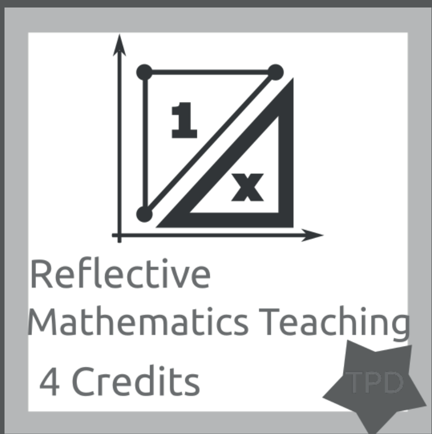
Reflective Teaching with ICT

S02 Reflective Mathematics Teaching
Course book



2017

CONNECTED LEARNING INITIATIVE

Mathematics –
Proportional Reasoning

This Handbook Belongs to

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CLIx Mathematics proportional Reasoning

CLIx MATHEMATICS

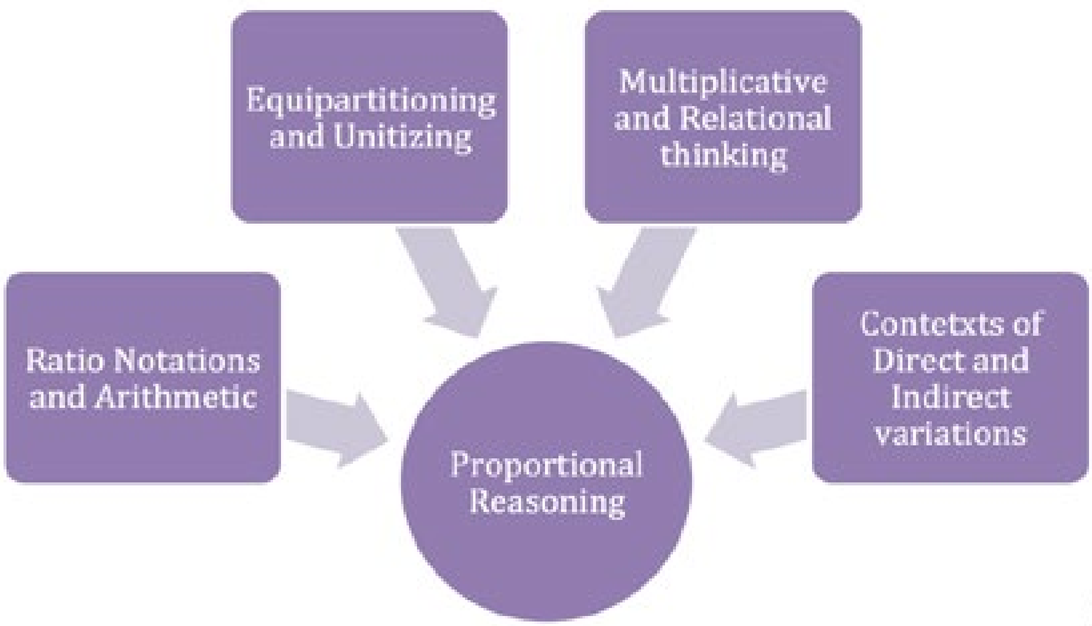
Proportional Reasoning

"Proportional reasoning is, most simply, the ability to compare two things using multiplicative thinking, and then apply this to a new situation."
- National Council of Teachers of Mathematics

Proportional reasoning is one of the crucial mathematical ideas students develop over the school years. It is a good indicator of learners' understanding of rational numbers and related multiplicative concepts, and at the same time, it lays the foundation for more complex concepts of mathematics. Proportional reasoning has been an umbrella term that covers the thinking behind many concepts, from fractions to algebraic representation of ratios, to variations and scaling and to probability. For the purpose of this handbook, proportional reasoning refers to an ability to scale up and down in appropriate situations and to supply justification for relationships that involve direct and inverse variations.

Proportional reasoning applies to almost all areas of the curricula and is considered a critical concept for success in secondary mathematics. However, there are only a few occasions where ideas of ratios and proportions are woven coherently into the curriculum to develop students' ability to reason multiplicatively.

The CLIx mathematics module on proportional reasoning attempts to bridge this curricular gap by using well-thought-out activities that are either hands-on or digitally interactive to take learners through the stages of development of proportional reasoning. These stages are sequenced in four units. This handbook describes how each unit and the lessons in it contribute to the development of proportional reasoning among students. It further discusses exemplars of students' thinking on some activities and elaborates potential mathematical discussion on those activities. These exemplars are given for both digital and hands-on activities, and teachers can use the two types of activities as per their convenience.



Objectives

The Proportional Reasoning (PR) module addresses three aspects of students' learning.

- Moving from additive to multiplicative reasoning: One of the challenges that school education faces is to move students from additive to multiplicative reasoning.
- Equipartitioning and unitising: Understanding the size of the unit and appropriating equal partition is a core skill required to develop relational and multiplicative thinking.
- Responding to situations involving ratios and proportions: Working with situations familiar to students that create a need to look at quantities multiplicatively helps students learn.
- Developing mathematical and contextual sense of direct and inverse proportions: Students make sense of the constant of proportionality to understand and navigate direct and inverse proportional situations.
- Applying proportional thinking: Using their understanding of ratio notation, students solve problems of probability and other mathematical contexts.

Organisation of Proportional Reasoning Module

	Unit Name	Description	Digital Resources
Unit 1	*Additive to Multiplicative Thinking*	This unit aims to respond to typical student thinking about relational situations and to bring out the conflict rising from their familiarity with additive situation. Often, due to overexposure to additive situations, students decode all situations additively. In this unit, students get exposure to situations where additive thinking would lead to unfair sharing of food packets. Students learn about ideas such as equipartitioning, where they figure out the appropriate number of cuts or unit sizes for food packets so that each worker receives the same amount of food. When students distribute shares to groups instead of individuals, they have to think multiplicatively about the groups' share.	The digital resources in this unit are tools for equal sharing. The cutting tool encourages students to think about the number of parts to be made of a whole, and the grouping tool helps them to adapt grouping strategies. The tools also provide opportunities to connect the visual figures of fractions to their numerical form. The choice of contexts, representations and numbers is made to guide students to the conclusion that additive thinking doesn't work in all situations and that some situations are proportional or multiplicative in nature.
Unit 2	*Multiplicative Thinking*	In this unit, students work on multiplicative situations that are geometric in nature. They work on pattern tasks where they scale the patterns keeping the visual appearance of the pattern the same. The unit exposes students to the idea of the scaling factor in two-dimensional contexts. They enlarge or shrink rectangular patterns and figure out the numerical change in the pattern.	The digital resource in this unit allows students to work on scaling and shrinking patterns so that the visual appearance of the pattern remains the same. The reasoning involved in these exercises is multilayered as students have to pay attention to the number of dots in the pattern as well as the pattern as a whole. The digital interface for adding rows and columns and the palette for dots facilitate such reasoning.

Unit 2	*Ratio Notation*	This unit builds on students' understanding in earlier units and connects it with the formal notation of ratios. The notation of ratios uses a similar format as the notation of fractions and therefore, it becomes all the more essential to work on symbolic manipulation of ratio notation to distinguish its arithmetic from the arithmetic of fractions.	In this unit, students work on a digital task that involves the understanding of volume. The aim is to let students construct the idea of constant of proportionality. The digital interface allows students to help a character cool her drink by adding ice cubes of different sizes. Again, as this task involves multilayered reasoning, the digital interface with different sizes of cubes and the flexibility of changing the volume of the liquid in the glass help students to do complicated reasoning.
Unit 2	*Applications*	This unit acts as a buffer unit and is optional. The main focus is on applications of proportional reasoning in other domains of mathematics such linear equations, probability, compound ratios and variations	There is no digital resource in this unit. However, the handbook provides some strategies for classroom discussion and extra activities.

Making Use of this Handbook

The handbook provides a brief overview of each lesson in all the four units of the Proportional Reasoning module. Each section in the handbook discusses one lesson from the module and a couple of examples from that lesson are discussed in detail. Various student strategies are discussed in relation to those examples. For the hands-on lessons, teachers should go through the handout chapters well in advance and prepare materials and pedagogical strategies based on the discussion of anticipated student thinking.

UNIT 1: ADDITIVE TO MULTIPLICATIVE THINKING
Lesson 1: Jamuni Learns to Share

UNIT OVERVIEW

Proportional reasoning is one of the best indicators that a student has attained understanding of rational numbers and related multiplicative concepts. At the same time, it lays the foundation for more complex concepts of mathematics. The process of multiplicative thinking is associated with situations that involve fair sharing, scaling, shrinking, duplicating and exponentiating. Beginning by strengthening students' ideas of equipartitioning and fair shares, this unit addresses the multiplicative thinking involved in situations that require knowledge of fractions. Simultaneously, the choice of examples brings forward the conflict between additive and multiplicative thinking. This unit, therefore, forms an essential foundation for developing any form of multiplicative thinking.

LESSON OVERVIEW

In this first lesson of the module, students will work on the idea of fair sharing through equipartitioning by using an area model. The digital unit involves students helping the character Jamuni to distribute food packets to workers such that everyone gets an equal amount of food. The food packets, circular or rectangular in shape, generate opportunities for students to cut equal sized portions. In a visual sense, students learn to divide the areas of the given food packets in equal shares. Further, they learn to identify situations that represent equal share. This unit exposes students to certain situations where, if they reasoned additively, they would arrive at an incorrect share. This exposure is an important milestone in moving from additive to proportional reasoning.

LEARNING OBJECTIVES

Students learn to

- Differentiate additive and multiplicative relationships between the quantities under consideration by comparing them across groups and within groups.
- Identify the relationships (multiplicative) among quantities that are relevant to decisions about the share of each person in a group.
- Compare ratios in two different sharing situations to identify the larger ratio by relating it to the share of one member of a particular group.
- Make decisions, and give reasons for those decisions, about increasing or decreasing a quantity based on mathematical relationships among quantities.

UNIT BREAK-UP

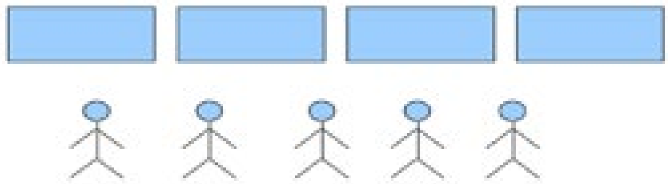
	Lesson name	Lesson Type	Lesson description
1	Jamuni Learns to Share	Digital Lesson	Students understand the concept of fair and equal distribution in this lesson. They learn to use the **Cutting Tool** to ascertain how a cake can be cut into different parts and be distributed in different ways to the same number of people. The problems require students to relate the cut parts with the fraction representation and to identify the composite share of one person by adding the parts given to that person.
2	Let Us Compare and Distribute	Digital Lesson	Students learn to distribute parathas among groups by using the **Grouping Tool**. They compare ratios of food received per person using their knowledge of fractions. In this, students reason about quantities and the relationship between quantities to achieve equal ratio in all food sharing situations.

