and available in the classroom. Educational philosophers, such as Gandhi, thought of productive work as central to education, and developed a vision of education centred around work. At an education conference in India in 1937, he argued that "the proposition of imparting the whole of education through the medium of trades (crafts) was not considered [in earlier days]. A trade (craft) was taught only from the standpoint of a trade (craft). We aim at *developing the intellect also with the aid of a trade or a handicraft* ... we may ... educate the children entirely through them" (National Council of Educational Research and Training (NCERT), 2007, p. 4, italics in original). In the present Indian context, this perspective has had an influence on the new National Curriculum Framework (NCF) (2005) which urges educators to draw on work experiences as a resource for learning. It points out that "productive work can become an effective pedagogic medium by connecting with life experiences of children; by allowing children from marginalised sections of society, having knowledge and skills related to work, to gain a definite edge; and by facilitating a growing appreciation of cumulative human experience, knowledge and theories by building rationally upon the contextual experiences" (NCERT, 2005, p. 6).

Measurement in the Out-of-School Context

Previous research on measurement within work-contexts or in other everyday settings was carried out alongside or within the research on out-of-school mathematics, with a particular focus on the alternative ways of thinking in different everyday contexts. Such research provided evidence of how mathematical ideas were developed and framed within work-contexts. These studies have highlighted the use of different measurement modes and units (e.g., Lave's study (1985) with Liberian tailors); mental estimation techniques markedly different from those learnt in school; extensive use of conventional mathematical concepts like congruence, symmetry, proportional reasoning, optimisation, and use of spatial visualisation (Millroy's study (1992) with South African carpenters in their everyday woodworking activities); multiplicative thinking in everyday work-contexts using proportions and inversion techniques, and use of scale-drawings which drew on measurement knowledge and proportional reasoning (study of Nunes, Schliemann, and Carraher (1993) with construction foremen); use of spatial visualisation, estimation skills and indigenous tools in Mukhopadhyay's work (2013) on "vernacular boat-making" in India. Saraswathi's study (1989) on agricultural labourers' measurement practices reported use of variety of measurement modes and units to describe the linear dimensions of routine objects used in everyday contexts. The units were standard (old British, metric) and non-standard (body parts, indigenous units). Linear dimensions often served as an object's identity and description. Estimation skills depended more on experience and mental measurement.

Most of the above studies have focused on participants' measurement knowledge in their singular work-contexts. We have not come across studies that looked at the varied contexts in the everyday settings that students from low socio-economic backgrounds are exposed to and the affordances of these settings for school learning about measurement. The implications of the above studies have led to a cumulative understanding of the skills, procedures, and strategies based on mathematical principles that are acquired in out-of-school work contexts. In this paper, we take a broader view of not only what our participants know or can do, but also what they have observed and are familiar with, even if the mathematical knowledge associated with these aspects is partial and fragmented. Our perspective is to explore what aspects can serve as starting points or building blocks for mathematical exploration in the classroom and unpacking the underlying mathematical concepts embedded in measurement practices. We are also interested in how mathematical learning can strengthen the understanding of measurement practices in real world contexts.

Measurement Learning as a School Curriculum Topic

Research on the teaching and learning of measurement as a school curriculum topic has been influenced greatly by the work of Piaget. Measurement refers to the quantification of an attribute of interest for purposes of comparison and for using in a calculation. Piaget stressed the key notions of conservation, transitivity, equi-partitioning, displacement, and iterative covering as underlying length measurement. Subsequent research has added the notion of accumulation of distance and additivity and the role of the origin on scales (Sarama & Clements, 2009). These ideas have also been extended to the learning of area and volume measurement. A look at textbooks prescribed by the central and state governments (followed by the vast majority of students in India) reveals that the dominant emphasis is on acquiring measurement skills and on knowledge of the international system of units for measurement (e.g., Maharashtra state mathematics textbooks for Grades 5, 6, 7 (Maharashtra Textbook Bureau, 2006). Conceptual issues are dealt with briefly under the rubrics of "use of non-standard units" and "need for standard units", before the treatment moves over wholly to the development of skills. These include familiarity with common measurement instruments, use of standard measurement procedures, interconverting between smaller and larger international units, and computing with units. Classroom teaching in the schools that formed part of the study revealed that there is even greater emphasis on paper and pencil computation skills with very little treatment of either conceptual matters or even of practical measurement.

