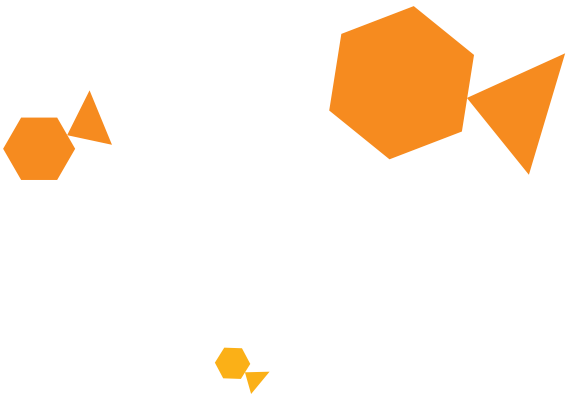
Teacher's Handbook



CLI CONNECTED LEARNING INITIATIVE

सत्यमेव जयते

This Handbook Belongs to

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The Connected Learning Initiative (CLIx) is a technology enabled initiative at scale for high school students. The initiative was seeded by Tata Trusts, Mumbai with Tata Institute of Social Sciences, Mumbai and Massachusetts Institute of Technology, Cambridge, as founding partners.

Collaborators:

Centre for Education Research & Practice - Jaipur, Mizoram University - Aizawl, Eklavya - Madhya Pradesh, Homi Bhabha Centre for Science Education - Mumbai, National Institute of Advanced Studies - Bengaluru, State Council of Educational Research and Training (SCERT) of Telangana - Hyderabad, Tata Class Edge - Mumbai, UNICEF Chhattisgarh - Raipur, Govt. of Rajasthan, Govt. of Mizoram, Govt. of Chhatisgarh and Govt. of Telangana.

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CLIx MATHEMATICS

Geometric Reasoning Module

"To create learning communities where all students and teachers engage with mathematics, and move collaboratively towards an understanding of mathematics as a discipline."
- Vision for Clix Maths

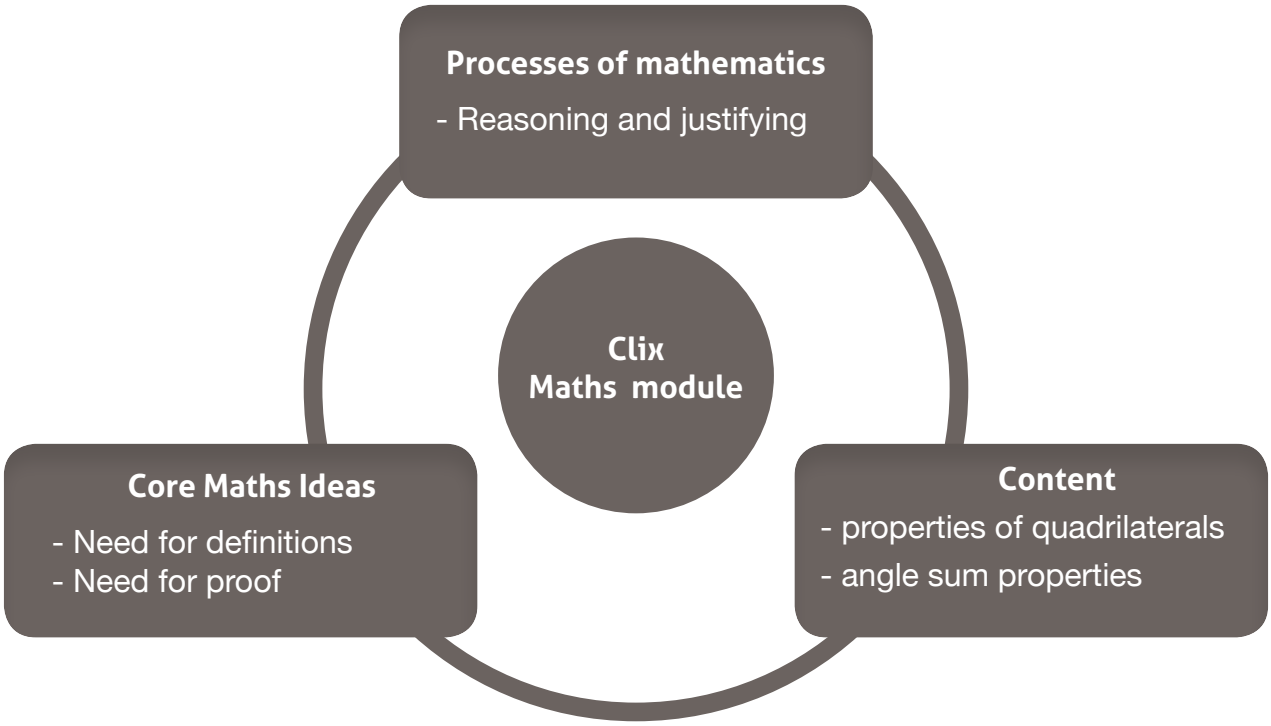
Objectives

The Clix Maths modules aim to touch upon three aspects that are interrelated, and yet deserve separate mention.

- *Content: the geometric concepts and ideas that the students are supposed to learn.*

- *Core Mathematical Ideas: for instance - what is the nature and purpose of definitions in maths, what are proofs, why are they needed, and what it means to 'prove' or 'disprove' something, etc*

- *Processes of Mathematics: the students are expected to actively engage in - reasoning, making and testing conjectures, discussing and debating mathematical ideas, articulating their mathematical ideas and trying to prove or disprove them to others,etc.*



Organisation of Module

	Unit Name	Description	Digital Resources
Unit -1	Concept of shape	This unit addresses some foundational concepts that are mandatory to build further geometric ideas on. The focus of this unit would be to develop a robust understanding of 2D shapes - which are the primary objects of thought in the Euclidean geometry that forms the bulk of the high school Geometry curriculum.	Turtle Logo Activity
Unit -2	Analyzing and Describing shapes	Based on the van Hiele levels of Geometric Thinking, this is the next level of reasoning. The lessons in this unit are woven around a digital game, which encourages students to start analyzing and talking about the attributes and properties of shapes, rather than view them as a whole object or symbol.	Mission 1 & 2 Police Quad Game
Unit -3	Classifying and Defining shapes	From looking at properties of individual shapes, the lessons in this next unit help students progress to an understanding of classes of shapes, and develop shared definitions collaboratively.	Mission 3 Police Quad Game
Unit -4	Property-based reasoning	This is a critical stage, where students step into the realm of relational understanding and inferential if-then reasoning. Alternate definitions and the need for definitions are also discussed here. The game extends up to this unit.	Mission 4 Police Quad Game
Unit -5	Understanding the need for proof	The final unit starts with a lot of conjecture making, testing and informal reasoning activities. Students come up with hypotheses through inductive reasoning, verify them, but also learn why verifications do not count as 'proofs'. They understand that though inductive reasoning is an important way of doing mathematics, it is fallible - and hence the necessity for deductive proofs.	Geogebra Activity

UNIT 1: CONCEPT OF SHAPE
(90 min)

OVERVIEW

After going through this unit, the students will develop a robust understanding of 'shape' by exploring which factors change shape and which don't (change in orientation, for instance) and why. The unit will help students grasp the concept that in mathematics, a 'shape' is defined by a set of defining properties, and not by its visual appearance. For this, the students will go through two activities, followed by a whole class discussion. In the first activity they construct shapes with matchsticks, and examine the effect of different actions on the shapes. In the second, they go to the computer lab and work on Logo Turtle programming to construct squares, and develop a working definition of a square. In the whole class discussion that follows, the students participate in a teacher facilitated discussion to tie it all in.

LEARNING OBJECTIVES

After going through this unit, the students will

• Develop an understanding of 'shape' and the effect of different actions like translation, rotation, reflection and deformation on shape

• Be able to identify squares and triangles, distinguishing clearly between examples and non-examples

• Develop working definitions of 'square' and 'triangle'

PRIOR MATHEMATICAL UNDERSTANDINGS REQUIRED

• Initial notion of shape

• Basic (visual) identification of squares and triangles

PRIOR UNDERSTANDING IN ICT REQUIRED

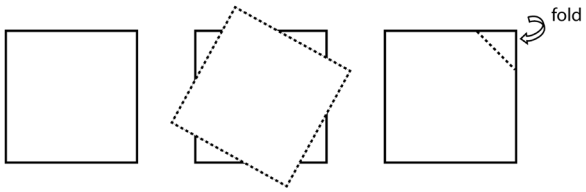
• Familiarity with basic actions of Logo Turtle programming *(only simple commands – Forward, Right, Left and Clear Screen are required)*

LESSON & ACTIVITY-WISE BREAK UP

Lesson	Activity	Activity Name	Duration (Recommended)	Mode
Lesson 1: What is Shape	1.1	Exploring Matchstick Shapes	35	Hands-on (Self learning)
	1.2	Constructing Squares	35	Digital
	1.3	Class discussion	20	Class discussion
Assessment 1	1.4	Assessment : Unit 1	35	Worksheet

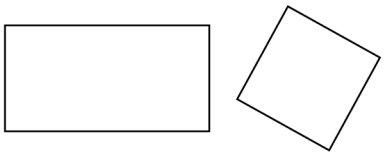
Ideas for Facilitating Classroom Discussions (20 minutes)

In this classroom discussion you will connect and consolidate learnings from Activity 1.1 & 1.2 so that students understand the idea of shape and actions that preserve or change a shape. Through discussions students should develop a working definition of the square. Some activities that you can perform in the classroom are mentioned below.



