

Student Workbook



2017



CONNECTED LEARNING INITIATIVE

An initiative seeded by

TATA TRUSTS



Founding Partners



MATHEMATICS

Geometric Reasoning

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The Connected Learning Initiative (CLIX) is a technology enabled initiative at scale for high school students. The initiative was seeded by Tata Trusts, Mumbai with Tata Institute of Social Sciences, Mumbai and Massachusetts Institute of Technology, Cambridge as founding partners

Collaborators:

Centre for Education Research & Practice – Jaipur, Mizoram University – Aizawl, Eklavya – Madhya Pradesh, Homi Bhabha Centre for Science Education – Mumbai, National Institute of Advanced Studies – Bengaluru, State Council of Education Research and Training (SCERT) of Telangana – Hyderabad, Tata Class Edge – Mumbai, UNICEF Chhattisgarh – Raipur, Govt. of Rajasthan, Govt. of Mizoram, Govt. of Chhattisgarh and Govt. of Telangana

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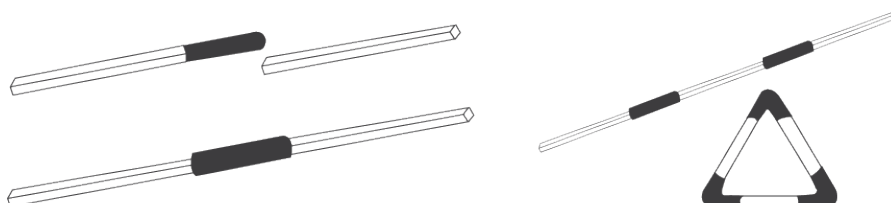
UNIT 1: CONCEPT OF SHAPE

Lesson 1: What is Shape?

ACTIVITY 1.1 MATCHSTICK SHAPES

Work in your group. You will need a set of (used) matchsticks and cycle valve tubes.

Task 1: Make a triangle, a square and pentagon using the matchsticks and valve tubes.



Task 2: Perform these actions (mentioned in the table) on the square. Discuss in your group and note whether the shape changes or not.

	ACTION	DOES THE SHAPE CHANGE?	WHY DO YOU THINK SO?
1	Sliding the shape on the floor/desk		
2	Rotating the shape on the floor/desk		
3	Flipping the shape on the floor/desk		
4	Pressing on the opposite vertices of the shape		

Task 3: Try pressing on the opposite vertices of the pentagon. Does it change shape? Now try doing the same with the triangle. Does it change shape?

Extension Task 1: Try making as many different shapes as possible by deforming/twisting the pentagon (without breaking it or opening up the joints). In particular, try to make

- a triangle that has exactly two sides equal
- a four sided polygon
- a star shape

Extension Task 2: Try making a triangle in which all three sides are of different length. What would be the minimum number of matchsticks needed for this?

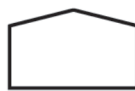
UNIT 2: ANALYSING AND DESCRIBING SHAPES

Lesson 2: Analysing Shapes

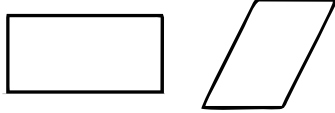

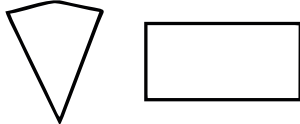
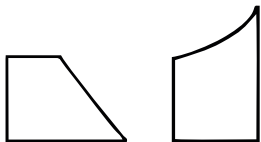
ACTIVITY 2.1 SORTING SHAPES

Work individually on the following tasks and then discuss with your group.

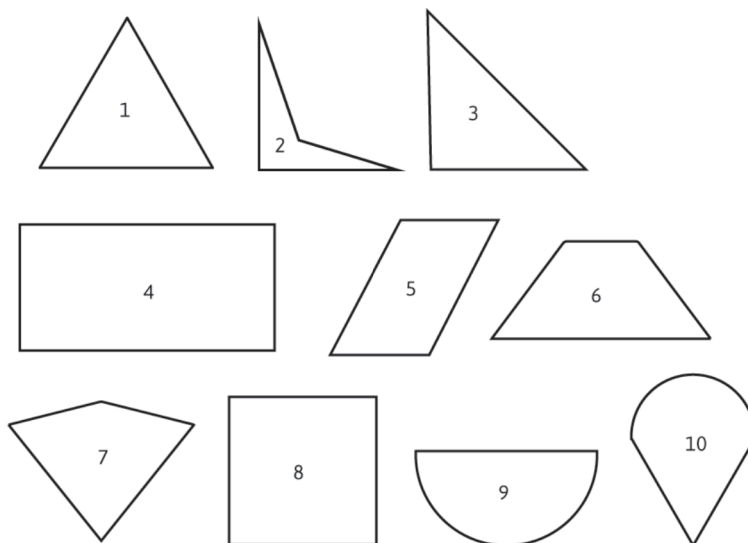
Task 1: Write 1-2 lines about (or properties of) each of the following shapes.



Task 2: Observe the two shapes given in each of the following sets. List as many similarities and differences that you can between the two. One example is given for the first set.

Set	Similarities	Differences
	1) Both have exactly 4 sides 2) Both have 2 pairs of parallel sides	1) The first shape has right angles, the second does not
		
		
		

Task 3: Look at the collection of shapes below. Based on their properties, sort them into two groups in as many different ways as you can. An example is shown in the table.



Property	These shapes have it	These shapes don't have it
Has exactly 4 straight sides	2, 4, 5, 6, 7, 8	1, 3, 9, 10

Extension Task 1: Draw 3 different shapes that have the following property 'all sides equal'. 'All sides equal' is one way which the shapes that you got are similar. What is one difference between them?

Extension Task 2: Draw a shape that has both these properties:

- i. exactly 5 sides
- ii. exactly 2 right angles

Extension Task 3: Draw a shape that has all these properties:

- i. exactly 4 sides
- ii. exactly 2 right angles
- iii. exactly 1 pair of sides parallel

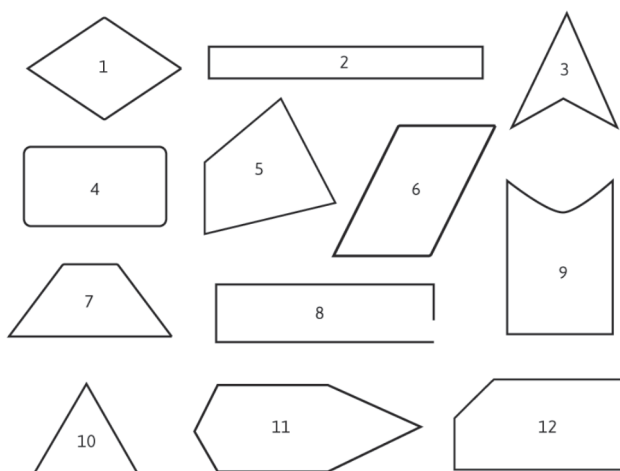
UNIT 3: CLASSIFYING AND DEFINING SHAPES

Lesson 5: Defining Shapes

ACTIVITY 3.3 WHAT IS A QUADRILATERAL

Work individually on the following task and then discuss with your group.

Task 1: Look at the collection of shapes and sort them based on their properties into two groups – 'Quadrilaterals' and 'Not quadrilaterals'.



Now, Fill in the table below.