3	Make Equal Shares Across Groups	Digital Lesson	Students learn to find values in equal proportion situations. They share food among not individuals but among groups of people. This task creates another opportunity for students to reason multiplicatively

DISCUSSION ON MATHEMATICAL IDEAS

In this section, we discuss a few examples from the lesson and talk about students' thinking about those examples. We provide prompts for teachers to build a discussion around these tasks and, at the same time, understand the mathematical trajectory of the lesson.

Tasks and their mathematical relevance	Students' thinking and possible strategies for facilitating
Activity 1: The aim of this activity is to introduce students to the *Cutting Tool* and help them make sense of fractional quantities. Here students cut three cakes fairly and distribute them among four children. Certain terms, such as *share or fair share*, are also used in colloquial language and therefore require special attention. Fair share in common usage means allocating more to a needier person. However, in the mathematical sense, *fair* means giving exactly the same to everyone. The mathematical sense will be used here. Learning to *fracture or equipartition* is a core skill that provides a basis for multiplicative thinking, and that is why it is the focus here. Therefore, bring to student's notice the size of the cake when they make equal parts.	Students' are familiar with sharing in the real world, and therefore, this activity provides the opportunity to merge their ways of *doing* mathematics with formal mathematical practices. Some of the expected students responses are given below with suggested pedagogical actions. **a.** *Sharing by half or quarter*: We tend to make halves or quarters of anything when it comes to sharing. Students will do the same. Opportunity to ask: How many total halves or quarters have they got? How many halves or quarters will each child get after fair sharing? Is making halves or quarters always a good strategy? In which situation would making halves and quarters not work? What happens to the share of each person when you make smaller or bigger pieces of the cake? **b.** *Unequal Sharing*: Students might begin sharing the whole with each child and end up giving unequal pieces. In such cases, the teacher could discuss what it is that the *cutting tool* can allow them to do. Discuss here how a smaller unit size leads to a greater number of parts of a whole.
Activity 2: The aim of this activity is to introduce students to the Grouping Tool. The task is to let student visualise the share and then validate their answer by actually grouping and finding the share.	*Equipartitioning* in everyday life is accompanied by the idea of grouping. As adults, we tend to find the price of a half or a quarter of a product before finding the price for one unit. We do not always find the price of one, rather try to find price of a chunk. For ecample, if we know the price of 12 eggs, to find the price of 3 eggs, we generally first calculate for 6 eggs (half of it) and then halve that value to find the value of 3. However, the textbook approach, always suggest to find the price of one and then find the price of many. We base this task on such grouping skills. The teacher can facilitate discussion about maintaining equivalence. When people are formed into two groups—that is, the total is halved—the cake required for each group would be half of the entire group

Activity 3: As students have used the Cutting and Grouping tools in the earlier activities, this activity goes one step further where students cut the cake in multiple ways and learn to add the share of one individual.	The process here is similar to Activity 1 with sharing as not a simple fraction. However, the numbers chosen allows possible distribution of the cakes, and most importantly, none of it can be obtained by sharing halves or quarters. This is purposely designed in contrast with Activity 1. If needed, the teacher could draw the following diagram, and discuss different partitioning possible—Unit size , Unit size . 
Activity 4: This activity lets students use the idea of ratio implicitly. They are prompted to make groups of people and then distribute the given number of cakes among the groups and the individuals in groups. Without being numerically explicit, students are working on the idea of that is equivalent to .	The core of this activity is making unequal groups and understanding the respective number of cakes required in each group so that the share of each child remains the same. The common error would be to distribute the cakes equally to the two groups without regard to the number of people in each group. The teacher can highlight the different group sizes and therefore the different shares of cakes. Another common concern that is targeted here is that students might notice the difference between group sizes of 4 and attempt additive reasoning to decide the shares. One possible wrong answer is 4 cakes and 8 cakes for 2 and 6 children, respectively. The right way is to find the multiplicative relation between group sizes, that the group of 6 is 3 times larger than the group of 2, and use that to find the answer for shares (3 cakes and 9
Activity 5: As students are familiar with all the tools from the digital source, this activity focuses on allowing them to design multiplicative structures of their own.	This activity is an assessment activity, with space for creativity. It contains elements of each earlier activity. Students might propose complicated sharing, such as 9 cakes among 10 children, and this activity may take time. To create some constraint, students first design a multiplicative situation for fair sharing and then follow the prompt to make groups and find the share of each group.

TEACHER REFLECTIONS AND OBSERVATIONS

1. Anticipate one of the strategies that student might use in your class for Activities 1 and 3. Design a set of questions that you will ask about that strategy.

2. For Activity 4, figure out all possible distributions (right and wrong) that students might come up with for assigning cakes to each group.

3. After all the activities are done, reflect on the following.
- List three challenges student faced while working on this lesson.
- List three strategies that you did not expect from students (right or wrong).

UNIT 1: ADDITIVE TO MULTIPLICATIVE THINKING
Lesson 2: Let Us Compare and Distribute

LESSON OVERVIEW

In this lesson, students will continue working on the idea of fair sharing through equipartitioning using an area model. They help Jamuni to distribute food packets to workers such that everyone gets equal amount of food. The food packets, circular or rectangular in shape, generate opportunities for students to cut equal sized portions. In a visual sense, students learn to divide the areas of the given food packets in equal shares. In this lesson, Jamuni visits her parents' workplace and uses her understanding of equipartitioning to distribute food packets to workers there. Here, the builder and a few others engage in additive reasoning that Jamuni has to recognise as wrong. Further, students work with two groups, creating scope for understanding proportions and their comparison.

LEARNING OBJECTIVES

Students learn to

- Distribute a given quantity among groups by using the cutting tool.
- Compare ratios in two different sharing situations to identify the larger ratio by relating it to the share of a particular group and its members.
- Make decisions, and give reasons for those decisions, about increase or decrease in a quantity based on mathematical relationships among quantities.
- Read a situation multiplicatively when appropriate and not always additively.

DISCUSSION ON MATHEMATICAL IDEAS

In this section, we discuss a few examples from the lesson and talk about students' thinking related to those examples. We provide prompts for teachers to build a discussion around these tasks and, at the same time, understand the mathematical trajectory of the lesson.

Tasks and their mathematical relevance	Students' thinking and possible strategies for facilitating
Activity 1: The builder in this task applies the additive relationship to the question of people and food packets and concludes that 4 parathas distributed among 5 people and 5 parathas distributed among 6 people are equivalent situations. As the difference between the number of people and the number of parathas is 1 in both cases, this is a common mistake. Again, seeing this difference between two quantities is an additive way of looking at the situations. Our explicit goal in this lesson is to train students to look at the situation multiplicatively.	a. One less paratha: In each group, the number of parathas is just one less than the number of people. Additively, these two situations are equivalent, but that doesn't mean that each worker in the group will get exactly the same share of parathas. Let us say in two groups there are m and n people respectively. Now distributing parathas among people indicates that each person got little less than the whole paratha, and that little less of each person's if added together will be one whole paratha. When $m - 1$ parathas are shared among m people, each person gets less than 1 paratha, less by the amount $1/m$ parathas. Similarly in group of n people, the same reasoning applies and each person gets $1 - 1/n$ parathas. However, as $m \neq n$, $(1 - 1/m) \neq (1 - 1/n)$, therefore eventhough these situations are additively identitcal, they are not multiplicatively same, and share of each person across the group is not equivalent.

Activity 2: This is a similar activity and follows the same reasoning as Activity 1. Here, the number of parathas are three less than the number of workers in each group.	There are again, there are two ways in which the students needs to figure out the non-equivalence between these two situations. a. *Finding the share*: Using the cutting tool, students figure out the share of each worker in the each group. Then, using the measure tool or visual cues, students see that and are not equivalent. Different possible ways of cutting involve dividing the parathas into halves or quarters to notice that in a group of 8 people, each will get half and 4 halves would remain for them to share (10 halves in 5 whole) whereas in the group of 6, each person will get only a half. b. *Distributing the difference*: This strategy cannot be worked out in the digital tool but could be used for complimentary thinking. In each situation, there are three parathas fewer than the number of people. However, the missing three parathas create different multiplicative effects for 8 people and for 6 people, even though the additive effect is the same. [Read the discussion of Activity 1.]
Activity 3: This is a more straightforward task than the others. Here, one group has one more paratha than the number of people, and another group has one less. These situations are even additively non-equivalent. Mathematically, it is a direct understanding that something more than a whole would always be greater than anything less than a whole.	Students might only see that the difference between the numbers is 1, but we need to train them to see that the situations are nonequivalent in the multiplicative context. Different cues that could be used are: i. The number of parathas is greater than the number of people. What does this signify? ii. The number of parathas is less than the number of people. What does this signify? Iii. Refer to the previously learned rules about the numerator being greater or the denominator being larger. What does that actually mean in terms of the magnitude? iv. Does this mean that all the proper fractions (numerator less than denominator) are always less than one whole? v. Find the largest proper fraction.
Activity 4: This one is the only activity in which two food distribution situations appear additively non-equivalent, but they are multiplicatively equivalent.	Students who are still seeing the situation additively will find the situations here non-equivalent. However, dividing by quarters, they will see that each person's share is . Students would have to find some careful ways to carry out these distributions. As in Activity 1, students might begin with halves and then further divide into quarters. As the numbers are slightly bigger, it might take more time. The teacher could initiate a discussion for using a grouping strategy in this activity. 9 parathas 12 workers can be seen as three groups of 4 people with 3 parathas for each group. Similarly, 6 parathas for 8 people can be seen as 2 groups of 4 people with 3 parathas for each group
Activity 5: This is an assessment for this lesson, which helps us understand how much students have learnt from the activities.	Let students work on this on their own. By taking rounds in the class, track students' strategies.

TEACHER REFLECTIONS AND OBSERVATIONS

- Describe the differences that you understand between additive and multiplicative situations.
- Can you find two situations that are multiplicatively equivalent and also additively equivalent? Describe your process.
- What are fractions? How are they the same as or different from ratios?
- List students' strategy, at least one, for Activities 2 and 3. Give your commentary on each strategy.