Using a paper cut-out of a square transform and deform the square and discuss if the shape changes or remains a square in each of these instances

Draw a rectangle and a turned square on the board and ask the class. The lesson aims to address this.



Ask students to justify or give reasons for their answers.

Ask them what transformations change the properties of a shape (stretching, deforming etc) and what transformations do not change the properties. (Rotation, reflection, translation). Let them go through these transformations on a shape and convince themselves which of them preserve properties and which ones don't. Help them understand that shapes are determined by their properties and as long as properties don't change, the shape doesn't change too. Ask the students to describe a square in terms of its properties in their own words and help them to form a definition. Similar exercise could be done for triangles.

Things to Remember

The Turtle logo activity can also be done outdoors, with some students giving commands and other students following the commands (like the turtle on the computer) and tracing their path. Some students may still be describing shapes based on their appearance. In such cases, emphasise on what happens to the properties when a shape is transformed. Students might have the misconception that when a square is turned it does not remain a square. Help students to understand that the appearance of the shape does not define it but its properties do. You may give some real life examples to establish this fact.

Student Observations	
Did the students engage in discussions? Give some examples	
Were there students who did not understand the concept of a shape? Why do you think so?	
What are some of the geometry mistakes that you were able to identify among students? Were you able to address them with the students and how? Describe at least one.	
What are some of the working definitions of a square that were developed during discussions? Note down at least 3.	

Self-Reflection	
Did the students understand what was being discussed? How can you be sure ?	
What problems arose? What did you do to solve them ?	
Did all students participate in the discussion? What did you do to include all students in the discussions ?	
Would you like to share something interesting that happened while teaching this class?	

UNIT 2: ANALYSING AND DESCRIBING SHAPES

(235 min)

OVERVIEW

After going through this unit, the students will able to analyse and describe shapes based on their properties, instead of the visual whole. They will understand and start using appropriate mathematical terms and vocabulary while talking about shapes. This unit will also help students develop an initial notion of deductive reasoning. Most of the lessons in this unit are situated in a game environment. The unit starts with a brief hands-on sorting activity to recap some useful terms and properties. After this, the students play the first two missions of the game Police Quad over at least 3-4 sessions, interspersed with focused whole class discussions. The first mission helps students analyse and understand important properties of shapes, and the second mission helps them use these properties in a game of strategy.

LEARNING OBJECTIVES

After going through this unit, the students will

- Be able to look at shapes analytically, based on their component parts and properties instead of what they 'look like' – and in the process, move from lower (or less sophisticated) levels to higher (or more sophisticated) levels of thinking about 2-D shapes.

- Understand some important terms and concepts related to attributes and properties of shapes (meaning of sides and angles, different types of angles, concept of parallel lines, etc.) And use them appropriately and precisely as and when required.

- Acquire an initial understanding of deductive reasoning – including the idea of arriving at a definite conclusion based on the given facts.

PRIOR MATHEMATICAL UNDERSTANDINGS REQUIRED 4

- Understanding the concept of shape, and the idea that a shape has properties.

- Understanding some basic properties of 2D shapes. For example:

- Distinguishing between straight and curved line/side

- Knowing and recognizing the number of sides and vertices of a shape

- Understanding the meaning of 'angle'

- Knowing the terms right, acute, obtuse and reflex angles and identifying them

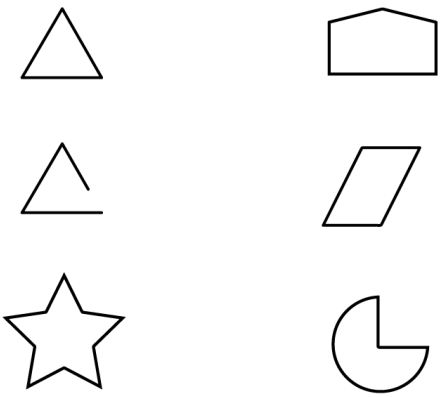
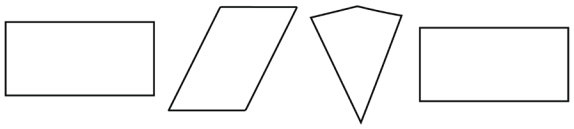
- Understanding the concept of parallel lines and identifying pairs of parallel sides in a shape

PRIOR UNDERSTANDING IN ICT REQUIRED

- Familiarity with drag, drop and other common interface use functions

Lesson	Activity	Activity Name	Duration (Recommended)	Mode
Lesson 2: Analysing Shapes	2.1	Sorting Shapes	35	Hands-on
	2.2	Police Quad – Mission 1	70	Digital
	2.3	Class discussion	40	Class discussion
Lesson 3: Analysing Shapes	2.4	Police Quad – Mission 2	70	Digital
	2.5	Class discussion	20	Class discussion
Assessment 2	2.6	Assessment : Unit 2	35	Worksheet

Lesson 2

Ideas for Facilitating the classroom Discussions (40 minutes)	
In this classroom discussion you will help students • crystallize the learnings from the game • enable them to develop more effective strategies for the next session of the game. Alternate conceptions that students hold can be understood and addressed. • Looking through student work before the discussion, will enable you to understand student thinking to guide the discussion. Some suggested tasks to help students learn to describe shapes in terms of their properties are outlined below. You could let students work through these tasks in groups and engage in group discussions.	You can draw shapes below on the board and ask them to describe these shapes. 
To give students further opportunities to talk about shapes and their properties, ask students to identify the similarities and differences between the shapes. • Draw the following shapes (and a few more pairs) and ask students to find similarities and differences in each pair. • While doing this exercise, please ask students to look at all possible attributes.	

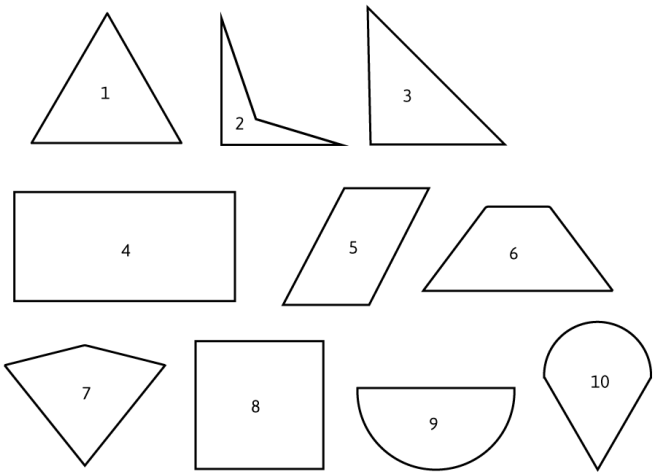
In order to discuss shapes more and understand the idea of classification based on the properties of shapes, the following task can be useful:

• Draw a set of shapes on the board. See the shapes on the right as an example

• Ask students to classify these shapes based on different criteria. (They have done this activity in their workbooks). Some examples of these criteria are ‘shapes that have curved sides and those that do not have’. OR ‘shapes that have at least one right angle and those that have no right angle’, etc. You can also select a different set of shapes and perform the same exercise.

In order to discuss shapes more and understand the idea of classification based on the properties of shapes, the following task can be useful:
• Draw a set of shapes on the board. See the shapes on the right as an example

• Ask students to classify these shapes based on different criteria. (They have done this activity in their workbooks). Some examples of these criteria are ‘shapes that have curved sides and those that do not have’. OR ‘shapes that have at least one right angle and those that have no right angle’, etc.
You can also select a different set of shapes and perform the same exercise.



Things to Remember

• Give students opportunities to talk about these properties and sort shapes on the basis of these properties.

• Students may have alternate conceptions like triangles have to be ‘up right’, or that one arm of a right angle has to be horizontal always and the other vertical etc.

• Address these notions by giving appropriate figures that would challenge these notions. For example you could include a triangle with a very small side, or a turned right triangle in the figures that you choose to draw.

My Notes

Student Observations	
How did students discuss the game in the class? Give some examples.	
What are the different criteria that students came up with while doing the sorting activity? Note down atleast 4.	
What are some of properties that students understood? Note down at least 3.	
What were the common mistakes observed by you? How did you address them?	

Self-Reflection	
In the group activity,did the students discuss the given task? Note down some discussions.	
Was it easy for you to organize the group activity? Note down some of the challenges that you faced.	
How were you able to use students' experience of playing the game in the classroom discussion?	

Lesson 3

Ideas for Facilitating the classroom Discussions (20 minutes)

You can ask students describe their game strategies by asking them:
- how they decided on the relevant questions to
- how they eliminated the shapes based on responses given by the system.
- why they were getting the response - 'I don't understand!' from the system. Help them identify the mathematical error in such questions.Help them see the need for precise and accurate language in formulating the question and discuss the strategies that will enable them identify the culprit using minimum number of questions.Encourage students to talk about the different strategies that they used while playing the game and discuss the merits and demerits of these

Hints: In order to identify the culprit using minimum number of questions,one should ask a question which divides the set of shapes into roughly two equal groups - one that has the mentioned property and one that does not have that property. Further questions will be guided by the same principle.Notice that now they are looking for properties of sets/classes of shapes - Each time a question is asked and answered in yes or no, a set/class of shapes is eliminated and another class remIn this classroom discussion students share their experiences from the game play.