These are Quadrilaterals	These are not Quadrilaterals

For each shape, discuss why you think it is a quadrilateral, or not. Now complete the following:

I think 'a quadrilateral is

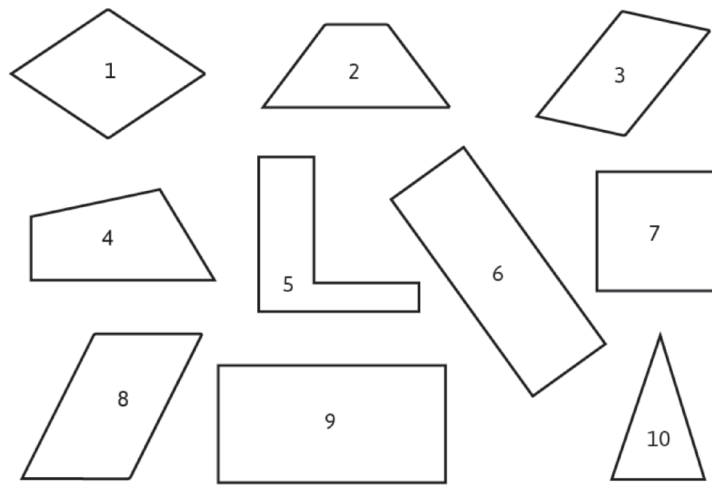
Lesson 6: Defining Special Quadrilaterals

ACTIVITY 3.5 EXPLORING SPECIAL QUADRILATERALS

Work individually on the following tasks and then discuss with your group.

Task 1: Look at the collection of shapes and sort them into those that are parallelograms and those that are not. Fill in the table.

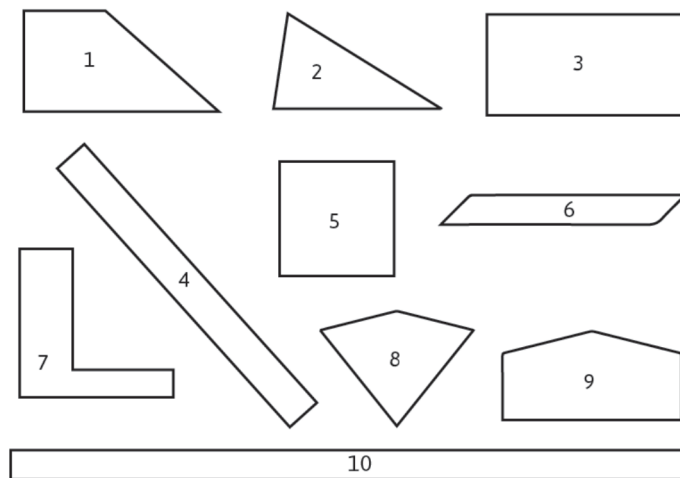
These are parallelograms	These are not parallelograms



For the shapes which are not parallelograms, explain why. Based on this, write your definition of a parallelogram.
I think 'a parallelogram is

Task 2: Look at the collection of shapes and sort them into those that are rectangles and those that are not. Fill in the table.

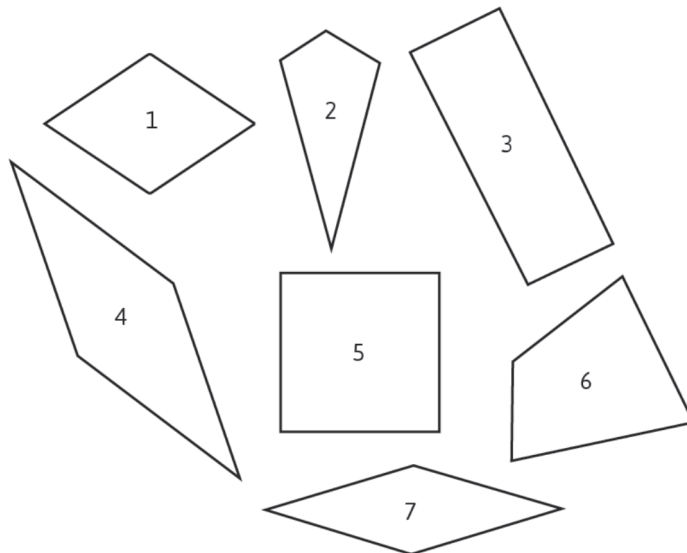
These are rectangles	These are not rectangles



Based on this, write your definition of a rectangle.

I think 'a rectangle is

Task 3: Look at the collection of shapes and sort them into those that are rhombuses and those that are not.



Now, Fill in the table below.

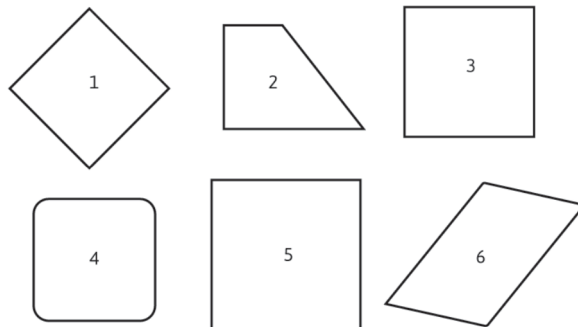
These are rhombuses	These are not rhombuses

Based on this, write your definition of a rhombus.

I think 'a rhombus is

Task 4: Look at the collection of shapes and sort them into those that are squares and those that are not. Fill in the table.

These are squares	These are not squares



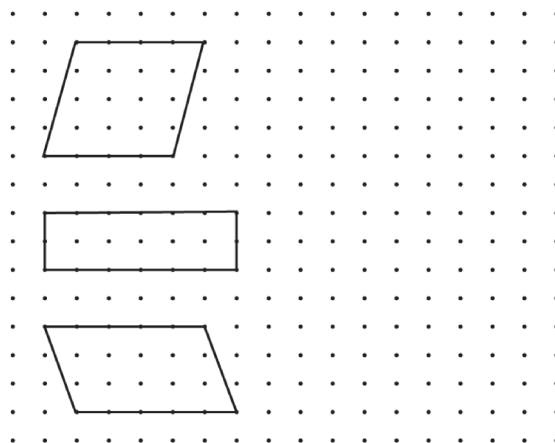
Based on this, write your definition of a square.

Lesson 7: Properties of Special Quadrilaterals

ACTIVITY 3.7 MAKING PROPERTY LISTS

Work individually on the following tasks and then discuss with your group.

Task 1a: Shown here are some examples of parallelograms. Draw 2 more examples of parallelograms on the dot paper. (Make sure that your parallelograms are different from the ones already given!)

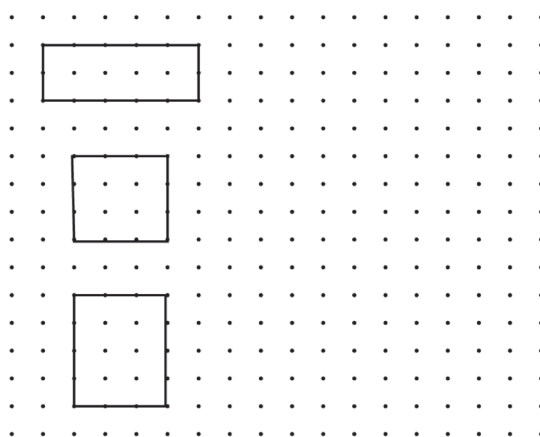


b. Write down the ways in which these parallelograms are different from each other.

Task 2: Observe the parallelograms in Task 1 and make a list of as many properties as you can. Remember, the properties should be common to ALL the examples!

Properties of a Parallelogram		
Side Properties	Angle Properties	Diagonal Properties

Task 3a: Shown here are some examples of rectangles. Draw 2 more examples of rectangles on dot paper. (Make sure that your rectangles are different from the ones already given!)

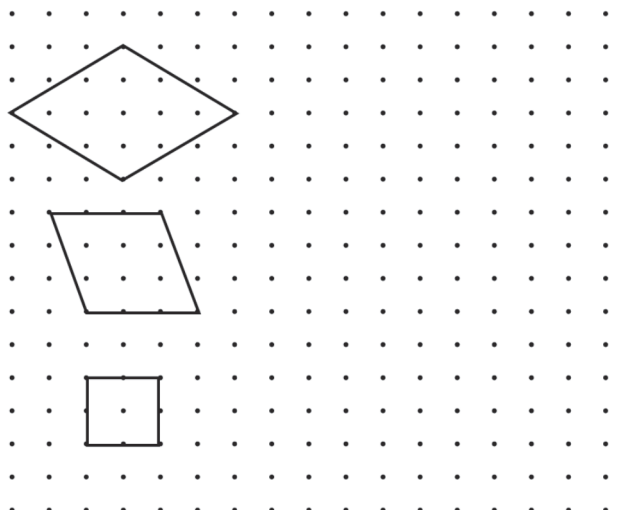


b. Write down the ways in which these rectangles are different from each other.