We note that the curriculum and research agenda also need to include concepts that connect with and illuminate the diversity of measurement-related practices encountered in work and everyday contexts. It needs to focus on the idea that quantification is at the heart of measurement and quantification is achieved in different ways for different attributes and for different purposes. It needs to develop an appreciation of the difference between scientific measurement and measurement in the everyday world. This paper, therefore, argues for the inclusion of conceptual aspects that have so far not been included either in the curriculum or in the research on measurement learning. We argue that the diversity of measurement experiences in work contexts and everyday settings justifies inclusion of these aspects in the curriculum and that the knowledge that children bring into the classroom from out-of-school contexts supports learning of these ideas.

The Study

The large ethnographic study was conducted in a low-income neighbourhood in central Mumbai that has as vibrant economy household based micro-enterprises and small scale manufacturing units, which provide employment to the dense population living in the locality Even within a single class, we find students engaged in a variety of income-generating work both within house-holds and in the neighbourhood. Some common micro-enterprises that students participate in are embroidery, *zari* (needle work & sequin stitching), stitching and garment-making, making plastic bags, leather goods (bags, wallets, purses, shoes), dyeing, button-stitching, making of *rakhi* (decorative wrist bands) and stone-fixing work on ornaments. Recycling work is also a major occupation in this locality. Being an old and established settlement, it receives immigrants from different parts of India, mostly unskilled workers who find jobs in the workshops and some of them become apprentices in the small factories.

The study done over two and a half years time, which forms the setting for this paper, was conducted in several phases. Beginning with the ethnographic exploration and classroom observation of Grades 5 and 6 of two municipal corporation-run schools, the researcher did informal discussions with the students to understand the nature and extent of their everyday mathematical knowledge. It helped in knowing about the opportunities available to gather such knowledge and the extent of their involvement in economic activities. In the next phase, data was collected through semi-structured interviews of a representative sample of 31 students (one-third of the two Grade 6 classes) to understand their family-background, socio-economic status, parental occupations, productive work done at home/elsewhere, and student's involvement in them. The interview included questions aimed at understanding students' basic arithmetical knowledge. In the third phase, a sub-sample of 10 students and an additional 7 students from the same grade who volunteered, were interviewed to obtain a detailed understanding of their work-context knowledge. Interviews were transcribed and transcripts were coded at first and second levels to review what they indicated about the nature of work students are involved in, and what they know about aspects of the work. Students have been designated with the letter "E" or "U" (for English and Urdu medium school respectively) followed by a numerical subscript. The data used for this paper is drawn from the interviews for measurement aspects and from other phases of the study including informal visits to the house-holds, manufacturing units and discussions held with adults in these locations.

Characterising Out-of-School Measurement Experience

Features and nature of students' involvement in work practices shape what the contexts demand of the students and the richness of the knowledge that they acquire. Diversity of out-of-school settings gives rise to diverse experiences of measurement. A characterisation of such diverse knowledge is presented below from the point of view of portraying the inherent richness of concepts implicated in such experiences.

Comparison and Estimation in Measurement

Measurement in everyday contexts including work and domestic settings is different from measurement in the scientific world. Precision and accuracy are not as important as convenience. In many situations approximate measurements suffice. However, many of the processes and concepts that underlie measurement in the everyday world are centrally relevant to a conceptual understanding of measurement. Everyday measurement contexts present diverse and extensive use of comparison and estimation, and varied processes of quantification. Templates for length measurement are often used in tailoring and leatherwork. Tailoring work begins by cutting "*futta*" - stiff fabric or a canvas cut as per the dimension specifications of the garment to be stitched and made into a template called "*farma*". *Farma* of shirt-collars, pockets, of wallets and purses are commonly used. Comparison is done following a *farma* and its design and specifications for making new products. Wallet making often involves cutting square shaped leather pieces of dimension 4" × 4" referred to as "*desi*" often cut from a rexin piece of size 33" × 39". A "*desi*" is a template and also used as a measuring unit. Although "*desi*" is an area measure, it is used as a discrete length unit and often leather pieces are measured in terms of number of *desi*. For example, "*nau desi se ek foot banta hai*" [nine desis make a foot]" implying 9 desis cover and are equal to a square foot. What may seem improper or ambiguous use of measurement units is commonly used and understood in the community, possibly from the context.