Draw a set of shapes on the board to play mission 2 in the class. Think of one shape as a culprit and ask the students to ask you questions to enable them to guess the culprit. Answer 'yes/no/ I don't understand' and let the students identify the culprit that you have thought of. One set of shapes is drawn for you below..
- Discuss terms like right angle, parallel lines, reflex angles, adjacent sides etc, that you find relevant, using more figures.
- Ask the students what they understood by the words 'at least'. Also discuss the relevance and importance of qualifiers like 'at least', 'more than', 'less than', 'exactly' etc.. You may use contexts that are familiar to students.
- Discuss questions where the response is 'yes' and those where the response is 'no'. Elaborate on the decision that they look when the response from the system was 'no'. Specifically discuss questions like ''does it have at least one right angle?' - what would they do if the system gives a response 'yes' ? Which shapes can be eliminated? How would they respond to a system response 'no'? Which figures would be eliminated now?

Things to Remember

- Students may have a misconception where they think that parallel lines have to be of equal length. In order to address this, draw parallel sides which are of unequal length and ask them if they are parallel. Also, draw parallel lines in different orientations. Identify if there are other misconceptions like this and address them.
- Encourage students to look for properties shared by classes of shapes, rather than properties of individual shapes. Help them see that shapes classes are identified by the properties shared by all members of the class.

My Notes

Student Observations

Mention some of the strategies used by the students while playing the game that they shared in the classroom.	
What are the new concepts that students learnt in this game?	
Did they understand the difference between guesswork and thoughtful strategies? Give reasons.	
Mention some of the students, difficulties while playing this game.	

Self-Reflection

How do you think mission 2 of the game and the classroom discussion will help students to develop deductive thinking abilities?	
How will deductive reasoning skill help students to learn geometry better?	
Do you think students are learning from their own mistakes? Give examples.	

UNIT 3: DEFINING AND CLASSIFYING SHAPES (235 min)

OVERVIEW

In this unit students learn to look beyond describing individual shapes and their properties to the common properties shared by a class of shapes. Thus they acquire the ability to define and classify shapes. They also understand the need for precise and accurate definitions. For this, the students will go through a series of activities. In the first activity they play a game involving identifying the common property(ies) of a set of shapes, given sets of examples and non-examples. In the second activity, the class arrives at a shared definition of a quadrilateral through a Socratic dialogue, realising the need for precise definitions in Mathematics. Then they explore their understanding of special quadrilaterals parallelogram, rectangle, square and rhombus and arrive at working definitions for the same. This is followed by two activities where they construct special quadrilaterals using Turtle Blocks and on dot paper. They then use the ideas used in the constructions to develop working definitions for special quadrilaterals. In the final activity of the unit, students create property lists for of each of these classes of special quadrilaterals. These activities ar e interspersed with whole class discussions where the teacher draws attention to the key points.

LEARNING OBJECTIVES

After going through this unit, the students will
- be able to classify shapes on the basis of common properties
- understand that a 'class of shapes' is defined by a set of common properties
- be able to look for and identify the common properties shared by a class of shapes and use these properties to arrive at definitions for a class of shapes
- develop shared (working) definitions for quadrilateral, parallelogram, rhombus, rectangles and square, and be able to provide examples and non-examples of each
- understand the need for precise definitions

PRIOR MATHEMATICAL UNDERSTANDINGS REQUIRED

- basic understanding of properties of shapes like 'has equal sides', 'has a right angle' etc.
- ability to describe a shape in terms of its properties

PRIOR UNDERSTANDING IN ICT REQUIRED

- Familiarity with basic actions of Logo Turtle programming (only simple commands – Forward, Right, Left and Clear Screen are required)
- Familiarity with drag, drop and other common interface use functions

LESSON & ACTIVITY-WISE BREAK UP

Lesson	Activity	Activity Name	Duration (Recommended)	Mode
Lesson 4: Classifying Shapes	3.1	Police Quad Mission - 3	35	Digital
	3.2	Classifying and Defining Shapes - Class Discussion	20	Class discussion
Lesson 5: Defining Shapes	3.3	What is a quadrilateral	45	Hands on
Lesson 6: Defining Special Quadrilaterals	3.4	Constructing Rectangles	30	Digital
	3.5	Exploring Special Quadrilaterals	30	Hands on
	3.6	Defining Special Quadrilaterals	30	Class discussion
Lesson 7: Properties of Special Quadrilaterals	3.7	Making Property Lists	45	Hands on
Assessment 3	3.8	Assessment - Unit 3	35	Worksheet

Lesson 4

Ideas for Facilitating the classroom Discussions (20 minutes)

The purpose of this activity is to consolidate the learnings from the game and to give more practice in looking for properties that are common to a class of shapes and those that distinguish them from another class. Students understand that these properties shared by a class of shapes lead to definition of shape classes. In the class, you can discuss questions like:
- What defines a shape?
- What is a class of shapes? How do we decide whether a given figure will belong to a particular class? You can also perform a small activity in the class: Draw two sets of shapes on the board. Ask the students to identify a property that all members of one set have and none of the members of the other set have. Please draw students' attention to the common properties of a set of a shapes.
You can draw another set of shapes on the board and ask the students to identify all the common properties. Students can be asked further to come up with a name for the set of shapes. Draw some other shapes and ask students whether these shapes are a part of that class.

Ask students to draw their own shapes having properties like ‘having at least one right angle‘ or ‘having a pair of parallel sides’ etc. Note that all shapes that they draw should have the given property. Observe the variations they bring into their set of shapes keeping one property fixed.

Things to Remember

Help students to understand that:
Shape classes are defined by shared properties. All shapes that belong to a particular class share some properties. Shapes that do not have that property do not belong to the class. For example all shapes that are 1) closed and made up of 4 straight sides only 2) all sides are equal 3) All angles are right angles belong to the class of squares. If a shape is such that it does not have property 3, then it CANNOT be a square.

My Notes

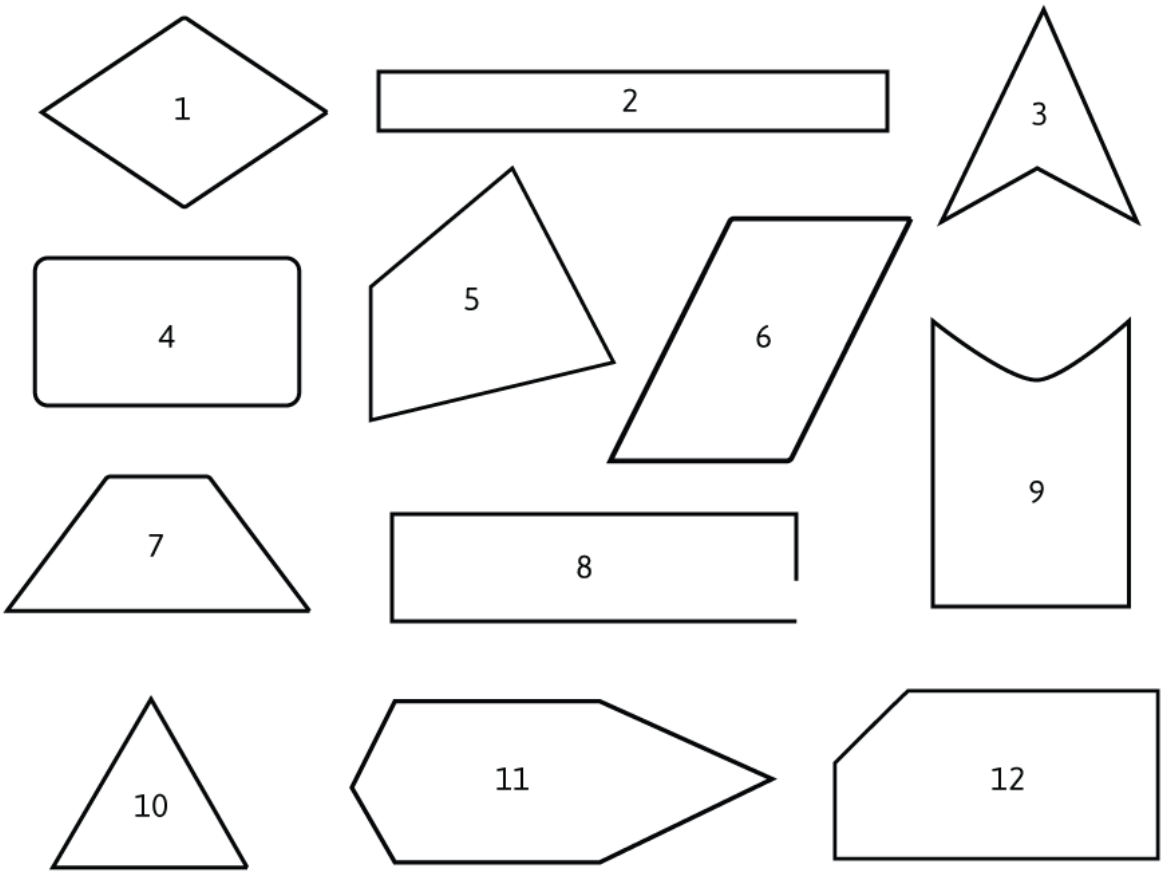
Student Observations	
What did students understand by ‘class of shapes’ in this class discussion?	
How do you think mission 3 of the game helped students to understand classes of shapes?	
Were students looking at all the attributes of the shape like parallel sides, equal sides,, angle properties etc, while deciding the class? How do you know?	
Do you think that students have started looking at the shapes analytically?	