Task 4: Observe the rectangles in Task 3 and make a list of as many properties as you can. Remember, the properties should be common to ALL the examples!

Properties of a Rectangle		
Side Properties	Angle Properties	Diagonal Properties

Task 5a: Shown here are some examples of rhombuses. Draw 2 more examples of rhombuses on dot paper. (Make sure that your rhombuses are different from the ones already given!)

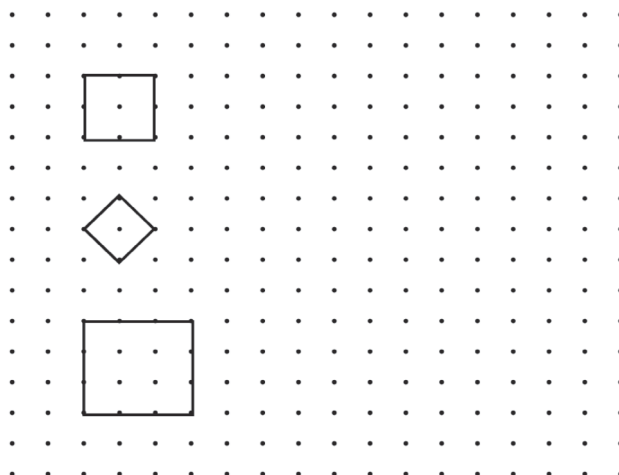


b. Write down the ways in which these rhombuses are different from each other.

Task 6: Observe the squares in Task 5 and make a list of as many properties as you can. Remember, the properties should be common to ALL the examples!

Properties of a Rhombus		
Side Properties	Angle Properties	Diagonal Properties

Task 7a: Shown here are some examples of squares. Draw 2 more examples of squares on dot paper. (Make sure that your squares are different from the ones already given!)



b. Write down the ways in which these squares are different from each other.

Task 8: Observe the rhombuses in Task 7 and make a list of as many properties as you can. Remember, the properties should be common to ALL the examples!

Properties of a Square		
Side Properties	Angle Properties	Diagonal Properties

UNIT 4: PROPERTY-BASED REASONING

Lesson 8: Relationships among Special Quadrilaterals

ACTIVITY 4.2 CREATING PROPERTY STACKS

Work individually on the following task and then discuss with your group.

Task 1: Study the table of properties below. Put a tick (✓) mark in a cell if the corresponding shape has the given property. Leave the cell blank if it doesn't.

The first row is done for you.

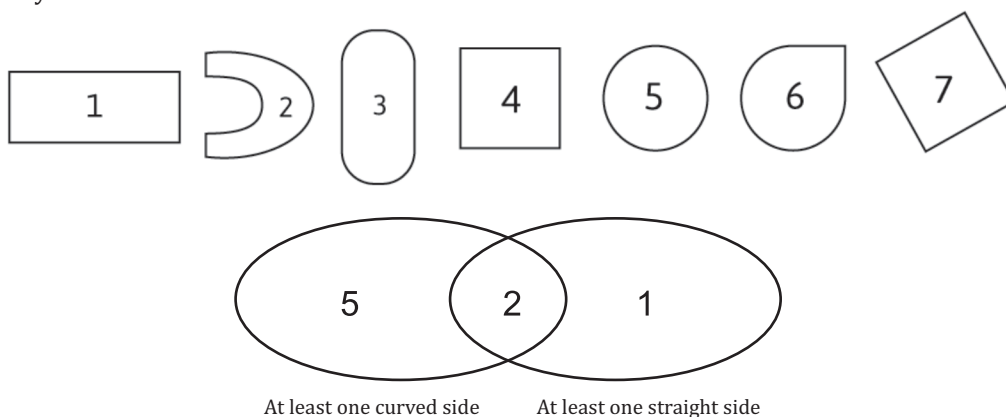
	Properties	Square	Rectangle	Parallelogram	Rhombus
1	Closed figure made of 4 line segments only	✓	✓	✓	✓
2	Pairs of opposite sides equal in length				
3	Pairs of opposite sides are parallel				
4	Opposite angles are congruent				
5	Diagonals bisect each other				
6	All angles are right angles / are equal				
7	Adjacent angles are equal				
8	Diagonals are equal in length				
9	Adjacent sides are equal in length				
10	All 4 sides are equal in length				
11	Diagonals are perpendicular to each other				

Lesson 9: Representing Relationships

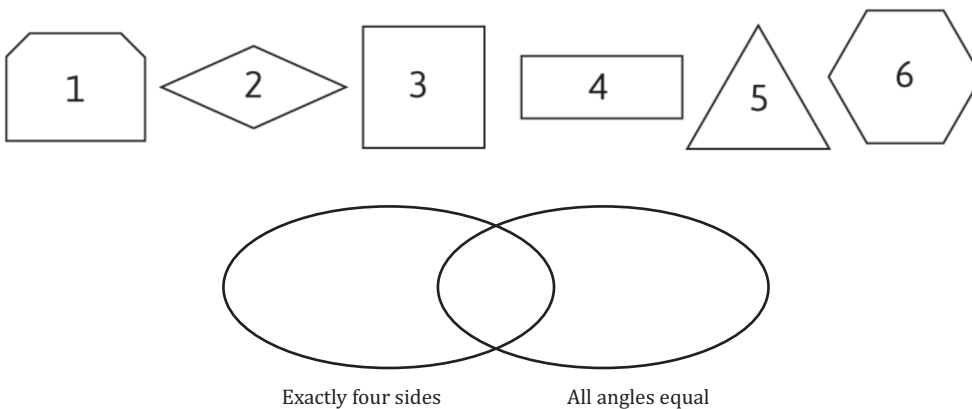
ACTIVITY 4.4 REPRESENTING RELATIONSHIPS 1

Work individually on the following tasks and then discuss with your group.

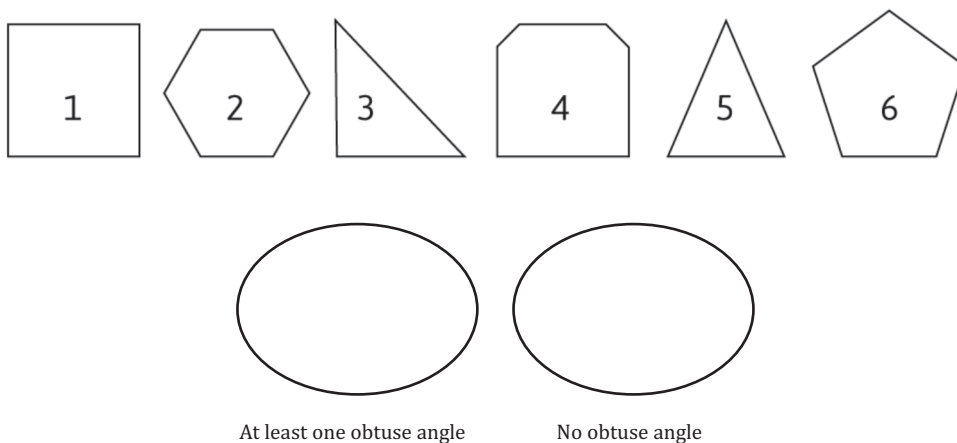
Task 1: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram. Some are done for you.