UNIT 1: ADDITIVE TO MULTIPLICATIVE THINKING
Lesson 3: Make Equal Share Across Groups

LESSON OVERVIEW

The digital unit designed involves students helping Jamuni to distribute food packets to workers such that everyone gets an equal amount of food. The food packets, circular or rectangular in shape, generate opportunities for students to cut equally. In a visual sense, they learn to divide the areas of given food packets into equal shares. In this lesson, students help Jamuni create multiplicatively equivalent situations. In Lesson 2, they learnt to identify whether two situations are equivalent. This lesson builds on that understanding and moves one step further towards an understanding of ratios.

LEARNING OBJECTIVES

Students learn to

- Find values in equal proportion situations.
- Apply multiplicative reasoning as emphasised through grouping strategy.
- Extend the idea of finding shares for individuals to finding shares for groups of people.
- Design a question requiring equiproportional sharing for a given situation.
- Develop ways of thinking to create equiproportional situations.

DISCUSSION ON MATHEMATICAL IDEAS

In this section, we discuss a few examples from the lesson and talk about students' thinking on those examples. We provide prompts for teachers to build a discussion around these tasks and, at the same time, understand the mathematical trajectory of the lesson.

Tasks and their mathematical relevance	Students' thinking and possible strategies for facilitating
Activity 1: Jamuni has to find equivalent distribution of 3 parathas among 4 people and, for the same distribution, the number of parathas when the number of people is 12. Mathematically, this problem is similar to finding the missing fourth number in proportional pairs. This contextual illustration shows the exact meaning of those proportional numbers.	We can use two ways of reasoning to find equivalent situations. *Sharing:* One method is to focus on the share of an individual and how that remains ¾ in an equivalent situation. In this case, students must pay attention to the multiplicative relationship between 3 and 4, and achieve the same relationship for 12 people. For example, a quarter of 4 is 1 and that is taken three times; so a quarter of 12 is 3, and taking that 3 times would give us 9. Therefore, 9 parathas for 12 people provides an equivalent situation. *Grouping*: In this method, students try to find groups of four people in the group of 12. And then they work out the number of parathas required to make each group equivalent to the situation of 3 parathas for 4 people. The tool uses this particular strategy. While using this strategy, keep the following in mind. Groups for which distribution is possible: Grouping 1: 4, 4, 4 Correct distribution: There are 3 sub-groups, each with 4 workers. The total share of each group should be equal to 3 parathas. Grouping 2: 2, 4, 6 Correct distribution: There are 3 sub-groups with 6, 4 and 2 workers. The total share of these groups would be 4½, 3 and 1½ parathas respectively. Grouping 3: 2, 2, 8 Correct distribution: There are 3 sub-groups with 2, 2 and 8 workers. The total share of these groups would be 1½, 1½ and 6 parathas respectively. Grouping 4: 4, 8 Correct distribution: There are 2 groups, 1 group of 8 workers and 1 group of 4 workers. The group of 8 workers gets 6 parathas, and the group of 4 workers gets 3 parathas.

	Grouping 5: 6, 6 Correct distribution: There are 2 groups of 6 workers each. Each group gets 4.5 parathas. Grouping 6: 2, 10 Correct distribution: There are 2 groups. 1 group of 10 workers and 1 group of 2 workers. The group of 10 workers gets 7.5 parathas, and the group of 2 workers gets 1.5 parathas.
Activity 2: : This is a similar problem as in Activity 1. There are 2 parathas and 4 workers in Group A. Group B has 10 workers. Mathematically, the problem poses a new challenge as students need to figure out a multiplicative relationship between 4 and 10	Two ways of reasoning can be used to find an equivalent situation. *Sharing*: Students see that, in the first situation, 2 parathas are given to 4 people, that is each one gets $\frac{1}{2}$ paratha. To give $\frac{1}{2}$ paratha to 10 workers, they would require 5 parathas. *Grouping*: In grouping, to make an equivalent group, if students begin with a group of 4 workers, they will arrive at two groups of four workers and one group of two workers. One could understand that the group of two workers could be made equivalent by assigning half number of parathas that they will assign to the group of 4 people. This problem illustrates the beauty of multiplicative thinking. When each group of 4 workers get 2 parathas, the one group with 2 workers in it will get one paratha, adding altogether to 5 parathas. The tool uses this particular strategy. While using this strategy, keep the following in mind. Groups for which distribution is possible: Grouping 1: 8, 2 Correct distribution: There are 2 sub-groups with 8 and 2 workers. The share of each group should be 4 and 1 parathas respectively. Grouping 2: 5, 5 Correct distribution: There are 2 sub-groups with 5 workers each. The share of each group is 2.5 parathas. Grouping 3: 3, 3, 4 Correct distribution: There are 3 sub-groups of 3, 3 and 4 workers. The share of each group will be 1.5, 1.5 and 2 parathas respectively. Grouping 4: 2, 2, 6 Correct distribution: There are 3 sub-groups with 2, 2 and 6 workers. Share of each group will be 1, 1 and 3 parathas respectively. Grouping 5: 4, 4, 2 Correct distribution: There are three sub-groups with 4, 4 and 2 workers. Share of each group will be 2, 2 and 1 paratha respectively. Grouping 6: 6, 4 Correct distribution: There are 2 sub-groups with 6 and 4 workers. Share of each group will be 3 and 2 parathas respectively
Activity 3 : Here, there are 2 parathas and 3 workers. Group B has 9 workers. Mathematically, the problem is the same as the one in Activity 1.	Again, two ways of reasoning can be used to find an equivalent situation. *Sharing*: Students see that in the first situation, 2 parathas are given to 3 people, that is, each one gets $\frac{1}{3}$ paratha twice. For 9 workers to be given $\frac{2}{3}$ paratha each, they would require $\frac{2}{3}$ paratha 9 times. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = 18/3$, which is 6 parathas. *Grouping*: Students will try to find groups of 3 workers among 9. After getting three groups of 3 workers, and to maintain the equivalence, if they give 2 parathas to each group, they would require 6 parathas in all. The tool uses this particular strategy. While using this strategy, keep the following in mind. Groups for which distribution is possible: Grouping 1: 3, 3, 3 Correct distribution: There are 3 sub-groups, each with 3 workers. The share of each group should be equal to 2 parathas. Grouping 2: 6, 3 Correct distribution: There are 2 sub-groups with 6 workers and 3 workers. The total share of these groups would be 4 parathas and 2 parathas respectively.

Activity 4: This is an assessment question for the lesson. Give students time to work on their own.	It is expected that students use either grouping or sharing reasoning and, if needed, work using paper and pencil. They do not get tool support in this question. This is intentional as we want students to develop ways of thinking and not just be dependent on the digital tool.

TEACHER REFLECTIONS AND OBSERVATIONS

- Anticipate one of the strategies that student might use in your class for Activity 4. Design a set of questions that you will ask for that strategy.

- For Activity 3, figure out all possible groupings (right and wrong) that students might come up with for assigning parathas to each group.

- After all the activities are done, reflect on the following prompts.
- List three challenges student faced while working on this lesson.
- List three strategies that you did not expect from students (right or wrong).

- Take the following situation: Divide 6 parathas among 8 people. Find equivalent situations where the number of people is 18. Discuss all possible groupings and the correct distribution for each.

UNIT 2: Multiplicative Reasoning

UNIT OVERVIEW

In this unit, students will engage in activities that encourage them to reason and move towards thinking multiplicatively. Students will build on their understanding of sharing and equal division and start thinking in terms of proportions. The unit includes hand-on as well as digital activities. The digital activity on pattern reproduction will help students to develop their intuitive ideas of patterns and scaling up and down. A few daily life situations in one of the digital activities help students to find the relevance of proportional reasoning in day-to-day situations. The mixture tasks are at the core of proportional reasoning. In this unit, we address some of those and students' reasoning about it. Hands-on activities like making maps make students aware of applications of ratios and highlight the relevance of the topic. The hands-on activities provide opportunities for students to engage in the tasks using concrete materials. Students get many opportunities to explore the idea of multiplicative thinking in many contexts and through different types of tasks.

LESSON BREAK-UP

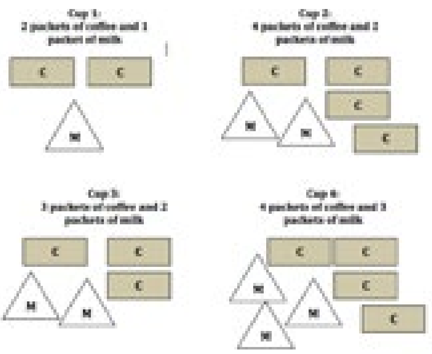
	Lesson name	Lesson Type	Lesson description
1	Coffee and Milk	Hands-on	The lesson introduces students to multiplicative thinking using discrete quantities. Students will use strategies that involve various kinds of grouping to maintain the equality of proportions. Here, continuous quantities are manipulated in discrete form, which allows students to develop flexible reasoning across discrete and continuous quantities. One activity has one context to introduce calculations requiring maintaining of equal ratios, and a similar context is used in another activity for reasoning related to multiple ratios.
2	Jamuni Solves Puzzles	Digital	This digital lesson introduces students to tasks which involve scaling up and scaling down along with finding a scaling factor. The activities also encourage students to think about the changes in the units of original and scaled patterns. This is a digital lesson which involves a tool called the Pattern tool. This tool allows students to scale and shrink the patterns in such a way that the visual appearance of it remains the same.
3	Jamuni Goes to the Bazaar	Digital	The activities in this lesson help students to use multiplicative thinking strategies. It uses quantities that are continuous but can be made discrete and allows students to use scalar and vector ratios. The activities allow students to engage in multiplicative thinking and help them to use proportional relationships with quantities. The activities also allow students to use different strategies in proportional thinking to compare continuous quantities. The best-buy problem connects with the day-to-day life of students. Students' understanding of proportions is strengthened by applying it to the concept of geometric shapes. It helps students to think about 'internal' and 'external' ratios with respect to length and breadth of geometric shapes.
3	Sahir Makes a Poster	Hands-on	In this lesson, students develop multiplicative reasoning using continuous quantities. Students also relate with the application of multiplicative thinking and ratio-proportion.

UNIT 2: MULTIPLICATIVE REASONING
Lesson 1:Coffee and Milk

LESSON OVERVIEW
In this lesson, students use various multiplicative thinking strategies. Grouping and finding proportion and comparing proportions are important concepts in this lesson.

LEARNING OBJECTIVES
Students learn to
- Develop strategies for various kinds of grouping that maintains the equality of proportions.
- Develop various kinds of grouping strategies.
- Manipulate continuous quantities in discrete form.
- Develop flexible reasoning across discrete and continuous quantities.