Self-Reflection	
What were the challenges that you faced while discussing the idea of classes of shapes?	
How do you think students mistakes can be used as an input for your classroom teaching?	

Lesson 5

Ideas for Facilitating the classroom Discussions (45 minutes)

In this classroom discussion you will connect and consolidate learnings from Activity 3.1 & 3.2 so that students understand the idea of a class of shapes and understand quadrilateral as a class. This classroom discussion is based on the the article by Lionel Pereira-Mendoza, NCTM. The article was written to discuss an activity to develop understanding of role of definitions in Mathematics
Reference: Lionel Pereira-Mendoza (1993). What Is a Quadrilateral? National Council of Teachers of mathematics, The Mathematics Teacher, Vol. 86, No. 9 (DECEMBER 1993), pp. 774-776.
Before conducting this classroom discussion read the document: What is a quadrilateral-document which is provided *** (location).

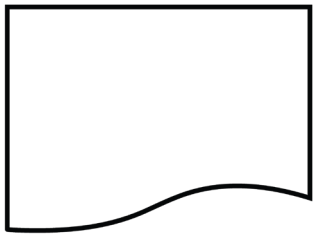


Ask the students to work individually and then in groups and classify the above shapes as quadrilaterals and non-quadrilaterals. You can add your own shapes during the discussion. Discuss following points with them:
- Why did they classify some shapes as quadrilaterals and others as non-quadrilaterals?

- Ask the students to justify their answers.

- Ask the students to define a quadrilateral.

Conduct a discussion in the classroom in order to develop a precise definition of quadrilateral. Produce counter examples for imprecise definitions suggested by the students. E.g. When a quadrilateral is defined as a 4 sided figure by the students, you can draw an open figure with four straight lines and ask if they consider that a quadrilateral. If they say that it has to have four sides and it is a closed figure, then draw a shape with a curved side and so on.



Also help them see that all 4 vertices of the quadrilateral have to be on the same plane. Discuss the list of common properties shared by all quadrilaterals. You can also discuss triangles a class of shapes. Ask the students to revisit the worksheet that they worked on and if they would like to change some of their earlier responses. Ask the reasons for change in their response.

Things to Remember

Help students to form a precise definition of the quadrilateral. A definition has to include all members of a class and exclude those that don't belong to the class. A definition of the quadrilateral will be:
A quadrilateral is -

1) A closed figure

2) having 4 straight sides and

3) all vertices of which lie on a plane

You can ask them questions like is square a quadrilateral? Justify.
Is parallelogram a quadrilateral? etc.

My Notes

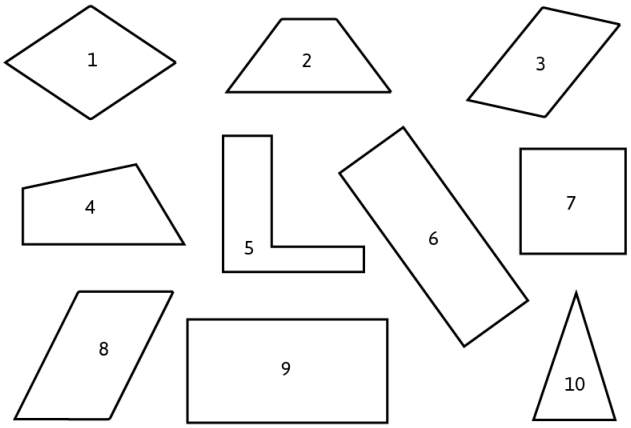
Student Observations	
While solving the worksheet for the first time, what mistakes did students make? Give 3 examples.	
Were students able to understand the importance of precise definitions?	
What was the final definition of a quadrilateral that the students arrived at?	
What did the students understand by the concept of 'class of shapes?'	

Self-Reflection	
Narrate your experience of conducting dialogue around definitions of quadrilaterals.	
How according to you was the presence of non-examples (in the worksheet) along with examples useful?	
Do you think the game is helping you better facilitate the class? How?	
How did you connect the students' learning from mission 3 to this classroom discussion?	

Lesson 6

Ideas for Facilitating the classroom Discussions (30 minutes)

In the class discussion you will help students to define special quadrilaterals and understand the properties of special quadrilaterals. They will arrive at shared definition of special quadrilaterals. Starting from the definitions the students give for these quadrilaterals, the class moves towards a shared definition for these quadrilaterals through a dialogue (similar to what was done for the quadrilateral activity.) You could refer to students' work in the worksheet and do following tasks in the classroom. Ask students to divide following shapes in parallelograms and non-parallelograms:



Ask them to form their own definition of a parallelogram. You can add some more shapes to this set or create your own shapes. Refine their definitions by using counter examples as suggested in the example of quadrilaterals. For other special quadrilaterals you could do similar activities and ask the students to create their own definitions for rectangles, rhombuses and squares.

Things to Remember

While helping the students to form precise shared definition of special quadrilaterals, have a variety of examples. Please notice if the students are going by the visual appearance of the quadrilateral or looking at its properties.
You will help students to understand:
- Definitions evolve from common properties of classes of shapes.
- Definitions are needed in order to have a shared understanding of classes of shapes, and be able to reason together about these shape classes.
- We evolve shared (working) definitions of squares, rectangles, parallelograms and rhombuses to use in class.

My Notes

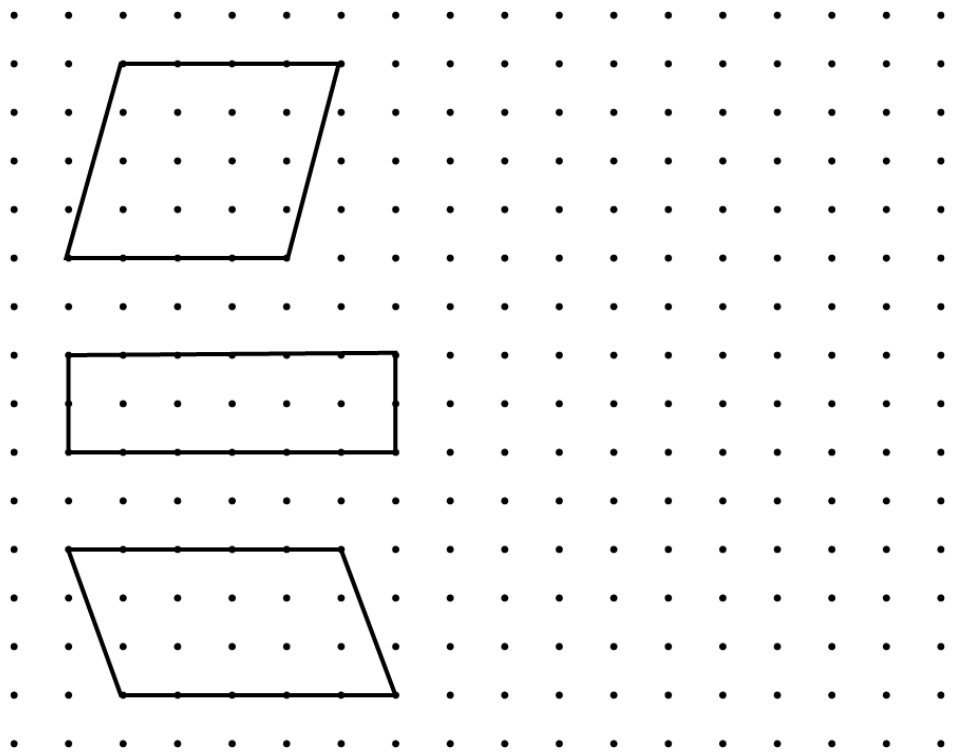
Student Observations	
What were the students' difficulties in constructing rectangle in Logo? Mention a few. .	
After doing the mentioned tasks, write two definitions of the parallelogram that the students came up with before coming to the final definition.	
final definition. Were there any misconceptions among students regarding rectangles and squares? If yes, then mention any two.	
Were there any misconceptions among students regarding parallelograms and rhombuses? If yes, then mention any two.'	

Self-Reflection	
What were your difficulties in handling student's misconceptions about special quadrilaterals? How did you handle them	
What according to you were the reasons for students' misconceptions?	
Mention some of the questions that students came up with in the discussion which according to you were very important.	

Unit 3 - Lesson 7

Ideas for Facilitating the classroom Discussions (30 minutes)

In this activity students work individually (or in predetermined small groups) to construct squares, rectangles, parallelogram and rhombus on dot paper. Students will draw the shapes on a dotted sheet.