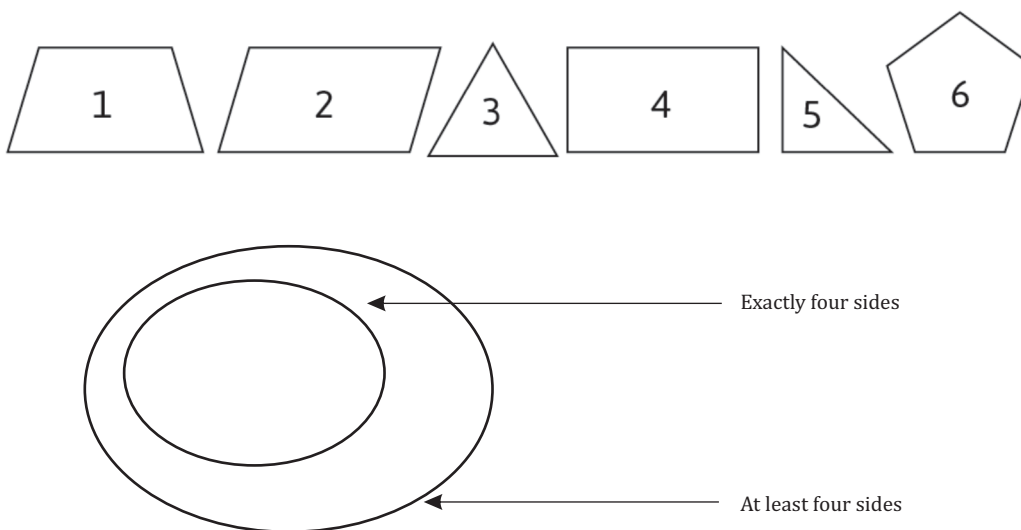
Task 2: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram.



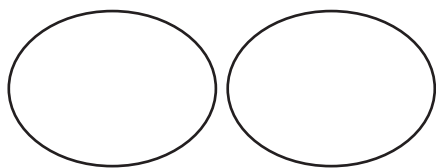
Task 3: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram.



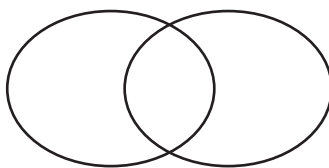
Task 4: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram.



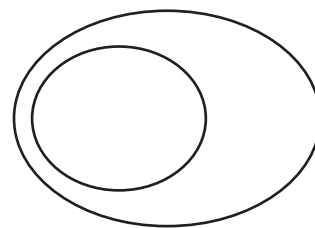
For Tasks 5 8, consider the three types of Venn diagrams Type 1, Type 2 or Type 3 shown below.



Type 1



Type 2



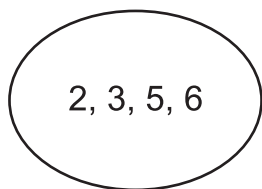
Type 3

Task 5: Consider two groups of shapes:

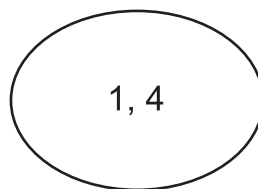
- Group 1: has at least 4 straight sides
 - Group 2: has less than 4 straight sides
- a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
- b. Now classify these shapes into the Venn diagram you selected. (Write the numbers in the appropriate place. Some are done for you.)



(This task is done as an example for you.)



At least four straight sides



Less than four straight sides

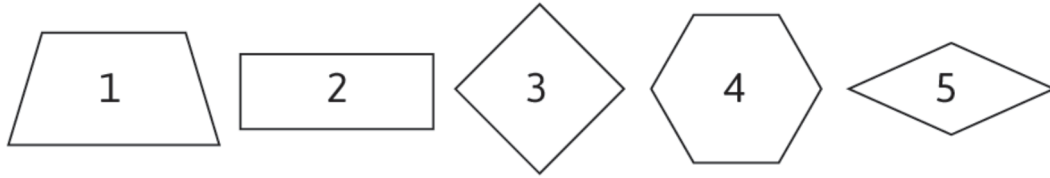
Task 6: Consider two groups of shapes:

- Group 1: has all sides equal
 - Group 2: has at least 1 right angle
- a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
- b. Now classify these shapes into the Venn diagram you selected. (Write the numbers in the appropriate place.)



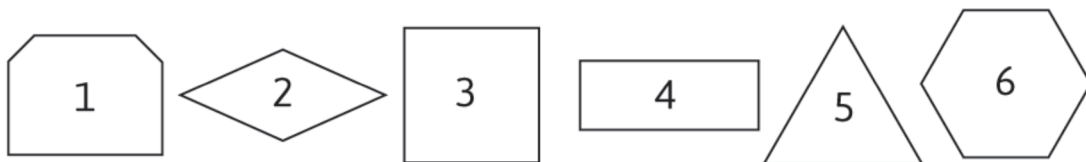
Task 7: Consider two groups of shapes:

- Group 1: has at least one pair of opposite sides parallel
 - Group 2: has no sides parallel
- a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
- b. Now classify these shapes into the Venn diagram you selected. (Write the numbers in the appropriate place.)



Task 8: Consider the property set

- at least 3 straight sides
 - exactly 4 straight sides
- a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
- b. Now classify these shapes into the Venn diagram you selected, by the given properties.

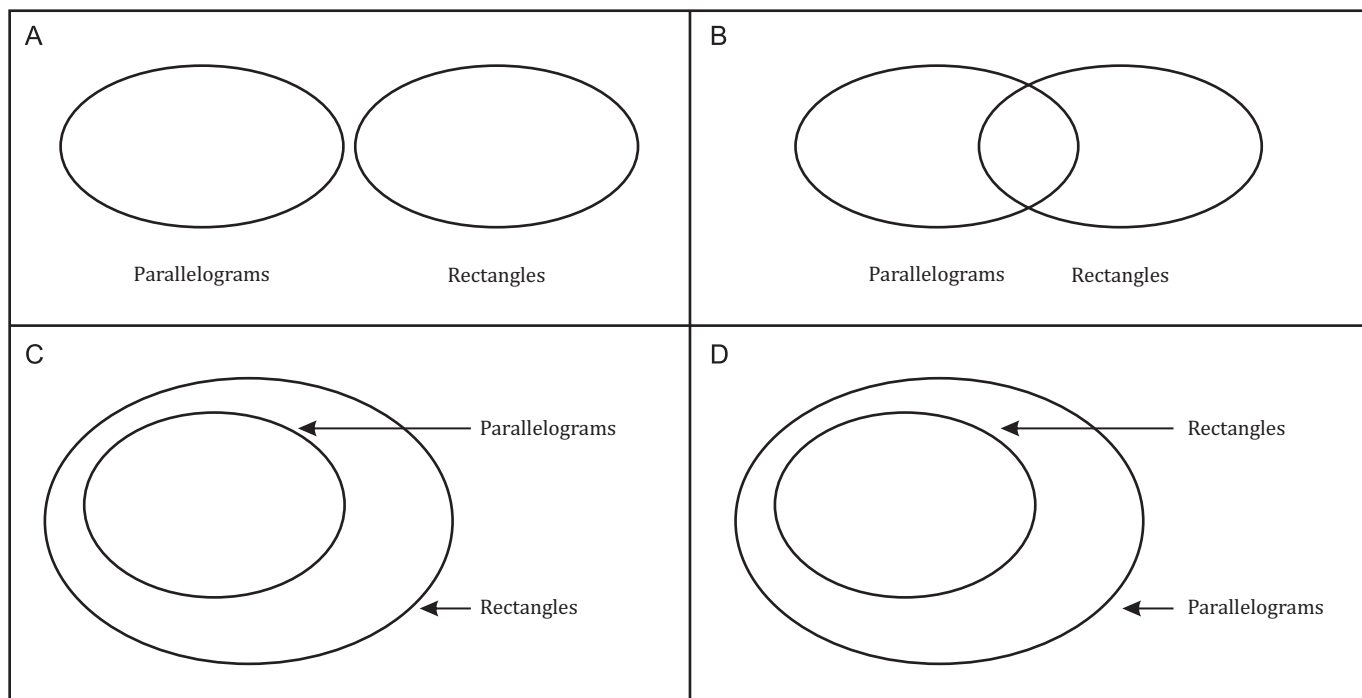


ACTIVITY 4.5 REPRESENTING RELATIONSHIPS 2

Task 1: Which of these is the correct representation of the set of parallelograms and quadrilaterals, and why? (You may want to use the definition of quadrilaterals and parallelograms and their properties to see how they are related.)