DISCUSSION ON MATHEMATICAL IDEAS
In this section, we discuss a few examples from the lesson and talk about students' thinking related to those examples. We provide prompts for teachers to build a discussion about these tasks and, at the same time, understand the mathematical trajectory of the lesson.
This lesson introduces students to multiplicative thinking using discrete quantities. Students will develop strategies for various kinds of grouping that maintains the equality of proportions. Here, continuous quantities like milk and coffee are manipulated in discrete form, which allows students to develop flexible reasoning across discrete and continuous quantities.
Prepare sets of two kinds of shapes representing milk and coffee. Rectangles represent coffee and triangles represent milk. You can make a drawing on the blackboard similar to the one shown on the screen where each number represents a cup of coffee. Students will arrange the cards as shown on the board at their desks. You can then ask the question and give time for students to figure out the solution in groups.

<table>
<tr><th>Tasks and their mathematical relevance</th><th>Students' thinking and possible strategies for facilitating</th></tr>
<tr><td>Activity 1: In this activity, students work on ideas of proportion from the point of view of uniform mixtures. A daily life activity such adding milk to coffee is used to facilitate different strategies of grouping to create or identify equiproportional situations.
</td><td>The task for students here is to study each cup of coffee carefully and find out which of the four cups of coffee is most milky. Students will make the following combinations on their desks using rectangles and triangles made of paper.

Students will see the combination in each cup to find out which coffee is milkier. For example, they might pair one packet of milk with two packets of coffee and check that pattern against the other cups. Let student explore on their own. Try to record their strategies as they work on the question.
If students find it difficult to proceed, draw their attention to the fact that Cup 1 and Cup 2 are equally milky as for every 2 packets of coffee there is 1 packet of milk in both the cups. In Cup 3, for 3 packets of coffee there are 2 packets of milk. When this is compared to Cup 4, we see that for 3 packets of coffee, there are 2 packets of milk and for the remaining 1 packet of coffee, there is 1 packet of milk. Hence, Cup 4 is milkier than Cup 3. Now ask students to similarly compare Cup 3 and Cup 1 (or Cup 2). Ask students to share their strategies with their peers.</td></tr>
</table>

Activity 2: This activity is an extension of the earlier activity, where students are given proportions for one cup of coffee and asked to make another equivalent situation. These situations represent the missing value problems of proportion, which are done only in a purely numerical and abstract form in the textbook.	Students are given 6 packets of coffee. How many packets of milk would they need to make a cup of coffee that is exactly the same as: • Cup 3 in Activity 1 • Cup 4 in Activity 1 There are two ways to look at this problem. External ratio: To make equiproportional quantities, one set of values is given. For example, in Cup 3, there 3 packets of coffee for 2 packets of milk. To make equivalent coffee using 6 packets of coffee, the students need to understand the ratio 3:2 in Cup 3, and ask ?:2 to maintain equivalence in the proportion. This is also called external ratio. Again, as ratio is a multiplicative concept, we do not see the relation between 6 and 3 as 6 is 3 more than 3; rather, we see it as 6 is twice 3. Therefore, to maintain equivalence in taste, we require twice the packets of milk, which is 4. Internal ratio: The other way to look at this problem is to ask how many packets of milk are needed for the given packets of coffee. So, in Cup 3, there are 3 packets of coffee for 2 packets of milk. Now with 6 packets of coffee we need 2:3 ratio of milk and coffee packets. Again, students must see the relationship between 2 and 3 multiplicatively. We know that 2 packets is 2/3rd of 3 packets, so to bring equivalence in taste, we need 2/3rd of 6 packets and that would be 4 packets of milk.
Activity 3: This activity increases the complexity of the earlier activity by constraining the number of packets for coffee and milk and also extends the problem to make equivalent tasting coffee for 4 cups unlike earlier where only one cup of coffee is made. The objective of this is to ingrain the idea of scaling with equal proportion.	Students are given 15 packets of coffee and 11 packets of milk and asked to make 4 cups of coffee that is as milky as the one in cup 4. They could find the internal or external ratio as discussed in Activity 2. Here, the teacher could ask various questions to make sure that students are engaged with the tasks and also to make sense of the equivalent ratio. • What is the ratio of milk packets with coffee packets in Cup 4? • To make 2 cups of equivalent tasting coffee as Cup 4 how many packets of coffee and milk do you need?

TEACHER REFLECTIONS AND OBSERVATIONS

- Narrate two different discussions that happened in your class today.
  - Why did you choose these two?
  - What is the mathematical importance of these discussions?
  - What are the social aspects of these episodes?
- In Activity 1, what were some of the methods that students use to find the milkier coffee?
- Describe at least two ways to find answers in Activity 3. Anticipate a possible incorrect response for this activity and explain the reasoning behind it.
- Did you find any repetitive pattern in students' thinking? Please describe in brief.

- Write in your own language what you believe proportional reasoning is. Now, elaborate how the coffee-milk task helps in developing proportional reasoning.
- There are various ways in which students can work on these problems. We saw how they used the grouping tool. Similarly, they can chunk the quantities. Design a coffee-milk task which will have the scope to apply some of these strategies.

UNIT 2: MULTIPLICATIVE REASONING
Lesson 2: Jamuni Solves Puzzles

LESSON OVERVIEW
The skill of scaling up and down is at the heart of proportional reasoning. In this lesson, we focus on this skill and reinforce the numerical representation of this skill. The activities in this lesson help students to think multiplicatively by scaling a given pattern up or down. There are two types of activities in this lesson. In the first type, the outer boundary of the scaled up or down pattern is given, and students have to find the factor by reproducing the pattern. In the second type of activities, the scaling factor is given and the students are expected to scale the given pattern up or down by that factor. Students will be asked to think about the relationship between the original and the newly created patterns. The digital tool created for this provides visual and semantic support for thinking proportionally.

LEARNING OBJECTIVES
Students learn to

- Apply multiplicative thinking to scale the given two-dimensional pattern proportionally.
- Explain and apply the process of scaling up and scaling down of a pattern keeping the visual appearance of the pattern the same.

DISCUSSION ON MATHEMATICAL IDEAS
In this section, we discuss few examples from the lesson and talk about students' thinking around those examples. We provide prompts for teachers to build a discussion about these tasks and, at the same time, understand the mathematical trajectory of the lesson.

<table>
<tr><th>Tasks and their mathematical relevance</th><th>Students' thinking and possible strategies for facilitating</th></tr>
<tr><td>Type 1 tasks: The grid boundary for scaled up or down pattern is given and students are asked to find the scaling factor after reproducing the pattern in the given grid.

<table><tr><td>G</td><td>G</td><td>G</td></tr><tr><td>G</td><td>R</td><td>G</td></tr><tr><td>G</td><td>G</td><td>R</td></tr></table>
A 9 x 9 outer grid is given. Observe students when they are filling up the grid. Discuss with them: How many times is the new pattern bigger than the original pattern?</td>
<td>The new pattern is scaled up by a factor of 3. You may ask students questions like the following:
• What is the total number of tiles in the original pattern?
• How many tiles are there in the new pattern?
• Why are there 81 tiles in the new pattern when we scaled up the pattern by a factor of 3 (when original pattern has 9 tiles)?

A couple of things we learned when we tried this activity with students are described here for your reference.
• When we say visually similar pattern, we need to discuss what that means. It means that the pattern could be scaled up or down but in proportion and while maintaining the design.
• The figures used here are two-dimensional, and so when we say scale the pattern by a factor of 3, it means scale both dimensions by that factor. So the length increases 3 times and the breadth also increases 3 times. So the number of tiles in the pattern actually increases 9 times.</td></tr>
</table>

Type 2 tasks: In these tasks, the factor by which the original pattern is to be scaled is given. The grid boundary of the new pattern is not given. Students create the new pattern and think about a few questions.

An example of this type of activity is: Scale down the pattern by a factor of 2 by dragging and dropping the green and red tiles into the grid.

R	R						
R	R						
R	R						
R	R						

Students will be asked to compare the original and the scaled up pattern and find the relationship between the green and red tiles in the two patterns.

You can ask students why they created a particular pattern and how they decided how many red and green tiles it should contain.
It is important that students see the relationship between the two patterns. It is also important that they understand that when a pattern is scaled up by a factor of 3, the number of units in the pattern increase 9 times (3 × 3) as it is a 2-dimensional pattern.
In one of the tasks, the scale factor is a fractional number. You might need to help students to understand this by taking examples of scaling by ½, 1.5 (1 and 1/2) etc.
Tessellation is like stamping where one just repeats the pattern one after another to fill up the given area. No scaling up or down is done in this process. Students might tessellate the given pattern instead of scaling it up or down. You can discuss with them the scaling up or down of the pattern, the meaning of the scale factor and its relationship with proportional reasoning. These activities will drive students' thinking towards multiplicative thinking as the use of additive reasoning is less intuitive here and they will not get the required pattern if they think additively.

Some complementary questions that you can ask are given here:

- If one has to represent scaling up in a mathematical sentence, how would you do it? For example, scale the rectangle of dimensions 6 x 2 by 4 times. How can you represent it mathematically?
- How would you represent the shrinking process mathematically?

TEACHER REFLECTIONS AND OBSERVATIONS

- Narrate two different discussions that happened in your class today.
    - Why did you choose these two?
    - What is the mathematical importance of these discussions?
    - What are the social aspects of these episodes?
- What were some of the students' difficulties while creating these patterns?
- Observe whether Type 1 tasks or Type 2 tasks were easier for students. Describe why any of the tasks was easier than others.
- While numerically scaling up by a factor of 3, students scaled up the tiles by what number? By 3 or by 9? Why do you think this is difficult for students to understand?
- How will the idea of direct proportions help students in developing problem solving skills?

UNIT 2: MULTIPLICATIVE REASONING
Lesson 3: Jamuni Goes to the Bazaar

LESSON OVERVIEW

In this lesson, students use multiplicative thinking strategies in the context of discrete quantities. The lesson tries to strengthen students' understanding of proportions by applying it in the concept of geometric shapes. Activities also allow students to use different strategies in proportional thinking to compare continuous quantities.

LEARNING OBJECTIVES

Students learn to

- Apply multiplicative thinking strategies.
- Calculate continuous quantities as discrete quantities and use scalar and vector ratios.
- Use proportional relationships with quantities.