Ask students to articulate in what way the given parallelograms are different. Ask them to draw a parallelogram that is different from the ones given. Ask them how they know that it is a parallelogram. What properties of a parallelogram do they use to draw it? Also, ask them to discuss the properties of parallelograms with respect to side, angle and diagonal properties. You may continue this exercise for other shapes that students have drawn. Have a discussion across smaller groups on what are the common properties of specific quadrilaterals like parallelograms, rhombuses, etc. Consolidate and make a list of all side, angle and diagonal properties of special quadrilaterals that they can come up with in the space provided in the worksheet. .

Things to Remember

Help students to explore the diagonal properties in detail. Unlike angles and side properties, this property is not explicitly (directly) visible. Diagonals is the extra attribute that we add to the shape. Encourage them to compare the property lists of the different special quadrilaterals that they have come up with. .

My Notes

Student Observations	
Mention some of the similarities and difference that students came up with in different parallelograms..	
Mention some of the discussions that students were having in groups while coming up with similarities and difference between different parallelograms, rectangles, squares and rhombuses that they drew.	
Was it easy for students to come up with diagonal properties?	

Self-Reflection	
How do you think working in groups helped students to learn better? You can think about your experiences for this activity or other activities.	
Do you think students were learning from each others' mistakes? How?	
Mention some of the challenges that you faced while doing this discussion.	

UNIT 4: PROPERTY-BASED REASONING
(270 min)

OVERVIEW

IThrough the activities in this unit, students will gradually begin to understand the complex relationships among classes of special quadrilaterals - in particular, hierarchical class relationships. This understanding of class inclusion relationships will be consolidated by representing these relationships in the form of Venn diagrams. The unit also sensitises the students to the existence of alternate definitions and the need for definitions - particularly shared definitions, for better communication of ideas. In the previous unit students made property lists of special quadrilaterals. In this unit, they organise these property lists in a manner that helps them see hierarchical relationships amongst the different classes of shapes. This is followed up by the game Police Quad (Mission 4) - which gives them an opportunity to use this informal understanding of class inclusion and leads to further questions, discussions and consolidation of this idea. Students then familiarise themselves with representing relationships between classes of objects pictorially (Venn Diagrams) and consolidate their understanding of class inclusion by representing these relations pictorially. This unit ends with a story-based lesson that helps students realise that it is possible to define a give concept (or term) in different ways, and appreciate the need for shared definitions.

LEARNING OBJECTIVES

Not inclusive of Assessment 4

After going through this unit, the students will
- understand the relationships between classes of special quadrilaterals (parallelograms, rhombuses, rectangles and squares).
- understand and justify class inclusion relationships among special quadrilaterals (for example, why a rectangle is necessarily a parallelogram).
- understand the basic logic of Venn diagrams and apply this logic to classify shapes based on their properties.
- be able to ‘see’ and represent class inclusion relationships among special quadrilaterals pictorially.
- appreciate the need for using shared definitions.

PRIOR MATHEMATICAL UNDERSTANDINGS REQUIRED

- Understanding of classes of shapes and their defining properties
- Ability to identify and list properties (including properties of diagonals) of parallelograms, rhombuses, rectangles and squares.
- Ability to analyse properties of shape classes to find out similarities and differences between them.

PRIOR UNDERSTANDING IN ICT REQUIRED

Familiarity with drag, drop and other common interface use functions.

LESSON & ACTIVITY-WISE BREAK UP

Lesson	Activity	Activity Name	Duration (Recommended)	Mode
Lesson 8: Relationships among Special Quadrilaterals	4.1	Police Quad Mission - 4	45	Digital
	4.2	Creating Property Stacks	45	Hands on
	4.3	Relationships among Special Quadrilaterals	45	Hands on + Class discussion
Lesson 9: Representing Relationships	4.4	Representing Relationships -1	45	Hands on + Class discussion
	4.5	Representing Relationships - 2	45	Hands on + Class discussion
Lesson 10: Discussing Definitions	4.6	What is a Trapezium?	45	Digital
Assessment 4	3.7	Assessment - Unit 4	45	Worksheet

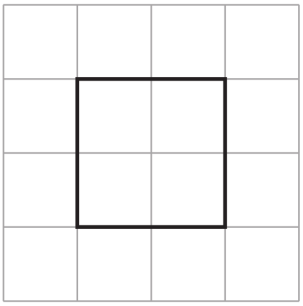
Lesson 8

Ideas for Facilitating the classroom Discussions (45 minutes)

The purpose of this activity is to consolidate learnings from the game and to assimilate the notions of class inclusion. Ask the students to describe the strategies that they used and help students to understand hierarchical classification in this class discussion.

You can draw following figure on the board:

Ask the students if the computer generated the above figure (a square) while generating rectangles. Ask them whether they think the computer was right or wrong. Encourage them to give reasons for their responses.



Help students understand the relationship between special quadrilaterals. Have a discussion on similarities and differences between the properties of two (or more) special quadrilaterals.) You can have a following list on the board and fill up the table with the help of students.

Properties	Square	Rectangle	Parallelogram	Rhombus
1 Closed figure made of 4 line segments only	✓	✓	✓	✓	✓
2 Pairs of opposite sides equal in length					
3 Pairs of opposite sides are parallel					
4 Opposite angles are congruent					
5 Diagonals bisect each other					
6 All angles are right angles/are equal					
7 Adjacent angles are equal					
8 Diagonals are equal in length					
9 Adjacent sides are equal in length					
10 All 4 sides are equal in length					
11 Diagonals are perpendicular to each other					

Ideas for Facilitating the classroom Discussions (45 minutes)

Draw attention to the properties that are common to classes of special quadrilaterals and the properties on which they differ. Ask students:

- What are the properties of a rectangle?

- What are the properties of a square?

- Are all the properties of a rectangle shared by a square as well?

- Can a square be called a rectangle? Why or why not?

- Can a rectangle be called as a square? Why or why not?

- What properties are present in the square but not in rectangle?

Repeat similar questions with different pairs of special quadrilaterals and sensitise students to notions hierarchical classification through the properties table.

Things to Remember

Students find it difficult to understand that a shape can belong to two different classes at the same time. For example, students may find it hard to understand that a square is also a rectangle. Some students may want to define rectangles and squares in such a way that the definition of a rectangle excludes a square from being a rectangle. Discuss the merits and demerits of doing this. Draw attention to more examples where a shape belongs to more than one class. You can have a discussion around if an equilateral triangle is also an isosceles triangle, or if a hexagon is also a polygon, or a quadrilateral is also a polygon etc.

My Notes

Student Observations

What was the initial response of students on the question 'is square a rectangle?' Mention some of the responses along with the reasons that they provided.	
Mention two strategies that students discussed while playing the game.	
What were the examples that you discussed while dealing with the idea of class inclusion?	

Self-Reflection	
Why do you think students find it difficult to assimilate that there can be two different names to one figure? (with reference to the idea of class inclusion.)	
Mention the challenges that you faced while facilitating this classroom discussion.	
How do you think understanding the idea of class inclusion will help the students further?	

Lesson 9

Ideas for Facilitating the classroom Discussions (45 minutes)

The purpose of this activity is to help students understand Venn diagrams and strengthen the understanding of relations of class inclusion among special quadrilaterals by representing them pictorially.
You can ask students to organize themselves in different types of groups. Some suggestions are:

a) boys and girls,

b) those who wear spectacles and those who have the colour blue in their dress,

c) Students of class 9 and girls of class 9.

Discuss with them representations using Venn diagrams and Represent these relationships on blackboard using venn diagrams. Help students understand how they use Venn diagrams to represent classes of shapes and the relations between them. There could be 3 possible types of relationships

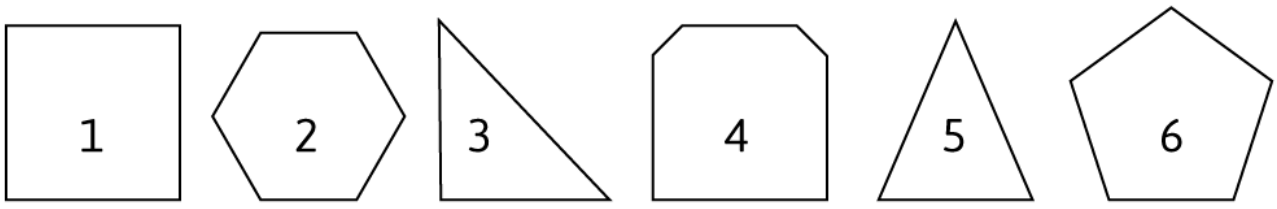
- where one class has some objects in common with another

- where one class is completely contained in another

-where one class does not have any element in common with another.