In everyday contexts, estimation is a common measurement mode used with continuous as well as discrete attributes. Children like adult workers learn different kinds of estimation skills based on their work requirement. Work-contexts like *zari* (decorative sequin stitching on garments) entail frequent use of estimation in choosing the quantity of sequins to be stitched in a marked area or a specified design laid out on a garment-part. Similarly, in leather and tailoring work, estimation skill is used while deciding the amount of adhesive to be used or while choosing the needle of a certain grade (called number) and amount and types of threads for stitching. "*Chindhi*" (garment recycling) work uses both estimation and visual comparison skills while sorting. Cloth pieces of similar size are sorted and collected together and the weight of the collection is estimated. Other work like textile printing requires estimating the lengths of cloth pieces on which block printing is done and choosing a suitable "stopper" (i.e., printing block) whose dimensions are known to the workers. During the interview with U23 (engaged in textile printing work), he gave detailed explanation about the estimation of the quantity of colour required in printing designs on cloth-pieces of different dimensions. For example, he said in simple designs, one kg colour is sufficient to print the design on 2000 small cloth-pieces. The researcher observed that some students like U23 had a strong estimation sense and were skilled in estimating the dimensions of different objects lying around. Estimates of quality are also a part of some work contexts, although these are rarely quantified. An exception is the practice of "grading" in plastic recycling work in which visual estimation and tactile senses are used to designate numbers to plastic wastes based on their quality.

Quantification and Construction of Measurement Units

All measurement depends on the use of measurement units. In school learning, children largely encounter standard units that are pre-given in the form of measuring instruments (tapes, weights, etc.). The choice of a unit and the construction of a convenient unit are the first steps towards quantification of an attribute, and are important aspects of the concept of measurement. In the classroom, these steps are rarely emphasised. In many classrooms, they may at best be explained verbally. However, there are several out-of-school contexts where children encounter construction of a unit and other abstract notions embedded in measurement processes.

Use of Body Parts in Measurement

The use of the body for purposes of length measurement using hand-spans, finger bands or finger widths is commonly practised. In tailoring, "finger band" (phalanx) and "finger width" are commonly used to estimate length and length intervals. E6 who regularly visits his father's button-stitching workshop and also manages its running at times, mentioned the use of finger bands to quantify and measure the distance between every two buttons – about four-seven fingers width distance is maintained between them. E6 knew that one “inch” is roughly equal to one “finger band” length.

Equi-partitioning of Units

Construction of sub-units from bigger units by equi-partitioning is a common feature in work-contexts, for example, convenient weight “templates” for small weights (50, 100 or 250g). Construction of convenient units or templates derived from standard units is a conceptually rich activity, since it may involve partitioning, combining or otherwise manipulating a given standard measure. It is a step beyond using ready made measuring instruments that are pre-encoded with standard units, in the direction of understanding measurement conceptually rather than learning it merely as a skill.

Iteration and Discrete Quantification

Most students were familiar with artefacts like measuring tapes and their iterative use in quantifying a length measure. Some were also familiar with folding of rope to make smaller lengths using equi-partitioning. Students also knew about templates (*farma*) and their use in the iterative covering of an area, for example for carving out smaller pieces of rexin from a bigger piece and to quantify it. Similarly, discrete quantification is also common in work-contexts, viz., the garment-sizes marked with a letter or a number. Although most adults and many children are familiar with these sizes, whether and how these numbers are obtained through measurement is not clear to most people. Students in our study interpreted these numbers as unrelated to any units like inch or centimetre, and as merely indicating increasing sizes. Only some tailors were aware that this indicates the person’s chest measurement (not chest measurement of the garment, which is larger) in inches. Here we have an instance of a measure familiar from experience, but whose origin in quantification is obscure.

Diversity of Objects, Measurement Instruments, and Units

Students are familiar with and handle diverse objects in a range of contexts with a variety of measurement units and tools. Length as a salient attribute of an object is measured in unary as well as in multiple dimensions. As a unary dimension, length may refer to length or distance, for example, length of a strap sewn on a bag, distance between two buttons, or the depth of a pouch or a bag. Sometimes area measures are indicated by specifying two length dimensions, as for example, when a rectangular textile printing frame is indicated by specifying the length of its sides (16”×12”) or when different sizes of rectangular plastic packets are given by their dimensions “*satrah paanch*” (seventeen by five), “*pandrah dus*” (fifteen by ten). Here the underlying connection between length of sides and area of a rectangle is implicit.

> Volume is commonly measured using both standard and informal units. Volume measures are often interchangeably used with weight measures. The word “kilo” commonly means “kilogram” and is a unit of weight. However “kilo” is often used as a synonym for “litre”, a unit of volume. For example, E_8 and U_{23} referred to kilos of milk and colour used in everyday shopping and textile printing work respectively, although they actually meant “litres”. Another common practice is to measure some quantities by volume instead of weight; for instance, shops sell *mutthi* or fistful of tea powder and grocery items. *Mutthi* is also a unit used in measuring sequins for *zari* work apart from other weight measures.