DISCUSSION ON MATHEMATICAL IDEAS

<table>
<tr><th>Tasks and their mathematical relevance</th><th>Students' thinking and possible strategies for facilitating</th></tr>
<tr><td>
Activity 1: This activity focuses on unitising and grouping. Here, students will either find the price of one and then find the price of many or make convenient groups to find the price of one and a half egg trays. Unitising allow students to also form units other than one. For example, finding the price of 6 eggs would be more useful than finding the price of 1 egg as one and a half tray has three halves.
The task is as follows:
Jamuni and her friends are at an egg shop. They see a tray of eggs. Each tray contains 12 eggs and costs Rs. 36. Now, if they buy an egg tray that has one and half times the eggs in this tray, how much will they need to pay?
</td><td>
Engage students in a discussion so that they understand the meaning of 'one and a half times'. One way of engaging with this task is to recognise that increasing the number of eggs one and a half times will also increase the cost of the egg tray one and a half times. Find one and a half times of 36 as the relationship between eggs and their price is proportional.
Another way is to find the cost of one egg, then find one and a half times of 12 eggs and then find their cost. Engaging students in these different strategies will help them develop mathematical modelling skills through the idea of proportional reasoning.
Shabana is Jamuni's friend and she wants to buy two egg trays. She finds that there is a mix of both white and brown eggs in each tray. The first tray holds 12 eggs of which 4 are brown and 8 are white. The second tray holds 18 eggs. If the proportion of brown and white eggs is the same in both trays, how many eggs of each colour does the second tray have?
<table>
<tr><td>Tray 1</td><td>Tray 2</td></tr>
<tr><td>Total no of eggs 12</td><td>18</td></tr>
<tr><td>No of white eggs 4</td><td></td></tr>
<tr><td>No of brown eggs 8</td><td></td></tr>
</table>
This task requires dividing the number 18 in two parts such that one part is twice the other. We need to draw students' attention to the fact that the proportion of white eggs and brown eggs in Tray 2 has to be the same as their proportion in Tray 1. Students may also adopt trial and error methods. They can double the number of white and brown eggs. The answer can be found by finding one and a half times the number of white and brown eggs in Tray 1 because 18 is one and a half times 12. The fact that both the number of white as well as brown eggs are to be made one and a half times is very important and should be highlighted.
</td></tr>
</table>

Activity 2: Here, the attempt is to provide familiar but different contexts than what students have been working with so far. To ensure mathematical learning, it is essential to change the context to get students to transfer a construct from one context to another.
Aman decides to buy a bar of chocolate to share with his friends. Help him solve some problems he faced when he went to a chocolate shop.

- A white chocolate bar contains 10 small pieces. If Aman decides to give each of his friends 2 such small pieces, how many children can share the bar?

- The shopkeeper sells 3 small sized packs of white chocolate for Rs. 4. If Aman spends Rs. 40, how many such pieces of chocolate can he buy?

- The shopkeeper charges Rs. 4 for a small piece of brown chocolate. If Aman decides to buy 10 such pieces, how much money he would need?

The purpose of this activity is to give students more contexts and more experiences to explore proportional reasoning strategies. Please help students to come up with different strategies.

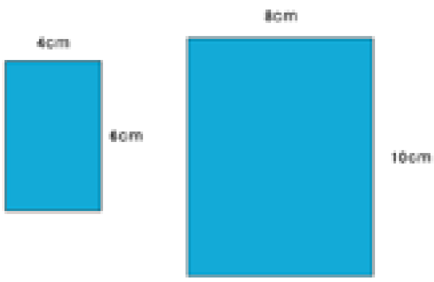
Problems related to proportions can be solved using two methods — finding the ‘between ratio’ and finding ‘within ratio’. Read this problem: If the cost of 4 chocolates is Rs. 12, what is the cost of 8 chocolates? Using the between ratio, we can see that the ratio of the number of chocolates is 1:2 (4:8) and so the ratio of costs will also be 1:2. Hence, the cost of 8 chocolates is twice 12, which is Rs. 24. In a method using within ratio, one has to find the cost per chocolate. It is also called vector function method. The cost per chocolate is then used to find the cost of 8 chocolates.

Activity 3: The purpose of this activity is to give students more contexts and more experiences to explore proportional reasoning strategies. Please help students to come up with different strategies.
Jamuni and her friends were thirsty and went to a juice shop. The juice shop had two options for orange juice: 6-litre cartons for Rs. 200 and 4-litre cartons for Rs. 150. Which of the two cartons is cheaper?

- 6-litre carton
- 4-litre carton

In research it has been found that students can use very sophisticated strategies to solve this type of problem. They can say that they can use the smaller juice box to pour juice in the bigger one. They will pour one full small box and half of it to fill up the bigger box completely. The logic is that the bigger box can hold 4 + 2 litres of juice. By this logic, the cost of the bigger box has to be 150 + 75 = 225. But its actual cost is Rs. 200 and so the 6-litre box of juice is cheaper. We can also use strategies like finding the cost of one litre of juice in both boxes and comparing.

Activity 4: This is scaling problem in another representation. Students by now should be able to reason about dimensions proportionally.
Jamuni wants to buy a square paper napkin but the shopkeeper only has rectangular ones. Look at the two rectangular paper napkins shown here. Which of them is more squarish? Why?



This question expects students to relate their understanding of geometrical shapes with the idea of proportions. In a square, as both the sides are of equal length, the ratio of length to breadth is 1:1. The closer the ratio of length to breadth to 1:1, more “squarish” the figure is. One can find the ratio of length to breadth in both the figures and compare. One can further compare the relationship between the length of figure 1 to the length of figure 2. This is similar but not exactly same as finding the internal and external ratio. In a scaled figure the ratio of breadth to length will be preserved and the ratio of length to length (figure 1 to 2) will be the same as the ratio of breadth to breadth (figure 1 to 2).

TEACHER REFLECTIONS AND OBSERVATIONS

- Mention some of the multiplicative thinking strategies that students used in Activities 1, 2, 3 and 4.
- What kind of things were students learning from each other while engaging in the activities in this lesson?
- Do you think that the contexts or problems in this lesson will help students to strengthen their understanding of multiplicative thinking? Can you think of more contexts where proportional reasoning is used?
- List five questions that you would like to ask students about these activities to help you understand how much they learned but also to help in furthering the discussion on proportional reasoning.
- Can you design a few more tasks to help students develop multiplicative thinking skills?

UNIT 2: MULTIPLICATIVE REASONING
Lesson 4: Sahir Makes a Poster

LESSON OVERVIEW

In this last lesson of Unit 2, the context of enlarging a photograph to make a poster is used. Various other problems are posed related to this context. Students will explore the relationship between a change in each dimension of a rectangle and its effect on the change in the area of that figure. The difference between stretching and scaling will be highlighted. The daily life situations in this lesson will help students to connect the idea of proportions with real life.

LEARNING OBJECTIVES

Students learn to

- Develop multiplicative reasoning using continuous quantities.
- Apply proportional reasoning and multiplicative thinking.

Tasks and their mathematical relevance

Activity 1: This task re-emphasises the stretching aspect of proportional reasoning and adds another component to it by bringing in the area of the figures as one more measure in direct proportion.

Question 1: Sahir wants to double the dimensions of the photo. What will the dimensions of the enlarged photo be?

	Length (cm)	Breadth (cm)
Photograph	7	5
Poster	?	?

Question 2: Find the relationship between the area of the photograph and the area of the poster. List other things you notice about the dimensions of the two figures.

Question 3: Interchange the dimensions of the photograph, make the width into the length and the length into the width. What do you see? Compare the two photographs and identify what is the same and what is different between the two.

Question 4: The original photo is scaled correctly, such that its breadth is 20 cm. Find the height of the scaled photo that is twice the original size.

New height: ________ cm

Students' thinking and possible strategies for facilitating

Students need to think about the fact that doubling each dimension will lead to an increase in area 4 times. You can do more such exercises with students using different geometrical shapes like triangle and parallelogram and discuss the change in the dimensions and its effect on the area of that figure.

By interchanging the dimensions of the figure, its area will remain the same. However, the image in the photograph will be distorted. Using the idea of proportions, you can discuss more such problems with students.

Activity 2: Here, linear growth is another context used to strengthen the concept of proportionality. Using a concept such as growth brings dynamism to the idea of proportionality. Question 1: If a plant grows three times taller every week, how tall will it be at the end of the 1st and 2nd weeks? Instructional text: Use GeoGebra to draw the growth of the plant at the end of weeks 1 and 2. Question 2: What is the relationship between the initial height of the plant and its height after two weeks? • The height of the plant will be 3 times its original height. • The height of the plant will be 6 times its original height. • The height of the plant will be 2 units more than its original height. • The height of the plant will be 3 units more than its original height.	Using a GeoGebra tool, students can explore this question and also be engaged in other questions of this type. You can pose some problems for them which they can solve using GeoGebra.

TEACHER REFLECTIONS AND OBSERVATIONS

- At the end of Unit 2, what are the things that you think students have understood? What events in the class or student responses make you believe that?
- What are some of the mathematical ideas on which you think further discussion with students is required?

UNIT 3: RATIOS AND PROPORTIONS

UNIT OVERVIEW

In this unit, students will begin engaging with the formal ratio notation, which describes how much of one thing there is compared to another thing. Students will build on their understanding of sharing, equal division and multiplicative thinking to work on ratios numerically and through a digital activity. The unit includes hand-on as well as digital activities. The digital activity on ice cubes being added to lemonade will help students to develop their intuitive ideas of direct and inverse variations. The connection to daily life contexts will help students to find the relevance of proportional reasoning in day-to-day situations. Hands-on activities like making maps will make students aware of applications of ratios and highlight their relevance. Students will have opportunities to explore the idea of ratio notation and direct and inverse proportions in multiple contexts and through different types of tasks.

LEARNING OBJECTIVES

Students learn to

- Accurately use and read ratio notation and explain its applications to relevant contexts.
- Explain and apply the idea of proportions, including direct and inverse variations.
- Solve problems in various contexts by applying ideas of direct and inverse proportionality.

UNIT BREAK-UP

	Lesson name	Lesson Type	Lesson description
1	Understanding Ratio Notation	Worksheet	In this lesson, Jamuni begins to apply her knowledge of ratios and proportions to things in her surroundings. She notices various ratios and writes them down and forms question about them. Such exposure is needed to our students as they themselves will start noticing proportional quantities in their surroundings. This lesson legitimises the students' mathematical interaction with the real world.
2	Map Reading with Jamuni	Hands-on	In this lesson, Jamuni learns to read, make and use maps. While helping Jamuni students will engage with finding scaled distances for the real distances on map. Students will apply the concepts of scaling—stretching and compressing while helping Jamuni. To do this they need to understand the scale factor and use multiplicative reasoning to find the distance between two places and vice versa.
3	Finding Length Using Strips	Hands-on	Students continue to work on ideas of scaling and shrinking, again in spatial mode. They learn to transfer their tactile understanding to numerical representations. In the beginning of this lesson, students compare strips of paper to find the multiplicative relationship in them. The measurements are made using different units, and that makes students think about relationship between a unit and the whole and further to see these relationships as either directly proportional or inversely proportional.
4	Ice Cubes in Lemonade	Digital	This is a digital activity based on the ideas of volume. Students will help Jamuni to decide the number of ice cubes required to either make something proportionally equivalent or non-equivalent.