In a group, ask students to discuss tasks from the worksheet like the following:

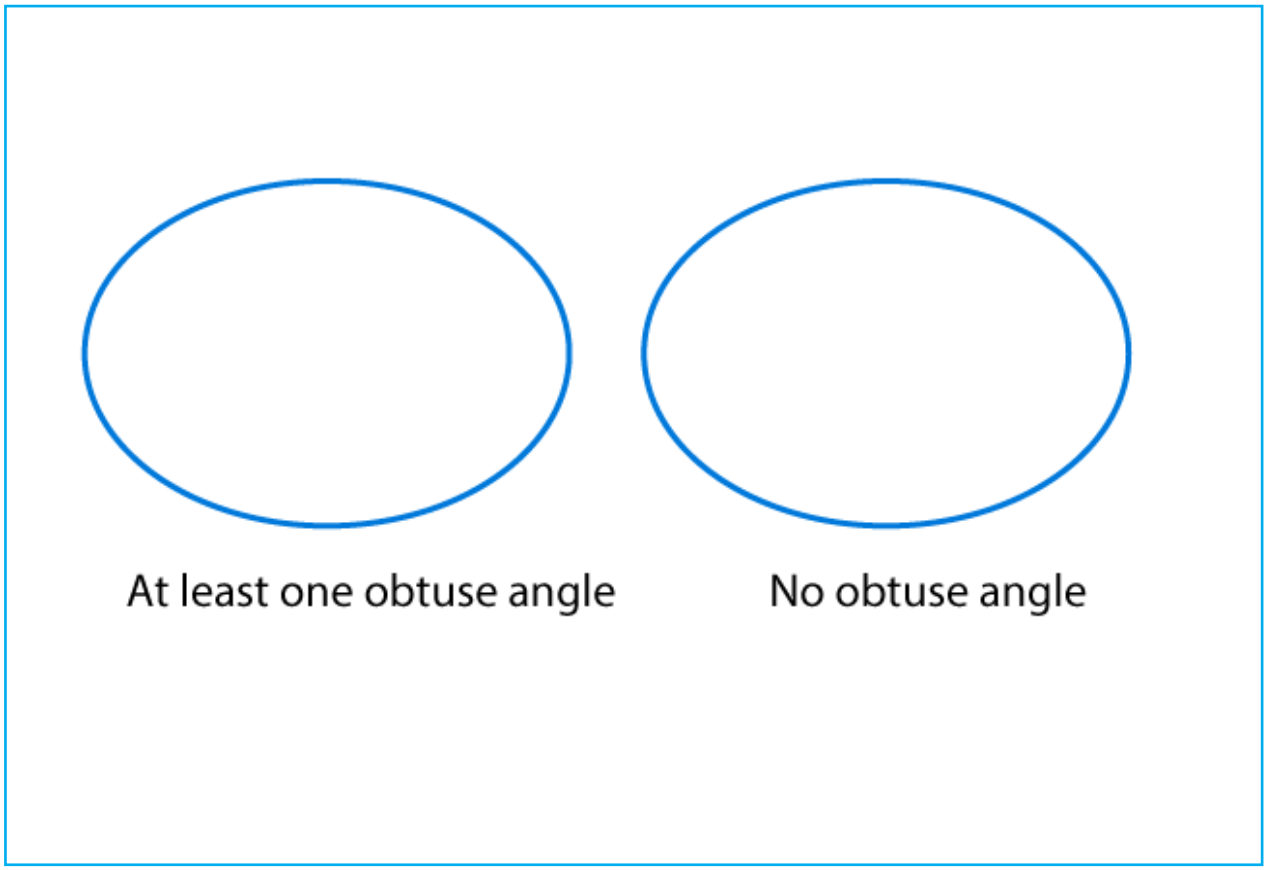
1. Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram.



Familiarise students with these 3 types of relationships with sufficient examples (as given in the worksheet) **Also give them opportunities to come up with examples of classes that are related in a given way.** Once they have understood the different types of relationships between classes and ways of representing them, move on to exploring the relationships between special quadrilaterals and ways of representing them. Tasks like the following from the worksheet may be helpful

Ideas for Facilitating the classroom Discussions (45 minutes)

2. Which of these is the correct representation of the set of parallelograms and quadrilaterals, and why? (You may want to use the definition of quadrilaterals and parallelograms and their properties to see how they are related.)



You can ask students to justify their answers.

Discuss with students - Are all quadrilaterals parallelograms? Are all parallelograms quadrilaterals etc. You may do similar exercises with other special quadrilaterals. Ask the students to explain the relationships between special quadrilaterals. E.g. A square is also a rectangle, squares and rectangles are also parallelograms, etc. All squares are rhombuses, Some rhombuses are rectangles etc Emphasize on looking at the properties of the special quadrilaterals and then deducing the relationships from them. Discuss the following task with the students and help them to come up with a diagram to represent relationships between special quadrilaterals in a diagram: Using the representations chosen in the above tasks, represent quadrilaterals, parallelograms, rhombuses, rectangles and squares in one diagram. Represent quadrilaterals using a rectangle and use appropriate circles to represent the remaining quadrilaterals.

Things to Remember

Help students to understand Venn diagrams starting with real life contextual examples like
- States and country
- Class 9 students and the entire school
- Those who like cricket and those who like football etc.

Students need to understand in this discussion that there are different types of relationships betweenthe sets - they could be disjoint (non- overlapping), overlapping or one set could contain another in it. Help them to understand how to represent these relationships in Venn diagrams with examples. Help them represent the relationships between special quadrilaterals using this understanding.

My Notes

Student Observations

Did students understand the idea of Venn diagrams? Was the use of real life examples helpful?	
How did student explain the relationships between special quadrilaterals? Give any two examples.	
Mention a few difficulties that students faced in this class discussion.	

Self-Reflection

What according to you makes it very difficult for students to understand the relationships between special quadrilaterals? What are the reasons for it?	
Do you think the activities that we performed will help students to develop reasoning skills? How??	
Mention some of the difficulties that you faced in this classroom discussion.	

Lesson 10

Ideas for Facilitating the classroom Discussions (45 minutes)

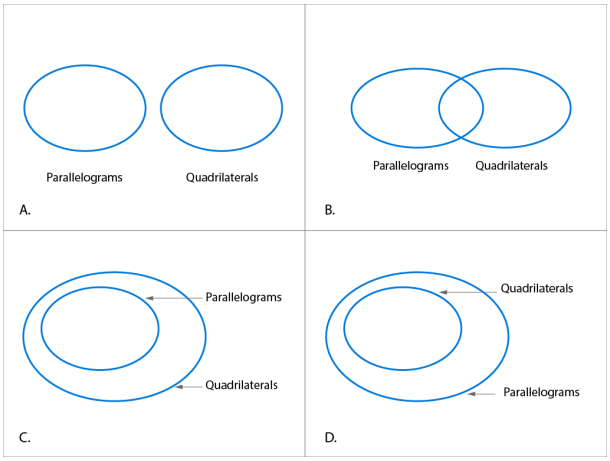
The purpose of this activity is to help students realise that different people may ascribe different meanings to the same word leading to misunderstanding and confusion. The activity emphasises the need for a shared definition. You can give real life examples where a word could mean different things - interest, left, balance, volume etc. You can also discuss some 'non-mathematical' terms like the definitions of city, mountain, goodness, etc. Discuss the story 'what is a trapezium.' Go through the entire story and discuss the definitions that the three children mentioned in the story use. Ask the students to form their own definition of the trapezium.

1. Trapezium is quadrilateral with exactly one pair of sides parallel (Kanasu's definition)

2. Trapezium is quadrilateral with at least one pair of sides parallel (Zo's definition)

3. Trapezium is another name for quadrilateral. (Sahir's definition.)

Ask the students whose definitions is correct and why? Ask students if more than one definition can be correct ?
Ask them to imagine what would happen if each of them has his/her own meaning of trapezium.
Ask the students to represent trapeziums in the following Venn diagram as per the three definitions given above:



Discuss with the students how each definition will lead to a different Venn representation.
Ask students to explain the Venn diagram they created in words by describing the relationships between trapeziums and parallelograms such as

A. All trapeziums are parallelograms.

B. All parallelograms are trapeziums.

C. Some trapeziums are parallelograms.

D. No trapezium is a parallelogram.

Discuss the above definitions in detail.
Ask the students to identify the definition provided in their textbooks. They can have a discussion around shared meanings of concepts.
Ask students why there is only one definition of a trapezium in the textbook ? Ask them what would happen if the textbook had multiple definitions ? Ask them who they think created the definitions they see in their textbooks ?

Things to Remember

You need to help students to understand that definitions are shared meanings and they need to be consistent. Help them see that different definitions are acceptable, but there should be a shared understanding amongst people and that once a definition is accepted, the logical consequences should be accepted too. Help them to understand that in case of trapezium, different definitions exists. Refer to the definition in your textbook and help them see that as the shared definition of the trapezium they would work with.

My Notes

Student Observations

Did students form their own definitions? Mention some of them.	
Did students think that there has to be only one definition for a concept? Mention some of the reasons that they gave.	
What are some of the arguments students made about having multiple definitions for a concept ? Give examples Who did they think created the textbook definitions. Mention some of their ideas.	

Self-Reflection

Were students accepting the idea that there can be different definitions of one concept? Was it easy for you to discuss this with students? Mention a few challenges that you faced.	
What is the definition of the trapezium that is provided in your textbook?	
After this lesson do you think about the purpose of textbook definitions differently ? How ?	
Mention some other concepts for which yo have come across different definitions.	