<p>A</p> <p>Parallelograms Quadrilaterals</p>	<p>B</p> <p>Parallelograms Quadrilaterals</p>
<p>C</p> <p>← Parallelograms</p> <p>← Quadrilaterals</p>	<p>D</p> <p>← Quadrilaterals</p> <p>← Parallelograms</p>

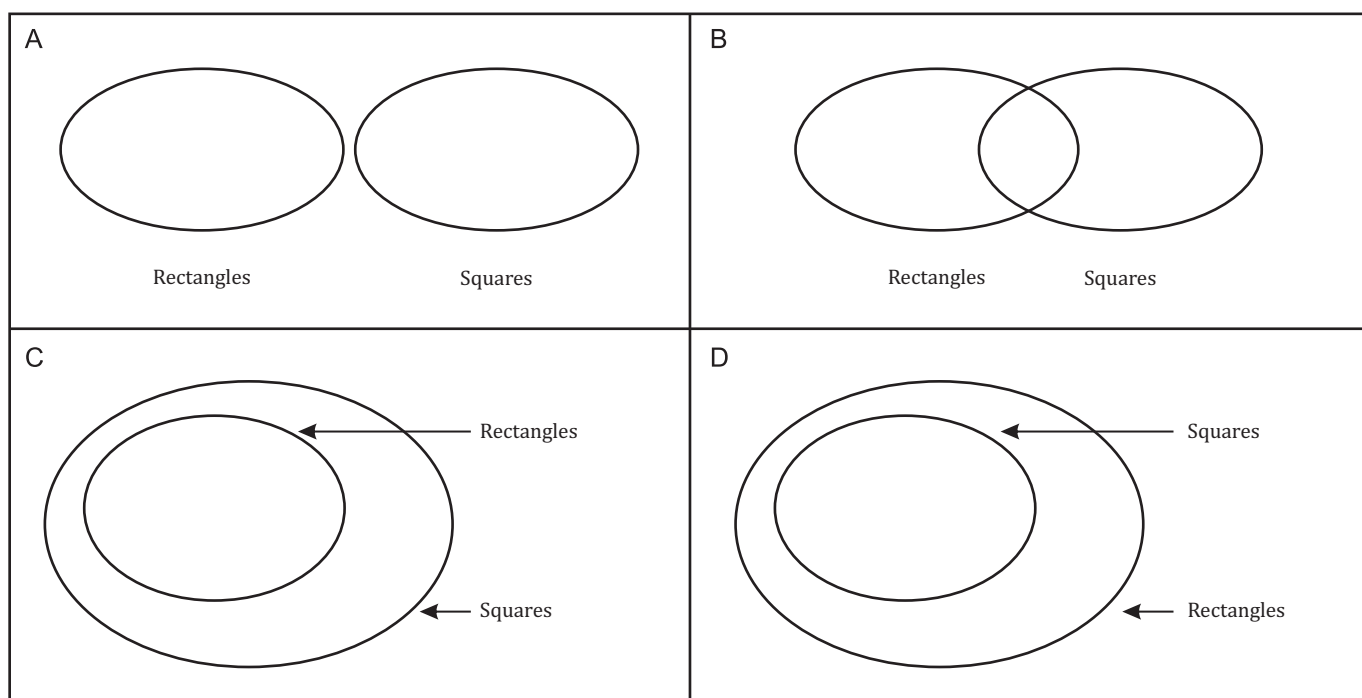
Task 2: Which of these is the correct representation of the set of parallelograms and rectangles, and why? (You may want to use the definition of parallelograms and rectangles and their properties to see how they are related.)



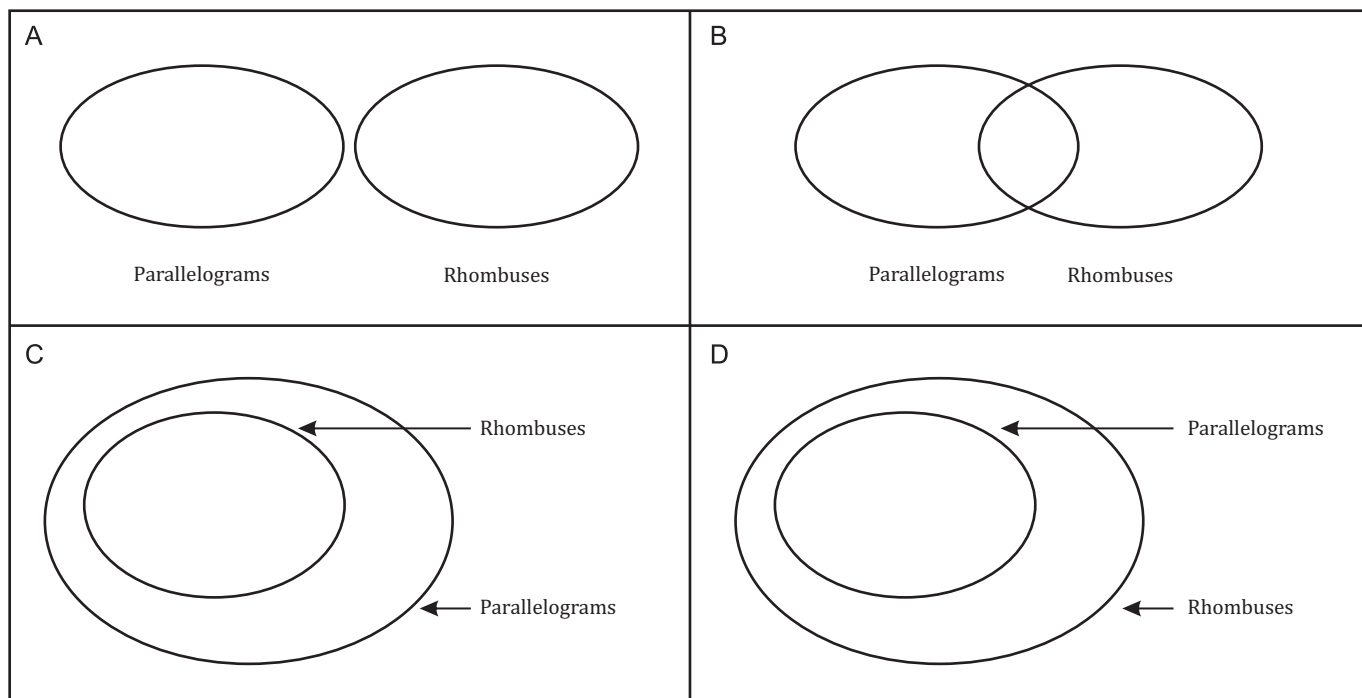
Task 3: Based on task 2, which of the following statements is correct? And why?

- a) All parallelograms are rectangles
- b) All rectangles are parallelograms
- c) Some rectangles are parallelograms (and some are not).
- d) No parallelogram is a rectangle.

Task 4: Which of these is the correct representation of the set of rectangles and squares, and why? (You may want to use the definition of quadrilaterals and parallelograms and their properties to see how they are related.)