In micro and small manufacturing units and in everyday contexts, students encounter a diversity of measuring instruments in the work-contexts. Weight measurement, for example, is done with the help of spring balances, two-pan balances of various designs, beam balances, electronic single pan balances, and platform weighing machines for large weights. Besides the use of tapes marked in both inches and centimeters for length measurement, shops and workplaces use steel rulers, which may contain other kinds of markings.

Shops selling cloth use steel meter scales with usually with markings for every 5 or 10 cms. Steel rulers often contain binary divisions of the inch up to 1/32 of an inch. Volume measures used to measure grain, oil or milk come in a variety of shapes and sizes. Often such volume measuring instruments are not properly calibrated or marked. There are diverse measurement units in practice that are both standard and non-standard, scientific and indigenous units (international, old Indian and British units) and known to the students. In most measurement practices (weight, length or volume), the underlying mathematical constructions remain implicit and disconnected between practices.

Opaque Quantification, Fragmented Knowledge

Students' familiarity with diverse measurement modes and instruments do not necessarily translate into sound knowledge, rather the knowledge remains fragmented and their understanding unclear. Although most students were familiar with the measuring tapes, they were unclear about the meaning and construction of the markings on the tape. In some instances, even if the measurement is fully quantified, the quantification remains opaque, and the measurement remains critically dependent on the integrity of the artefact. For example, as we noticed, some plastic scales that students were using perhaps were not marked with proper calibration. Understanding the construction behind a measuring scale, its meanings, and inter-connections between the different markings was not required. What has become important now in the school curriculum is to learn to use the scale and be able to measure a length. However, in this study we came across students (viz., U23, E6) who did not know the connection between inch and cm but still had a fair estimation of how much distance both signify. Archaeology of concepts can connect such skills for better learning.

In everyday contexts, ways of quantification are diverse. It is important to make sense of the quantified attributes. We argue that by drawing on students' familiarity with the range of objects and attributes that are quantified, students can explore questions such as what is common and what is different in how we quantify different attributes? How is an abstract attribute like monetary (exchange) value quantified? How do we quantify different aspects of labour such as time, effort and expertise? Such questions are important to build a holistic understanding of the measurement concept that students get to handle in different domains of their lives and in different manner.

Implications for Classroom Learning

We argue that archaeological exploration resists the processes of demathematization as well and stresses on the comprehension of the hidden underlying concepts. Such explorations therefore have strong potential to become effective pedagogic modes. Generalised forms of knowledge are neither about abstraction without the concrete content, nor is it about mere induction from a number of instances. Rather, generalisation is all about arriving at or holding an idea or a construct that can illuminate and be applicable in diverse instances. Valuing generalizability as an outcome of school learning in fact places greater importance to the diversity of out-of-school experiences, for such diversity actually creates contexts for school learning. From this standpoint, we understand that mathematical aspects are present in the work-contexts as hybridized and opaque embeddings and it would not be correct to look at such practices as reflecting mathematical thinking and understanding. At the same time, we argue that it would be fallicious to look for elements of school learning in a particular work-context or to expect school mathematics to illuminate such similar practices. We claim that formal mathematical learning can illuminate the diversity of practices as a whole and strengthen the understanding, not the practice.

A second aspect of out-of-school knowledge that makes for potentially powerful connections with school learning is the fact that artefacts and practices from everyday settings represent a sedimented and embodied form of mathematics. The measuring tape embodies the processes of unit construction, unit iteration and counting and partitioning of units into sub-units. These processes are however hidden from view and are opaque. The redundant inclusion of a second system of units in the form of inches and feet on the measuring tape incorporates a part of historical reality, and highlights the arbitrariness of the choice of the basic unit of length. The purpose of such embodiment is precisely to make the mathematical thought and processes behind the construction of the measuring scale unnecessary, and to reduce the practice of measurement to the simple act of reading off the scale. As long as we treat the learning of measurement as merely the learning of a skill, unpacking the mathematical ideas that are embodied in artefacts will remain unnecessary. However, if we view the learning of measurement as conceptual understanding, then such material artefacts present an opportunity for unpacking the mathematical constructions sedimented in them (Subramaniam, 2012). Such "archaeology" may have an important place in providing opportunities to learn powerful mathematics that illuminates the diverse aspects of everyday experience. An approach to the teaching and learning of measurement that aims to connect out-of-school knowledge with school learning will hence need to draw on the implicit as well as the explicit conceptual constructions that underlie measurement experiences in the real world.

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