UNIT 5: UNDERSTANDING THE NEED FOR PROOFS

(315 min)

OVERVIEW

IIn this unit, students understand that explorations and inductive reasoning are an important way of discovering mathematical ideas, but they are fallible - and therefore, the need for deductive reasoning and 'proofs'. To get to this idea, students start with activities where they have to come up with conjectures through observation of patterns. They verify their conjectures for different cases and begin to understand the difference between verification and proof. Through a hands-on activity they also realise that one counterexample is sufficient to disprove a conjecture. They further realise the inadequacy of inductive reasoning to 'prove' a conjecture conclusively. Students are also introduced to 1-2 proofs involving deductive reasoning. Students start with an exploration about the quadrilaterals formed by joining the midpoints of different types of quadrilaterals.Starting with guided explorations, they gradually start making conjectures more independently. Next, students explore the relationship between the number of sides and sum of the interior angles of a polygon. They first do this as a guided conjecture- making for quadrilaterals, justify it for the general case, and engage in building it into a 'proof' - led by the teacher. After this, they try to explore angle sums of different polygons in Geogebra, come up with a general 'rule', and justify the rule through informal deductive reasoning. In the next lesson, students are given a task where they can arrive at a (seemingly obvious) rule based on a
few examples, only to then encounter a counter example where the rule breaks down. This helps them realise the fallibility of generalising based on examples, and the need for deductive proofs in mathematics. To close the loop on the midpoint exploration activity, students are also introduced to the formal deductive proof that the quadrilateral formed by joining the midpoints of the sides of any quadrilateral is a parallelogram. The unit ends with an informal assessment activity to see whether students have indeed realised and are able to apply some of the core ideas (about reasoning and proof) that were discussed in the unit.

LEARNING OBJECTIVES

After going through this unit, the students will

- understand the term 'conjecture', be able to formulate conjectures through observation of patterns and verify them.
- start to articulate conjectures clearly.
- understand the difference between 'verifying' a conjecture and proving.
- prove conjectures using informal (deductive) reasoning.
- realise that one counterexample is sufficient to disprove a statement.
- realise that any number of examples is not sufficient to conclusively prove a statement in maths.
- realise the fallibility of inductive reasoning and hence the need for deductive proofs.
- understand and be able to use the angle sum property of a quadrilateral.
- prove some theorems/results given in the textbook chapter on quadrilaterals.

PRIOR MATHEMATICAL UNDERSTANDINGS REQUIRED

Ability to observe patterns

- Reasoning to arrive a conclusion based on the given assumptions/premises
- Understanding definitions of special quadrilaterals, polygons etc
- Know the angle sum property of a triangle, and be able to use it in problems
- Know the properties related to parallel lines and transversal (pairs of alternate angles being equal, etc.)
- Result of the Mid-point theorem (developed through a teacher designed exploration)

PRIOR UNDERSTANDING IN ICT REQUIRED

Familiarity with the basic Geogebra interface and tools

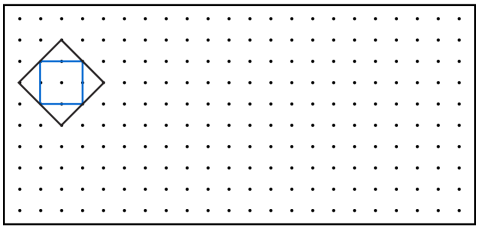
LESSON & ACTIVITY-WISE BREAK UP

Lesson	Activity	Activity Name	Duration (Recommended) Minutes	Mode
Lesson 11: Midpoint - Explorations	5.1	Mid-Point Explorations	45	Hands on
Lesson 12: Angle Sum Property	5.2	Angle Sum Property of Quadrilaterals	45	Hands on
	5.3	Exploring Angle Sum Property of Polygons	90	Digital
Lesson 13: Need for Proof	5.4	Need for Proof	45	Hands on + Class discussion
Lesson 14: Writing a proof	5.5	Proving Mid-point result for quadrilaterals	45	Class discussion
Lesson 15: Proving and Disproving	5.6	True and False Statements	45	Hands on
Assessment 5	5.7	Endline Assessment	45	Digital

Lesson 11

Ideas for Facilitating the classroom Discussions (45minutes)

Mid- point exploration activity will provide an opportunity for students to come up with conjectures and verify them.
Ask students to join the midpoints of each side of different squares on a dot paper and ask them which quadrilateral they get. Students will form conjectures like - The quadrilateral formed by joining the midpoints of sides of a square is a _____.



Ask students what shape they think they will get by joining midpoints of sides of a rectangle and help them to justify their conjectures. Ask them to verify the conjectures by drawing rectangles on the dot paper and joining the midpoints of the sides. Ask the students to make conjectures for rhombus and parallelogram in a similar way. Students will then generalize these conjectures for all quadrilaterals.
Ask the students:

- How did you make your conjecture?
- Are you sure that this conjecture will be valid for all the quadrilaterals? How do you know that?

Help students to make their conjectures independently and justify them. Students' understanding of different types of quadrilaterals can be used here.

Things to Remember

Help students see that:
By joining midpoints of sides of any square we get a square.
By joining midpoints of sides of any rectangle we get a rhombus.
By joining midpoints of sides of any rhombus we get a rectangle.
By joining midpoints of sides of any quadrilateral we get a parallelogram.
You can revisit the idea of a class inclusion. Joining midpoints of any quadrilateral we get a parallelogram. Joining midpoints of the special quadrilaterals, we get different types of special quadrilaterals and all of them are parallelograms.

My Notes

Student Observations

Were students able to identify all the special quadrilaterals by their properties?	
Write some conjectures that students made about joining midpoints of sides of squares and rectangles.	
Write some conjectures that students made about joining midpoints of sides of parallelograms and rhombuses.	
Were students able to justify the conjectures that they made ? Write at least two examples.	
Were students using precise language to communicate their conjectures ? What were some of the problems they were facing in terms of language and vocabulary while making their conjectures? Describe.	

Self-Reflection

What kind of problems did students face while making conjectures?	
Did all the students participate in this activity? What do you think are some of the reasons for non-participation ?	
Does, students making conjectures and justifying them help you in assessing students? How?	
What were some of the errors students made while making conjectures? Were you able to guide them to correct their mistakes? How?	

Lesson 12

Ideas for Facilitating the classroom Discussions

The purpose of this activity is to understand the angle sum property of quadrilaterals through observation of patterns. Students make a conjecture and generalise the result. They also understand and differentiate between verification and proof. This activity also helps them see the connection between pattern making and formal reasoning (and in this case, proof). Ask the students to draw different quadrilaterals and measure their interior angles by working in groups of 2 or 3. Encourage the students to talk to other groups and compare their answers with others.
Ask them:
Compare your quadrilateral with those of others in your class.

a. Do they appear the same?
b. Do you observe any pattern/regularity in the sum of interior angles across all the quadrilaterals? Write your observation in the form of a conjecture.
c. Do you think this pattern will hold true for ALL quadrilaterals?
d. Why or why not?
In the next part of the activity ask the students to draw a quadrilateral and draw any one of its diagonals. Ask them if they can find the sum of the interior angles of the quadrilateral without measuring the angles. Help the students identify that a quadrilateral can be divided into two triangles. Make use of the sum of the interior angle property of a triangle and prove in a step by step fashion that sum of the interior angles of a quadrilateral is 360 degrees. Please take inputs from the students while proving.

- Ask the students if this property will be true for parallelograms. Why?

- Ask them if they think that there is any quadrilateral in which sum of the interior angle is not 360 degree?

(Students will perform the following activity in their computer labs using geogebra.) Ask the students to draw polygons with the number of sides mentioned below. Ask them to measure the interior angles of the polygon and note the sum of it. Ask them to divide polygons in non-overlapping triangles and record the number of such

Number of sides in regular Polygon	Sum of interior angles of polygon	Number of non-overlapping triangles formed	Sum of angles in all non-overlapping triangles
4	360	2	360
5			
6			
7			

Things to Remember

While joining the vertices of a concave polygon, a precaution has to be taken, The vertices can be joined in more than one way. Choose only those ways that divide the polygon into non-overlapping triangles. Help students to form the generalized statements and conjectures by observing patterns. Always ask them to justify their conjectures to understand how they are thinking about it.

My Notes

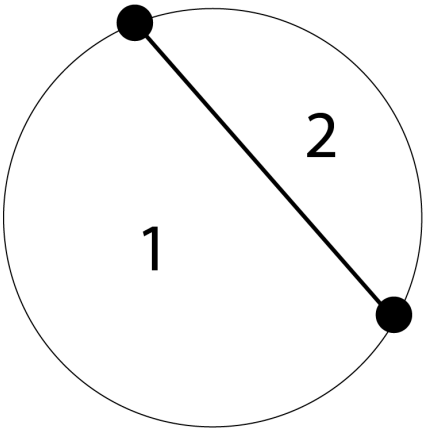
Student Observations	
Were the students able to generalize the pattern and make conjectures about the angle sum of the quadrilaterals? What were their conjectures? How did they justify these conjectures?	
Was the proof for angle sum property of quadrilaterals convincing for the students? Were they convinced that it will hold for all quadrilaterals?	
What pattern/s did students observe about sum of the internal angles of 'n' sided regular polygons? Did they realise that the same rule could be applied to the irregular polygons? Why (give the reasons provided by the students)?	
What was the patterns observed by students for angle sum of concave polygons?	
Mention some of the incorrect patterns/ conjectures formed by the students in any of the tasks above.	
Note some observations of students use of Geogebra tool in this lesson.	