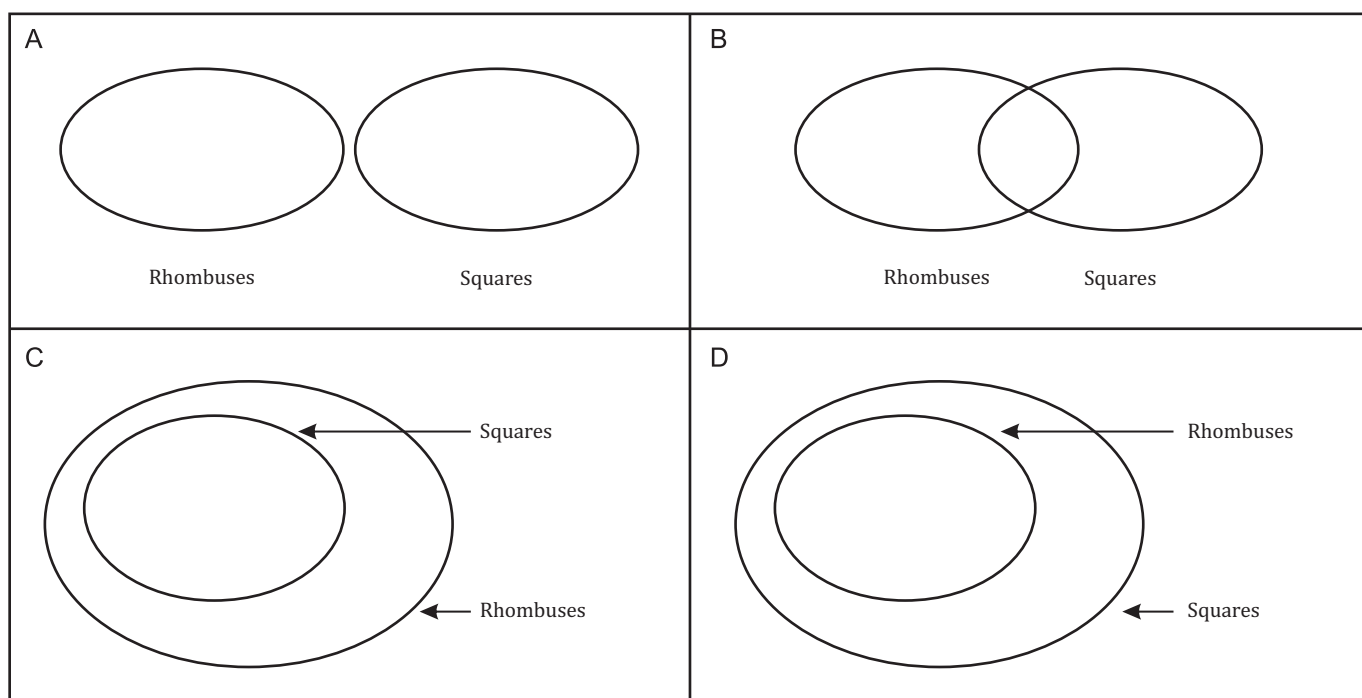
Task 5: Which of these is the correct representation of the set of parallelograms and rhombuses, and why? (You may want to use the definition of parallelograms and rhombuses and their properties to see how they are related.)



Task 6: Based on task 4, which of the following statements is correct?

- a) All parallelograms are rhombuses
- b) All rhombuses are parallelograms
- c) Some rhombuses are parallelograms (and some are not).
- d) No parallelogram is a rhombus.

Task 7: Which of these is the correct representation of the set of rhombuses and squares, and why? (You may want to use the definition of rhombuses and squares and their properties to see how they are related.)

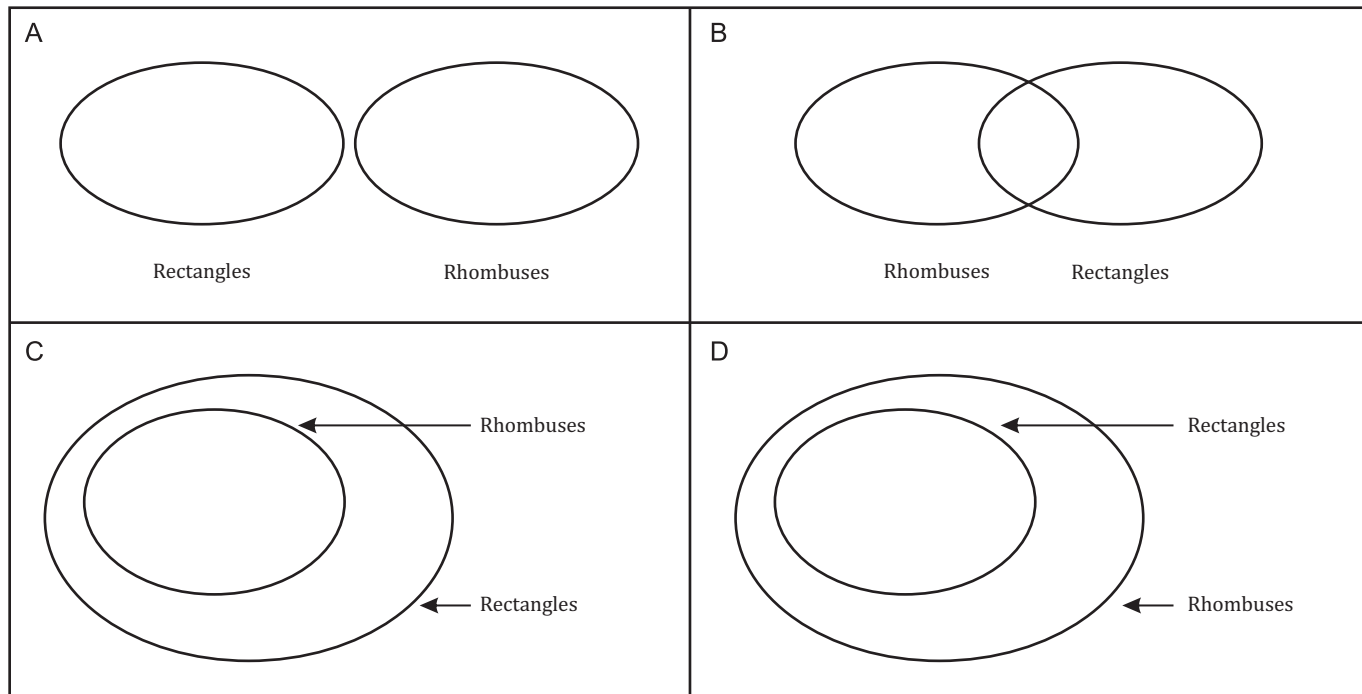


Based on this choose the correct word that completes the sentence.

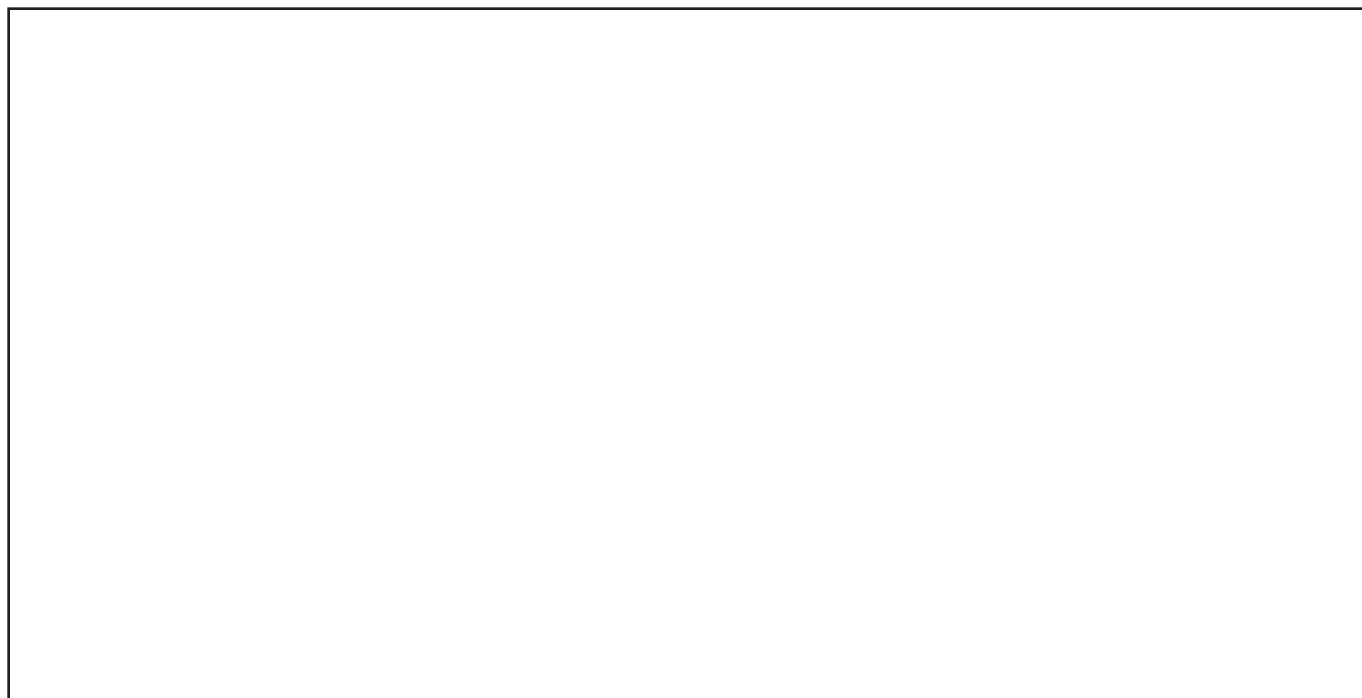
i) _____ rhombuses are squares. (All/ Some/ No)