UNIT 3: RATIOS AND PROPORTIONS
Lesson 1: Understanding Ratio Notation

Discussion on Mathematical Ideas

In this section, we discuss a few examples from the lesson and talk about students' thinking on those examples. We provide prompts for teachers to build a discussion on these tasks and, at the same time, understand the mathematical trajectory of the lesson.

<table>
<tr><th>Tasks and their mathematical relevance</th><th>Students' thinking and possible strategies for facilitating</th></tr>
<tr><td>

Activity 1: The notation for writing ratios is unusual compared to many other notations used in mathematics. Comparison between two quantities A and B can be expressed as A:B. The notation A:B represents the quotient when A is divided by B. Ratios are often written for quantities of the same type. One important thing we need to convey to students is that this comparison is not additive. For example, Jamuni's weight is 25 kg and her father's weight is 75 kg. In this case, the ratio of Jamuni's weight to her father's weight is represented as 25:75. And the ratio is not 75 – 25 = 50, but 25 divided by 75. This difference is crucial in understanding ratios. It is a multiplicative relationship between two numbers.

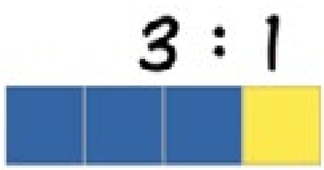
This activity requires Jamuni and her friends to write ratios for their observations.
After explaining with examples (given in the next column), students can be asked to solve the problems in the worksheet.

- There is 1 boy for every 2 girls at the giant wheel.
- A farmer had 10 chickens and 2 goats for sale.
- Leena's mother is 3 times taller than her. (You can emphasise here that height as a quantity is continuous, whereas the previous 2 problems dealt with discrete quantities.)
- Geo is 2½ times shorter than Inspector Kaata. (You can depict multiple ways to express this ratio - 2½ : 1, or 5:2, or 2.5:1.)

</td><td>

Like Jamuni, ask your students to notice ratios from their real life surroundings, ask them to represent ratios verbally as well as numerically. If there is difficulty in writing ratios numerally, help them to represent them using diagrams. Here is one such example. You can explain describing a statement in ratio notation with the help of one or two examples.

- 'There are 3 blue squares to 1 yellow square' can be shown as:



- A farmer is selling 4 cows and 8 pigs. So the ratio of cows to pigs is 4:8.

</td></tr>
<tr><td>

Activity 2: Interpreting ratio notation is a challenging task since the language used to read ratios matches many other concepts in mathematics, for example, chance, probability and fractions. We need to be watchful of different student interpretations.

There are 27 children inside a video game room at the mela. The ratio of girls to boys is 3:6. Which of the following statement/s is/are true?

- The ratio of boys to girls is 6:3.
- Half the children in the video room are girls.

</td><td>

Again, here, the attempt is to provide various contexts and situations so that students get the general idea of ratios and proportions by seeing them in various contexts.

The following complementary activities could be used to track students' understanding.

- Give students some ratios and ask them to design a situation around them.

This is a converse of the task that

</td></tr>
</table>

• We know exactly how many boys are present in the video room. • We know exactly how many girls are present in the video room. • If we choose 9 children at random, we can expect that 3 will be girls. • We can calculate how many boys there are if there are 36 children in the video room with the same gender ratio as above.	they were doing earlier. Engaging in this activity will allow students to understand how real life structures are represented using ratios. • Ask students to bring newspaper cuttings that make use of ratio notations. Let them elaborate on how ratios are used. • Many items such as hair oil, cream, toothpaste, have ingredients written on the packaging. The teacher can ask students to represent the quantities in ratio.
Activity 3: This is an assessment question and students will work on it independently.	A circus tent in the mela can accommodate 100 people. It is divided into two zones. Zone 1 has 30 seats, and Zone 2 has 70 seats. A total of 80 people came for the show and all the seats in Zone 1 were occupied. • What is the ratio of seats in Zone 1 to seats in Zone 2? • What is the ratio of empty seats to occupied seats in the tent? • What is the ratio of empty seats to occupied seats in Zone 2?

TEACHER REFLECTIONS AND OBSERVATIONS

- Design two assessment questions to check students' understanding of equal and unequal proportions. Make sure the contexts used are familiar.
- Describe any challenges students faced while working on this worksheet.
- List two topics of middle school mathematics that you think are connected with the ideas of ratio and proportion. Describe how they are connected.

UNIT 3: RATIOS AND PROPORTIONS
Lesson 2: Map Reading with Jamuni

LESSON OVERVIEW

The process of multiplicative thinking is associated with situations that involve fair sharing, scaling, shrinking, duplicating and exponentiating. Beginning with strengthening students' ideas of equipartitioning and share, this unit addresses the idea of scale and situates it in the numerical as well as spatial context of reading maps. In the beginning of this lesson, Jamuni works on reading and using scale on a map. Ultimately, she figures out drawing and making scales on the map. The scale factors on the map are another context for equal proportions. On a map, there is exactly the same ratio between the scale and the actual distance.

LEARNING OBJECTIVES

Students learn to

- Measure the length depicted on a map and, by applying scale, find the equivalent real distance.
- Draw maps and design scales that use spatial as well as numerical engagement.
- Apply the concepts of scaling—stretching and compressing.
- Read and write the notion of factor and begin using multiplicative reasoning by using a factor and a given scale to find the distance between two places and vice versa.
- Participate in classroom discussion to develop a deeper understanding of scaling and factor.

DISCUSSION ON MATHEMATICAL IDEAS

In this section, we discuss a few examples from the lesson and talk about students' thinking about those examples. We provide prompts for teachers to build a discussion on these tasks and, at the same time, understand the mathematical trajectory of the lesson.

Tasks and their mathematical relevance	Students' thinking and possible strategies for facilitating
Activity 1: In this question, students are given the opportunity to measure distance using scales. They are introduced to the idea of scale. On the map, scale means the ratio of the distance between two places on the map to its distance in the actual world. This ratio remains the same across the map.	Maps are not commonly discussed in a mathematics classroom, and their use brings novelty to this module. The concept of map-scale provides an opportunity to find the ratios between the distances in maps with real distances. There is a possibility that students might come up with various strategies to measure the length on the map in Question 1. The teacher will have to discuss strategies that provide near accurate lengths on the maps. It is not a common skill to be able to read maps. Let students spend some time on measuring various distances and come up with the idea of scale. Discuss the following in class: • Why are scales important? • What would happen if the ratio between the real distance and the distance on the map is not constant across the map? • Are there other ways in which the idea of scale could be used in the map? • How are maps made? Students then practice with various scales and use numerical calculations to arrive at the actual distance. At every opportunity, ask students how they came up with the distance and how they made use of the scale.

<table>
<tr>
<td>Activity 2: This is an extension of Activity 1, where students understand the idea of scaling in a numeric context.</td>
<td>Students understand scaling on the map as multiplying by the scale factor. To fill in the following table, anticipate different student responses.
<table>
<tr><td rowspan="3">Map-scale (1:25000)</td><td>Thread length</td><td>Real distance</td></tr>
<tr><td>10 cm</td><td></td></tr>
<tr><td>18 cm</td><td></td></tr>
<tr><td rowspan="2">Map-scale
(1 cm = 2.5 km)</td><td>12 cm</td><td></td></tr>
<tr><td>21 cm</td><td></td></tr>
</table>
Here thread lengths are the distances between two cities on a map. There is one very large scale and a decimal scale. This is done on purpose to provide students contexts that are generally found in various maps.</td>
</tr>
<tr>
<td>Activity 3: Mathematically, this activity is similar to missing ratio problems. Here, students find the missing value for the distance on the map or the distance in reality or the scale.</td>
<td>In contrast to general equiproportion problems where four quantities are used to indicate equal proportions, in this table, students work on six quantities. Therefore, the problems could be a little tough for students. Determining scale is similar to determining ratio, where determining the distance is the same as determining the missing value in given proportions. Students learn the idea of geometric scaling in numeric context.</td>
</tr>
<tr>
<td>Activity 4: Jamuni has been given a faulty map and attempts to find the error. This problem is an application of the understanding students have developed in the three earlier activities.</td>
<td>Jamuni found that the scale for the distance is represented differently in four different maps. Students are expected to help her locate the map in which the distance between the cities A and B is different from all the other maps.
<table>
<tr><td>Map</td><td>Map Distance Between A and B</td><td>Map-scale</td><td>Scaled Distance Between A and B</td></tr>
<tr><td>Map 1</td><td>25 cm</td><td>1:600</td><td></td></tr>
<tr><td>Map 2</td><td>12 cm</td><td>1:1250</td><td></td></tr>
<tr><td>Map 3</td><td>24 cm</td><td>3:1800</td><td></td></tr>
<tr><td>Map 4</td><td>30 cm</td><td>5:2500</td><td></td></tr>
</table>
The way to identify a faulty map is to see which measurements do not match with the actual distance as given by other maps.</td>
</tr>
</table>

TEACHER REFLECTIONS AND OBSERVATIONS

1. Measuring distances on graphs is challenging. Write three practices that you will use in the class to ensure accurate measuring.

2. For Activity 4, work out all possible reasoning that students might come up to say that Map 2 is a faulty one.

3. After all the activities are done, reflect on the following prompts:

- List three challenges student faced while working on this lesson.
- List three strategies that you did not expect from students (right or wrong).

UNIT 3: RATIOS AND PROPORTIONS
Lesson 3: Finding Lengths Using Strips

LESSON OVERVIEW
Students continue to work on ideas of scaling and shrinking, again in spatial mode. They learn to transfer their tactile understanding to numerical representations. In the beginning of this lesson, students compare strip of papers to find the multiplicative relationship between them. The measurements are made using different units, which makes student think about the relationship between a unit and the whole and, further, to see these relationships as either directly proportional or inversely proportional.

LEARNING OBJECTIVES
Students learn to
- Measure the length of a sheet of paper using paper strips as units.
- Explain how the smaller the unit, the more units are required for measurement.
- Manipulate units of measurement based on direct and inverse proportionality ideas.
- Apply ideas of direct and inverse variation

DISCUSSION ON MATHEMATICAL IDEAS
In this section, we discuss a few examples from the lesson and talk about students' thinking about those examples. We provide prompts for teachers to build a discussion on these tasks and, at the same time, understand the mathematical trajectory of the lesson.