Self-Reflection	
While measuring the angles of a quadrilateral, did the angles sum up to exactly 360 degrees every time? Did students ask you a query if they did not get exactly 360 degrees? How did you respond to that?	
How do you think conjecture making exercise will help students learn the above concepts (sum of internal angle of a polygon) better?	
Were students talking 'maths' with each other while doing the activities? Write down some of the peer discussions. Do you think peer discussions help you with your teaching ? How do you use them to your advantage ?	
Do you think it is useful to use Geogebra in such activities? List down the benefits and issues of using Geogebra	

Lesson 13

Ideas for Facilitating the classroom Discussions (45minutes)

Ask the students to draw a circle and mark any two distinct points on the circle and make a note of the number of regions created by joining those points. Ask them to repeat this for 3, 4 and 5 points.



Number of points on circle	2	3	4	5
Number of separate regions				

Each time ask them to count the number of separate regions formed and see if they can recognize any pattern/ relationship between the number of points and the number of separate regions formed. Ask them if they think that the rule will hold for any number of points on the circle with reasons. Ask the students to explore this with different number of points on the circle and change their response if required based on their observations. You may ask the students the following questions:

- What did they understand by generalization?
- Can a conjecture be proved only by giving examples? How many examples should one give?/ How many examples are
- enough to prove a conjecture?
- How can we be sure that there will not be any counter example?
- How do we disprove a conjecture? How many counter examples do we need?

You can also ask them why according to them proofs are important.

Things to Remember

Help students to understand that a conjecture cannot be said to be definitely true based on examples alone. In the first few cases, the number of regions happens to be 2 n-1 where n is the number of points marked on the circle.. (It is ok if students do not use this formal symbolic notation but use language to articulate the pattern and state it as a 'rule'). However the pattern fails at n = 6, showing that a conjecture arrived at from examples need not always be true. This helps get to the idea that any number of examples will not prove a conjecture – one needs to have a deductive argument to prove it.

My Notes

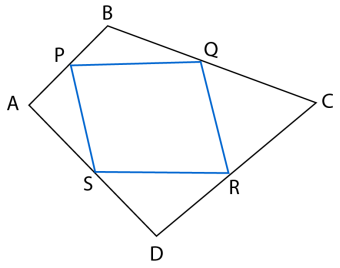
Student Observations	
What were the conjectures students made about the patterns formed? List at least two.	
How many students were able to show that the pattern failed at n=6 (n is the number of points on the circle.)? What are some of the reasons they gave ? List a few.	
Did students understand the difference between verification and proof? How?	

Self-Reflection	
Do you think students understood the need for a proof? Give some examples to demonstrate this ?	
Why do you think it is important for the students to understand the need for proof?	
What other activities you think you can do in order to highlight the need for proof?	
How will you connect this lesson with the formal proofs in the textbooks ?	

Lesson 14

Ideas for Facilitating the classroom Discussions (45minutes)

Help students to understand formal proof writing. Ask the students to prove that PQRS is a parallelogram in the following figure, where P,Q,R and S are respectively midpoints of sides AB, BC, CD and DA of quadrilateral ABCD:



Ask the students to explain the proof and give reasons. For every statement they make ask them how can they be sure and help them to know that all the proven statements are rooted in axioms and definitions.

Things to Remember

Help students to articulate the reasons for their statements and justifications. Highlight the logic of the proof and hence the deductive nature of mathematics.

Student Observations

Please write one sample proof given by a student.	
What were some mistakes commonly made by students?	

Self-Reflection

Why do you think students find it difficult to write their own proofs?	
Did you feel that this module helped students appreciate and understand geometry ? Explain.	

Lesson 15

Ideas for Facilitating the classroom Discussions (45minutes)

Ask the students to work in groups and find out if the given statements are true or false. Ask them to work in groups and present their answers to the class and justify their answers.. Ask the students

- How can they be sure that the statement is true in all the cases. i.e. It will be always true.
- How do they know that a particular statement is false?

If there is a disagreement between two groups on a particular statement, then ask them to have a dialogue and come to a common conclusion. Based on the interactions that happen between students/groups and based on how they try to convince others that a statement is always true (do they use examples, or do they use arguments) you could understand the extent to which students have understood the concepts targeted at by this unit. You can add some new statements to the provided ones.

Things to Remember

This activity is intended to be an informal assessment of students' understanding of the ideas focussed on this unit, by seeing how they apply these ideas to a slightly different context.

Student Observations

Was there a disagreement between any two groups/ individuals on the 'truth' of a particular statement? How did they resolve the issue and come to a common conclusion?	
Give some examples of the reasoning that students gave for for proving a statement 'true.'	
Give some examples of the reasoning that the students gave for proving a statement 'false.'	

Self-Reflection

Do you think that this activity will help students to develop deductive reasoning? How?	
After completing this module do you feel that students are able to explain, justify, make conjectures and communicate geometry ideas ? Do you think it is helpful for your teaching ?	
Do you think students are learning from their own mistakes and also from each other's mistakes? Please give some examples.	

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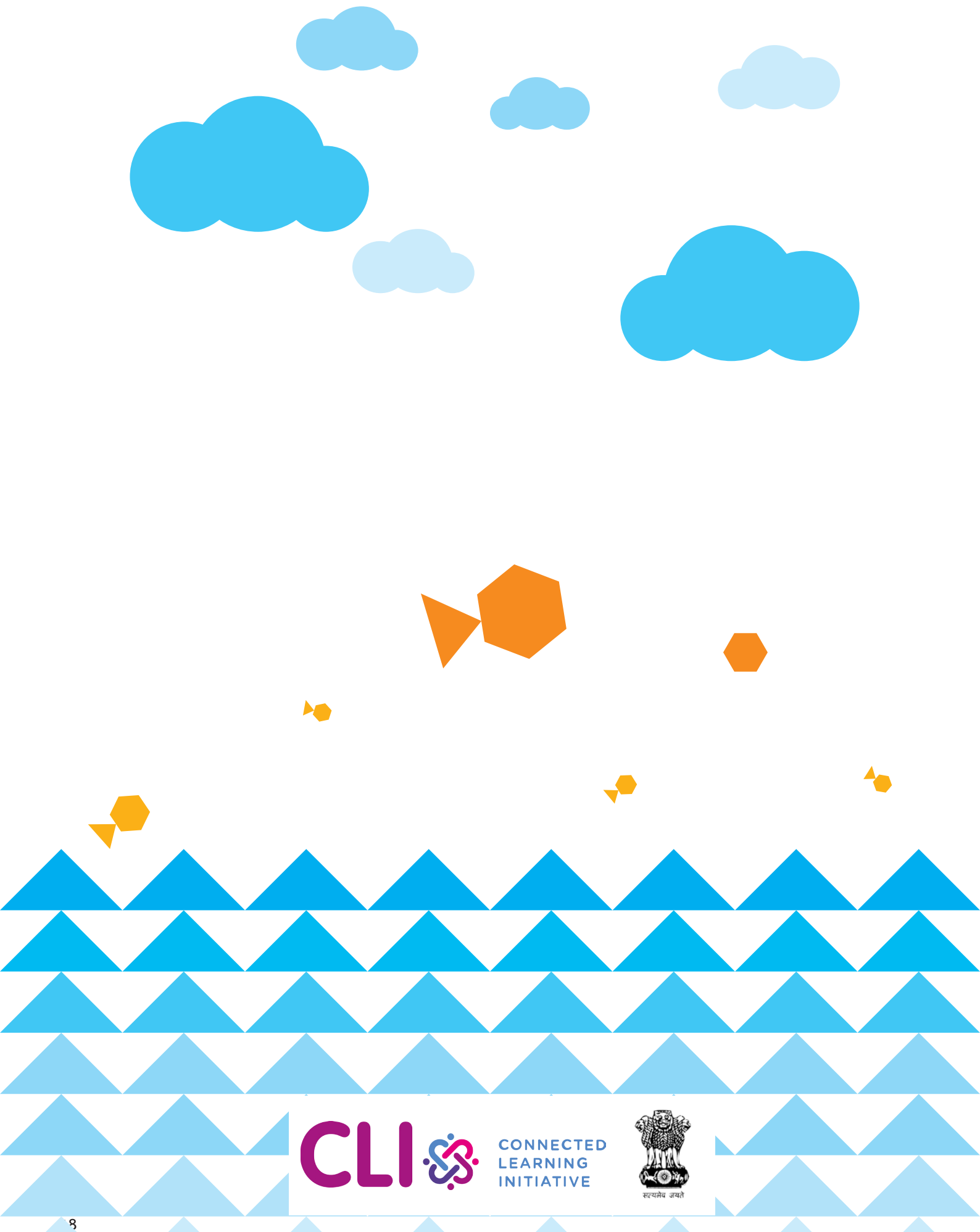
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