ii) _____ squares are rhombuses. (All/ Some/ No)

Task 8: Which of these is the correct representation of the set of rectangles and rhombuses, and why? (You may want to use the definition of parallelograms and rhombuses and their properties to see how they are related.)



Task 9: Using the representations chosen in the above tasks, represent quadrilaterals, parallelograms, rhombuses, rectangles and squares in one diagram. Represent quadrilaterals using a rectangle and use appropriate circles to represent the remaining quadrilaterals.



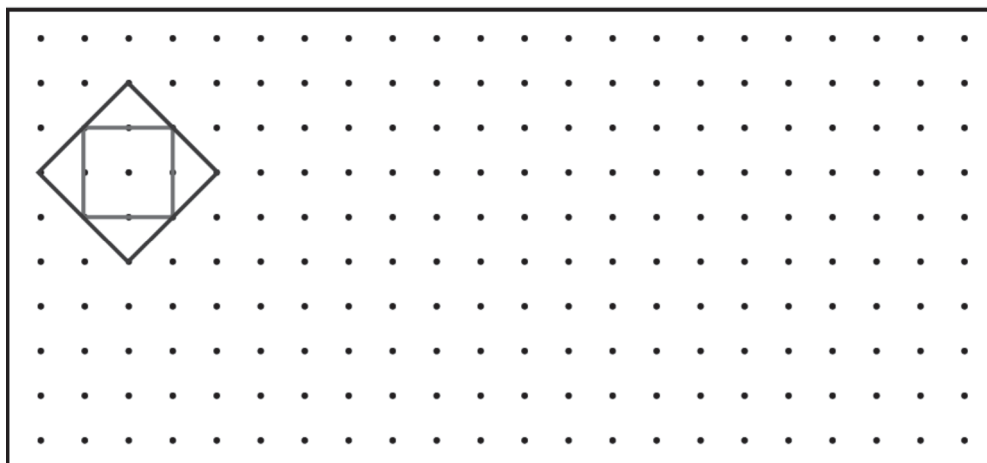
UNIT 5: UNDERSTANDING THE NEED FOR PROOF

Lesson 11: MidPoint Explorations

ACTIVITY 5.1 MIDPOINT EXPLORATIONS

Work individually on the following task and then discuss with your group.

Task 1: On the dot paper below, draw different squares. Join the midpoints of the sides of each of these squares (in order) to create a new quadrilateral. The first one is shown as an example.

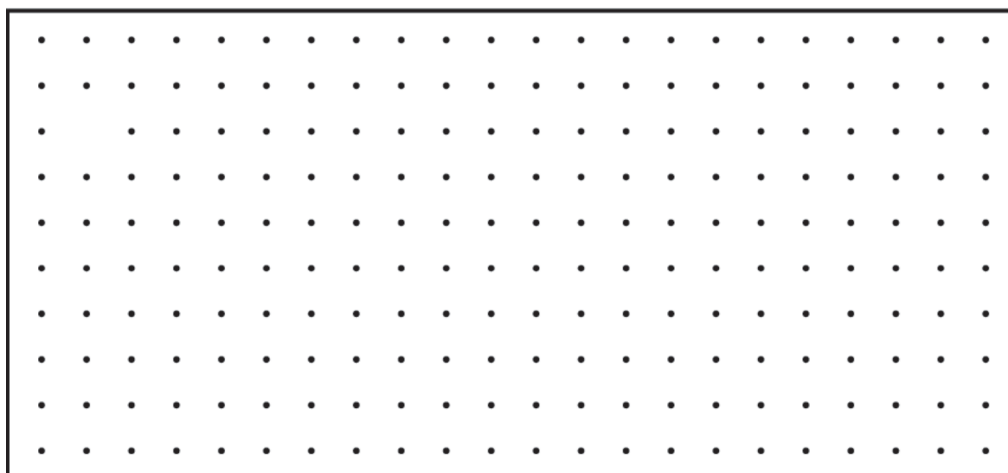


Observe each of the new quadrilaterals formed, and complete the following: The quadrilateral formed by joining the midpoints of sides of a square is a _____

Task 2: Suppose you were to join the midpoints of sides of a rectangle in a similar fashion. What shape do you think you might get? Think about it, and write your conjecture here:

The quadrilateral formed by joining the midpoints of sides of a _____ is a _____

Task 3: Now verify your conjecture by drawing different rectangles on the dot paper below and joining the midpoints of the sides.

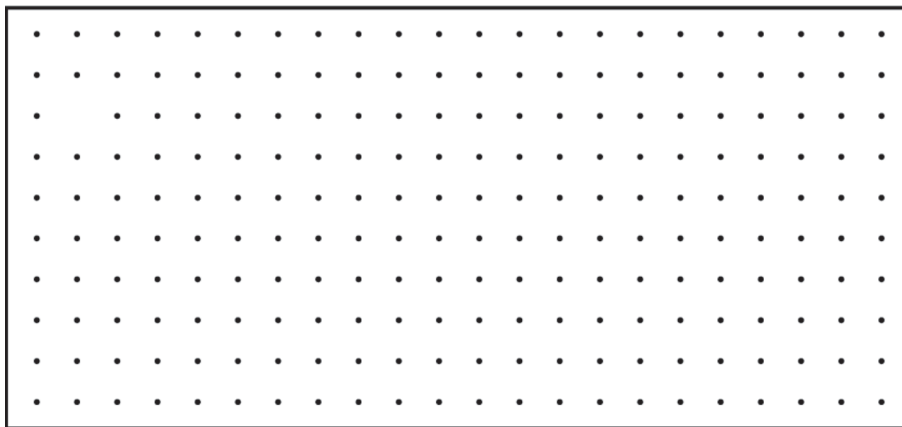


Task 4: Based on Task 3 does your conjecture hold? If not, how would you modify it?

Task 5: Now make similar conjectures about other special quadrilaterals rhombus and parallelogram, and verify them. Write your conjectures in the space provided, and use the dot grid for verifying.