Tasks and their mathematical relevance	Students' thinking and possible strategies for facilitating
Activity 1: In this activity, some students measure a strip of paper using three strips of paper of different lengths, meaning using different units. Now using this information, students use this tool to figure out the length of the strip measured. Students will have to use ideas of factors and common multiples for working on this.	The three students depicted in the problem find three different measures of the given strip. The reason is that they have been using different strips for measurement. Given the following information, students first predict who used which coloured strip and then they figure out the length of the strip. 4 cm 8 cm 2 cm A man found the length of the sheet to be 8 strips. Leena found the length of the sheet to be 16 strips. Sahir found the length of the sheet to be 4 strips. One conservation that students will must understand here is that measuring with different scales or different units cannot alter the length of the strip. The given information tells us that 8 times something, say , 16 time something, say and 4 times something, say is exactly the same quantity. If one strip is needed 16 times to measure something that another strip measures in 4 times, the first strip must be the smallest. Therefore, one can predict that Leena was using the strip with length 2 cm to measure the given strip. So the given strip is (2 cm × 16 =) 32 cm. The challenge here is to see whether students first assume the length of the given strip as the same across different units. If they don't, ask them how changing the measuring scale can change the measure. Also, what kind of change would that be? How can such problems be addressed in mathematics? (For example, universal scale.)

<table>
<tr>
<td>Activity 2: Actual measuring of the given strip and understanding the relationship between the unit and the whole are the two main goals of this activity. By this time learners must have understood that smaller the unit more number of strips are needed (inverse variance) and larger the strip less number of strips are needed (inverse variance). Mathematically, students would learn inverse proportion where the product of the two quantities remains the same.</td>
<td>In this activity, students work on measuring a given strip and finding the relationship between the size of the unit and the length of the unit.
<table>
<tr><td>Length of strip (l)</td><td>Number of strips used (n)</td><td>l x n</td><td>l/n</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>
This activity is a bridge between understanding ratios and the constant of proportionality. The pedagogical practice used here is to first collect various records for the table using different strips and then find what remains constant. From there, students and teacher together can discuss why it remains the same. Some questions that the teacher can ask in the class are:
• What happens when the measuring strip is smaller?
• What happens when the measuring strip is longer?
• What would remain the same and why?
By the end of the discussion, explain to students how quantities in the table are inversely proportional. To understand how increasing one of the quantities decreases the other and what remains the same, is crucial in understanding inverse proportions.</td>
</tr>
<tr>
<td>Activity 3: This activity is the converse of Activity 1. Here, the measuring units are kept the same, and the strips measured are different for different children. Mathematically, students would learn direct proportion here where the larger the number of strips longet the strip.</td>
<td>This activity is meant to illustrate direct proportion. Students will measure and find that with the same unit size, more measuring strips are needed if the given strip is longer and fewer measuring strips are needed if the strip is smaller. The length of the strip varies and with it the number of strips, creating the opportunity to explain direct proportion.</td>
</tr>
<tr>
<td>Activity 4:
This activity is a consolidation of all the earlier activites. Students will measure the length of a sheet of paper using paper strips as units. The ideas of direct and inverse variation will be re-emphasized.</td>
<td>In this activity, student work on measuring a strip and finding the relationship between the size of the unit and the length of the unit.
<table>
<tr><td>Length of strip (l)</td><td>Number of strips used (n)</td><td>l x n</td><td>l/n</td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
<tr><td></td><td></td><td></td><td></td></tr>
</table>
This activity is a bridging probe between understanding of ratios and the constant of proportionality. The pedagogical practice used here, is to first collect various records for the table using different strips, then find what remains constant across. From there the student and teacher together can discuss why it remains same. Some questions that the teacher can ask in the class are given here:
• What happens when the measuring strip is smaller?
• What happens when the measuring strip is larger?
• What would remain same and why?
By the end of the discussion, introduce to the student, how quantities in the table above are directly proportional. How increasing one of the quantities increases the other, and what remains the same, is crucial in understanding direct proportions.</td>
</tr>
</table>

TEACHER REFLECTIONS AND OBSERVATIONS

1. Narrate the challenges student face in understanding direct and indirect proportional situation.

2. Describe what remains the same and what changes in direct and indirect or inverse proportion situations.

3. Design one situation that involves quantities that are directly proportional and one where quantities are inversely proportional.

3. After all the activities are done, reflect on the following prompts.
- List three challenges student faced while working on this lesson.
- List three strategies that you did not expect from students (right or wrong).

UNIT 3: RATIOS AND PROPORTIONS
Lesson 4: Ice Cubes in Lemonade

LESSON OVERVIEW
Students continue to work on ideas of scaling and shrinking, again in spatial mode. They learn to transfer their tactile understanding in numerical representations. This is a digital activity based on ideas of volume. Students will help Jamuni to decide the number of ice cubes required to either make a quantity proportionally equivalent or non-equivalent. The activity challenges students' understanding by introducing ice cubes of different sizes and bringing in ideas such as percentage to represent the volume of the lemonade in each glass.

LEARNING OBJECTIVES
Students learn to

- Use ideas of proporitonality in the context of volume measurement.
- Connect ratio—proportion representations with other concepts such as volume of cubes, percentages.

DISCUSSION ON MATHEMATICAL IDEAS
In this section, we discuss a few examples from the lesson and talk about students' thinking about those examples. We provide prompts for teachers to build a discussion of these tasks and, at the same time, understand the mathematical trajectory of the lesson.

<table>
<tr><th>Tasks and their mathematical relevance</th><th>Students' thinking and possible strategies for facilitating</th></tr>
<tr><td>Activity 1: This activity aims at making students familiar with what happens when ice is added to a glass of lemonade, especially what happens when they add two different sizes of ice cubes.</td><td>There are two types of ice cubes. One type has volume 2 cubic cm, and the other has volume 1 cubic cm. For every problem, students will choose ice cubes. To do that, they have to predict its effect on the volume of the lemonade in the glass. If the lemonade overflows, the problem resets.
Students' guesses are important in this activity as it binds their visual as well as conceptual understanding of the concept of volume. The different sized ice cubes allow students to see direct and indirect variation.
<table>
<tr><td>Size of cube (s)</td><td>Number of cubes added (n)</td><td>s x n</td><td>s / n</td></tr>
<tr><td>2 cm^3</td><td>3</td><td>6</td><td>2/3</td></tr>
<tr><td>1 cm^3</td><td>6</td><td>6</td><td>1/6</td></tr>
</table>
To arrive at an understanding of direct and indirect variation, students describe patterns in the table. As the volume of the glass is fixed, will always be the same. That means the number of cubes needed and the size of cubes are inversely proportional to each other. On the other hand, volume per ice cube is directly proportional to the size of the cube.</td></tr>
</table>

Activity 2: This activity uses the same context and extends it by using the concept of percentage. The volume of the glass filled is given in percent. Students will have to figure out the remaining volume in percent and then decide the number of cubes to add. Mathematically, the complexity is to make the situations equivalent when one quantity is expressed in percentage and the other in volume.

The glasses are larger in size here. Aman and Jamuni are still thirsty and buy bigger glasses of lemonade this time. After 5 minutes, Jamuni's glass of lemonade is 40% full while Aman's glass of lemonade is 70% full. All the ice has melted since it is so hot. Students have to help Jamuni add more ice to her glass and fill it up to the brim.

Percentage volume of glass to be filled up (v)	Number of cubes added (n)	v x n	v / n
60	18		
30	9		

The actual working on the digital resource will give students the number of cubes that they need to add. However, the important part of this activity is to understand the sense these numbers make in the context of how much is added.

By the end of the discussion, explain to students how quantities in the table are directly proportional. Understanding how increasing one of the quantities increases the other and what remains the same is crucial to understanding direct proportions. Students should answer that, to fill the 40% full glass to the brim, they need 18 ice cubes and to fill the 70% full glass, they need 9 ice cubes. In this context, the number of ice cubes required to fill in the empty part of the glass are inversely proportional to the volume of lemonade present in the glass. Further, the teacher can ask students the following questions to see how well they have understood the mathematics of the context:

- What is the volume of the glass of lemonade?
- What would be 40% of the volume of the glass?
- How many ice cubes were added to completely fill the glass that was 40% full?
- Based on this information, can you find the volume of each cube? What is the process for finding that?

Activity 3: Here, we take an important step from the point of view of learning mathematics. We ask questions for the same concept but by changing the context. It is important that students transfer ideas from one context to another so that the essentials of the concept are highlighted and learned.

The work to be done and the number of hours of work is a context specifically used for inverse proportion, with the work done as the constant. This is also an assessment question here, where we are checking how successfully students can transfer what they learned from one context to another.

Number of Workers (w)	Number of Hours (H)
12	6
8	9
4	6

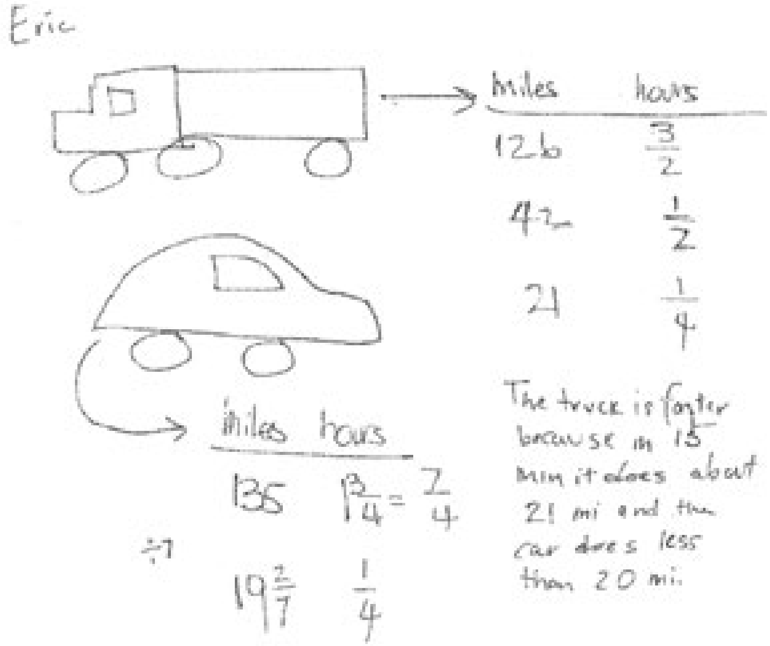
TEACHER REFLECTIONS AND OBSERVATIONS

- Design a problem with the lemonade and ice-cube activity context such that you use two different sizes of ice cube, provide the filled in volume of the glass in percentage and provide the opportunity to experience direct as well as indirect proportion.