Conjecture 1

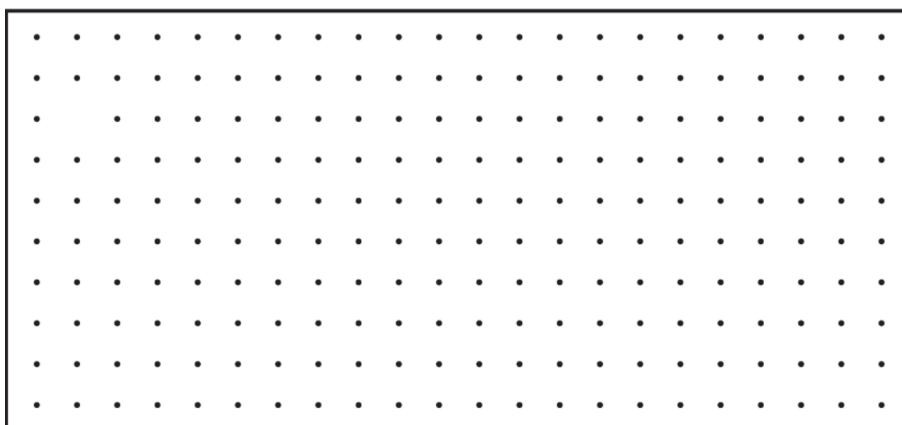
Conjecture 2



Task 6: Drawing on your observations in the 5 previous tasks, make a conjecture about the shape formed by joining the midpoints of sides of any quadrilateral.

Points to think about:

Would this be true for ALL quadrilaterals? How do you know? Explain your reasoning here. Use the dot grid below if necessary.



Extension Task 1: If possible, draw a quadrilateral, joining whose midpoints of sides in order gives a figure that is NOT a parallelogram. If not possible, explain why?

Lesson 12: Angle Sum Property

ACTIVITY 5.2 ANGLE SUM PROPERTY OF QUADRILATERALS

Work individually on the following task and then discuss with your group.

Task 1: Draw a quadrilateral ABCD and measure its (interior) angles. Record in the table below.

$\angle A$	$\angle B$	$\angle C$	$\angle D$	Sum of all (interior) angles

Task 2: Compare your quadrilateral with those of others in your class.

- Do they appear the same?
- Do you observe and pattern/regularity across all the quadrilaterals? Write observation in the form of a conjecture:

Task 3: Refer to the pattern observed in Task 2.

- Do you think this pattern will hold true for ALL quadrilaterals?
- Why or why not?

Task 4: Draw a quadrilateral and join any one of its diagonals. Without measuring, can you say what the sum of the interior angles of this quadrilateral will be? Write your reasons.

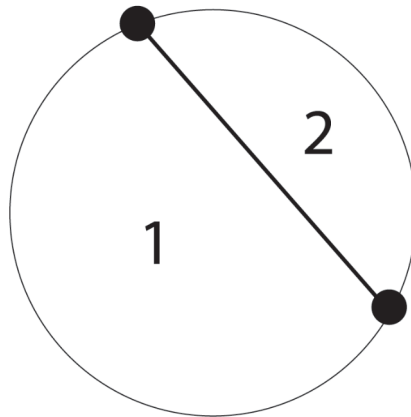
Extension Task 1: Do you think this property (sum of interior angles) will hold true for all parallelograms? Why or why not?

Lesson 13: Need For Proof

ACTIVITY 5.4 NEED FOR PROOF

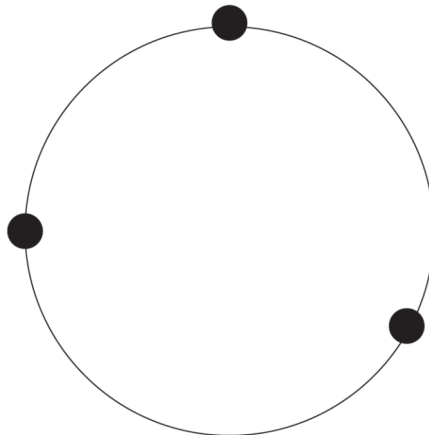
Work individually on the following task and then discuss with your group.

Task 1: Mark 2 distinct points on a circle and join them. Note how many separate regions the circle is divided into.



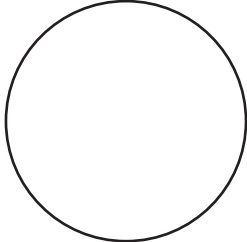
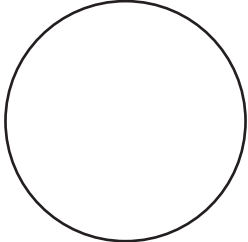
- Number of points on circle : _____
- Number of separate regions : _____

Task 2: Now draw another circle. Mark 3 distinct points on it. Join all possible pairs of points. How many separate regions is the circle divided into?



- Number of points on circle : _____
- Number of separate regions : _____

Task 3: Mark 4 distinct points on one circle, join all possible pairs of points. Note the number of (separate) regions the circle is divided into. In the other circle, do the same with 5 distinct points.

	
<p>For four points on the circle</p> <ul style="list-style-type: none"> • Number of points on circle: _____ • Number of separate regions: _____ 	<p>For five points on the circle</p> <ul style="list-style-type: none"> • Number of points on circle: _____ • Number of separate regions: _____

Task 4: Now record your observations from Tasks 3 in the table below.

Number of points on circle:	2	3	4	5
Number of separate regions:				

Task 5: What is the pattern you observe? Write it down. (You could write it as a 'rule' about the relationship between the number of points taken on the circle, and the number of separate regions the circle is divided into.)

Task 6: Do you think your 'rule' will hold true for ANY number of points taken on the circle? Why or why not?

Task 7: Verify your 'rule' by taking:

- i) 1 point on a circle
Number of separate regions: _____
- ii) 6 points on a circle
Number of separate regions: _____

Task 8: Does your rule hold true? Based on this, would you like to change your response to Task 7? If yes, put the new response here.

Point to think about

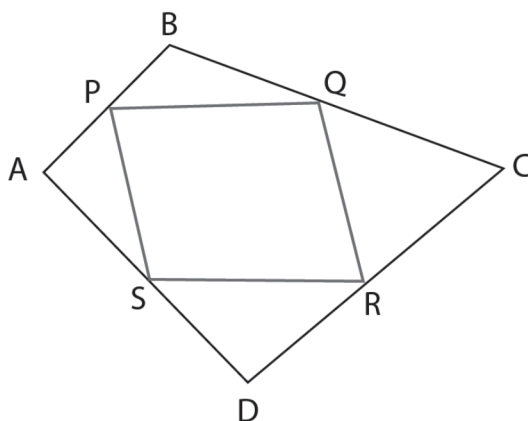
How many examples do you think are 'enough' to prove a conjecture?

How many examples do you think are 'enough' to disprove a conjecture?

Lesson 14: Writing A Proof

ACTIVITY 5.5 PROVING MIDPOINT RESULT FOR QUADRILATERALS

Task 1: In the figure, PQRS is formed by joining the midpoints of a quadrilateral ABCD. Prove that PQRS is a parallelogram.



(Hint: Use the result of the Midpoint Theorem. The line joining the midpoints of two sides of a triangle is parallel to the third side and half of it.)

Lesson 15: Proving And Disproving

ACTIVITY 5.6 TRUE AND FALSE STATEMENTS

Work these out in your group, and then present your solution to the class. Use the space provided for working out your solution.

Task 1: Examine each statement and state if it is true or false. In each case, work out the reasons for your answer. You could write, draw diagrams, tables etc. in the space provided to support your answer.

1. If you double a whole number, you get an even number.

True

False

2. If you add two odd numbers you will get an even number.

True

False

3. If you multiply two odd numbers you get an even number.

True

False

4. If you add 1 to a whole number you get a number less than 1,000,000,000,000,000,000.

True

False

5. If a parallelogram has one pair of adjacent sides equal, it is a rectangle.

True

False

6. If a parallelogram has at least one right angle, it is a rectangle.

True

False

7. If a quadrilateral has one pair of opposite sides equal, and the other pair parallel, then it is a parallelogram.

True

False

8. If a quadrilateral has one pair of opposite sides both equal and parallel, then it is a parallelogram.

True

False



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