- List three contexts that involve inverse proportion situations similar to work to be done and the number of hours needed.
  - After all the activities are done, reflect on the following prompts.
  - List three challenges student faced while working on this lesson.
  - List three strategies that you did not expect from students (right or wrong).

- Analyse the following response given by a student. Discuss with your peers what is happening in that child's mind.

- Which vehicle has a faster average speed, a truck that travels 126 miles in $1\frac{1}{2}$ hours or a car that travels 135 miles in $1\frac{3}{4}$ hours?



UNIT 4: APPLYING RATIOS AND PROPORTIONS

UNIT OVERVIEW

Proportional reasoning is one of the best indicators that a student has attained understanding of rational numbers and related multiplicative concepts. It lays the foundation for more complex concepts of mathematics. The process of multiplicative thinking is associated with situations that involve fair sharing, scaling, shrinking, duplicating and exponentiating. In this last unit, we introduce other mathematical ideas where ratio-proportion contexts are used. In this lesson, students work on linear equations and probability, making use of what they have learned about ratios and proportions.

Two examples are used to work on linear equation and probability —a schedule for a train and a game Jamuni plays while waiting for the train. In the train schedule activity, students will work on drawing a curve for the distance covered in a given time. Later, they will find the ratio of distance travelled per hour. In the bucket-ball game, students will write the probability for red or yellow ball being picked. In the second lesson, students work on the arithmetic of ratios. They find compound ratios, apply ratios to a recipe problem and practice working on real life contexts. Compound ratios require a logical understanding of why two quantities that are directly proportional to each other, when multiplied by another two quantities directly proportional to each other, remain directly proportional to each other.

LEARNING OBJECTIVES

Students learn to

- Relate their understanding of proportional reasoning to plotting linear equations.
- Apply their understanding of proportional reasoning to a situation involving probability.
- Apply concepts of ratio-proportion to other ideas in mathematics.

UNIT BREAK-UP

	Lesson name	Lesson Type	Lesson description
1	Proportions in Linear Equations and Probability	Worksheet	In this last unit, we introduce other mathematical ideas where ratio-proportion contexts are used. In this lesson, students work on linear equations and probability, making use of what they have learned about ratios and proportions.
2	Compound Ratios and Proportions	Worksheet	In the second lesson, students work on the arithmetic of ratios. They find compound ratios, apply ratios to a recipe problem and practice working on real life contexts.

UNIT 4: APPLYING RATIOS AND PROPORTIONS
Lesson 1: Proportions in Linear Equations and Probability

LESSON OVERVIEW
Two examples are used to explain the ideas of linear equation and probability — one is a schedule for a train and the other is a game Jamuni plays while waiting for the train. In the train schedule activity, students will draw a curve for the distance covered in a given time. Later, they will find the ratio of distance travelled per hour. In the bucket-ball game that Jamuni plays, students will calculate the probability for a red or a yellow ball to be picked.

DISCUSSION ON MATHEMATICAL IDEAS
In this section, we discuss a few examples from the lesson and talk about students' thinking about those examples. We provide prompts for teachers to build a discussion on these tasks and, at the same time, understand the mathematical trajectory of the lesson.

<table>
<tr><th>Tasks and their mathematical relevance</th><th>Students' thinking and possible strategies for facilitating</th></tr>
<tr><td>Activity 1: This activity brings in a train schedule, another context familiar to students, to build a linear equation. Students plot the line and understand changes in quantities.</td><td>
Here, students are presented with the following situation and asked to plot a curve with the distance travelled from Station A on the x-axis and the time taken on the y-axis.
<table>
<tr><th>Train 99991</th><th>Station Name</th><th>Arrival time</th><th>Departure time</th><th>Distance from Station A (in km)</th></tr>
<tr><td></td><td>A</td><td></td><td>10:00</td><td>0</td></tr>
<tr><td></td><td>B</td><td>14:30</td><td>15:00</td><td>225</td></tr>
<tr><td></td><td>C</td><td>16:30</td><td>16:40</td><td>300</td></tr>
<tr><td></td><td>D</td><td>18:40</td><td>19:00</td><td>400</td></tr>
<tr><td></td><td>E</td><td>21:00</td><td></td><td>500</td></tr>
</table>
Once students mark these points on the graph, ask them to highlight the points, A, B, C, D, and E. Students might face different challenges here. First of all, the table provides the time and the cumulative distance from Station A. Students will have to calculate the time and distance between two consecutive stations. Once that is done, students will plot a graph. Finding the time is a straightforward task but might bring out students' ideas about how to subtract times. Please be watchful of those. Once the plot is made, ask students to fill in the following table.
<table>
<tr><th>Stations</th><th>Distance travelled(x)</th><th>Time taken(y)</th><th>x/y</th></tr>
<tr><td>A to B</td><td></td><td></td><td></td></tr>
<tr><td>B to C</td><td></td><td></td><td></td></tr>
<tr><td>C to D</td><td></td><td></td><td></td></tr>
<tr><td>D to E</td><td></td><td></td><td></td></tr>
</table>
If you find it useful, you could ask students to make this table first and then plot the graph. Various prompts could be used here to lead the classroom discussion. We provide some examples here.

What is the shape of the graph?
Which two cities were farthest from each other?
In the table you prepared, do you notice any pattern?
What would you call the quantities represented in the column x/y?

</td></tr>
</table>

Activity 2: In this activity, students will work on the idea of probability and make use of notations of ratios. The game Jamuni plays with red and black balls in a bucket is one that you could actually play in the class to introduce ideas of probability.

Students are exposed to the following situation.
At a stall at the railway station, Jamuni finds three buckets. Bucket A contains 2 red balls and 4 yellow balls. Bucket B has 4 red balls and 8 yellow balls. Bucket C has 7 red balls and 14 yellow balls. Jamuni and her friends come up with interesting questions.
Aman asked Jamuni: 'If you pick one ball from each bucket, what is the probability that the ball will be red?"
At this moment, there are two ways to handle this. Teachers can actually make such buckets and ask students to see what happens empirically. Of course, with fewer number of trials, the probability won't be the same as the mathematically calculated number. The teacher could use this opportunity to talk about what probability means in reality. How many trials are needed to see that the probability for a red ball to be picked up from bucket A is actually ½.

	Red balls	Yellow balls	Probability of picking a red ball
Bucket A	2	4	
Bucket B	4	8	
Bucket C	7	14	

Once students fill in this table, ask the following questions.
- Do you see any pattern in the last column of the table?
- Can you explain why you see this pattern?
- Why do the values in the last column of the table remain the same, even though the number of balls in each bucket is different?

TEACHER REFLECTIONS AND OBSERVATIONS

- Describe the concept of linear equation and probability in your own words. Where do you think the ideas of ratios and proportions could be used in these concepts?
- Are there any other mathematical ideas that we can use to illustrate the concepts of ratios and proportions? Elaborate on these ideas.
- After all the activities are done, reflect on the following prompts.
- List three challenges student faced while working on this lesson.
- List three strategies that you did not expect from students (right or wrong).

UNIT 4: APPLYING RATIOS AND PROPORTIONS
Lesson 2: Compound Ratio and Proportion

LESSON OVERVIEW

In the bucket-ball game that Jamuni plays, students will write the probability for a red or a yellow ball picked. In the second lesson, students work on the arithmetic of ratios. They find compound ratios, apply ratios to a recipe problem and practice working on real life contexts. Compound ratio requires a logical understanding of why two quantities that are directly proportional to each other, when multiplied by another two quantities that are directly proportional to each other, remain directly proportional to each other.

Learning Objectives

Students learn to

- Apply their understanding of proportional reasoning to a situation involving probability.
- Describe and apply compound ratio and proportion.
- Relate concepts of ratio-proportion to other ideas in mathematics.

DISCUSSION ON MATHEMATICAL IDEAS

In this section, we discuss a few examples from the lesson and talk about students' thinking about those examples. We provide prompts for teachers to build a discussion on these tasks and, at the same time, understand the mathematical trajectory of the lesson.

Tasks and their mathematical relevance	Students' thinking and possible strategies for facilitating
Activity 1: As in the last lesson of this final unit, in this lesson, students work on problems that involve proportions and compound proportions found in real life. The context of work done and time spent on it is used.	Students work on a situation where work done is changed and the time allotted is also changed. Jamuni and her friends are back home from the mela. When Jamuni visits her parents' work site, she observes that a team of construction workers have constructed a wall 400 metres long in 12 days by working 8 hours every day. How long will it take to construct a wall 600 metres in length if the workers put in 9 hours every day? Here, students are expected to understand the ratio of work done with time spent and use that ratio to find the number of days for the task that involves building a 600-metre wall. Continuing with ratios, students work on the following task. Jamuni's mother deposited Rs. 4,500 in a bank and received an interest of Rs. 360 after two years. How much interest will she receive at the end of five years if she deposits Rs. 6,000? Here, students first find the rate of interest which is the ratio between the amount deposited and the number of years and then use that ratio to find the interest on Rs. 6,000. The teacher can discuss here how the interest rate is a ratio and, in this form, what other ratios students observe.

<table>
<tr>
<td>

Activity 2: This activity uses another familiar situation and allows students to practice their knowledge of ratios and proportions.

One evening, Jamuni, Aman, Leena and Sahir are sitting at the local tea shop. The recipe used by the shopkeeper to make tea for four people is provided here.

- Tea powder - 2 teaspoons
- Sugar - 4 teaspoons
- Milk - 12 teaspoons
- Water - 20 teaspoons

After half an hour, Jamuni's parents join the group and they all decide to have a cup of chai. List how much of each ingredient will be required to make tea for six people if the tea must taste exactly the same as that made earlier.

- Tea powder - _________ teaspoons
- Sugar - _________ teaspoons
- Milk - _________ teaspoons
- Water - __________ teaspoons

</td>
<td>

The situation presented to students involves understanding a recipe for tea and using that to scale up for six people. This is a familiar activity, and the teacher should make sure to ask questions about how students arrived at their answers.

List how much of each ingredient will be required to make tea for six people, if the tea must taste exactly the same as that made earlier.

- Tea powder - _________ teaspoons
- Sugar - _________ teaspoons
- Milk - _________ teaspoons
- Water - __________ teaspoons

To find the answers to these questions, students could make use of internal or external ratios.

</td>
</tr>
</table>

TEACHER REFLECTIONS AND OBSERVATIONS

- Write a reflection about what you think your students have learned throughout this module.
- Design a ratio-proportion problem using percentages and a daily life context. Describe how ideas of ratios are used in the problem.
- Design a geometry question that requires an understanding of ratios and proportions Describe your thinking process while you were designing the questions.
- Ask your peers to solve these designed questions.

NOTES

NOTES

CONNECTED LEARNING INITIATIVE

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