Professional Development Workshops for In-Service Mathematics Teachers in India

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Abstract

In this chapter we present the background of teacher professional development workshops conducted in the Indian context with the need for rethinking the goals of the workshops in light of new policy initiatives. We elaborate on the goals, principles and the framework adopted for design and enactment of the workshops conducted at Homi Bhaba Centre for Science Education (HBCSE) along with description and a few examples of the different types of tasks and sessions planned for the workshop. We describe three principle which guided the design and enactment of the workshop- (i) situatedness in the work of teaching, (ii) offering challenges to teachers' to revisit their knowledge and beliefs, and (iii) developing a sense of belonging to a professional community. Subsequently we present analysis of two episodes from a ten day professional development workshop to illustrate – (a) how the three principles and goals of the workshop design shaped the tasks and enactments of those tasks, (b) how authenticity of the tasks and enactment lead to agency of teachers and teacher educators and (c) how teachers demonstrated sensitivity towards students thinking

Keywords: In-service teacher professional development, professional development in India, professional community for teachers, framework for design and enactment of workshops, teacher agency, teacher beliefs, teacher knowledge

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Introduction

Teachers are central to any education system. There is growing realization across the world that reform in education cannot be brought about without adequately addressing teachers' role in it. In India, two major policy initiatives aimed at providing quality elementary education to all children have been launched in the last few years: the National Curriculum Framework for school education 2005 and the Right to Education Act 2009 (see Khan, this volume). As India struggles to implement these two initiatives, teachers face challenges like providing children quality education through student centered pedagogy, assessing students comprehensively and continuously, and relating school subjects to the daily lives of children. The National Curriculum Framework has been criticized for being silent on how teachers are supposed to bring about the change in their classroom and for not addressing the much needed teacher development to support curriculum renewal (Batra, 2005). Efforts undertaken thus far like changing textbooks and issuing directives to schools and teachers, sidestep the issue of developing adequate knowledge and enabling beliefs among teachers, which is needed to realize the vision conveyed in the new curriculum framework.

Several national committees have over the years recognized the need for continuing professional education of teachers and recommended “at least two or three months of in-service education in every five years of service” (Government of India [GoI], 1966, section 4.56). The New Education Policy of 1986 recommended a rapid expansion of the infrastructure for education of teachers at the elementary level through the setting up of institutions at the district and block levels, which would deal with both pre-service and in-

service teacher education (GoI, 1986). While the teacher education infrastructure has indeed expanded vastly, issues of poor quality and low relevance of teacher preparation remain (Sharma & Ramachandran, 2009). Further, teacher education institutions have tended to focus more on pre-service education leading to the neglect of in-service education (MHRD, 2009). A renewed attempt to address the problems of pre-service and in-service teacher education is made by the new National Curriculum Framework for Teacher Education (NCFTE), which re-affirms the importance of in-service teacher development, and puts forth several principles that should govern the design of in-service teacher education programs (National Council of Teacher Education [NCTE], 2009). Of these, we highlight the following three principles:

- Designing programs with clarity about aims and strategies for achieving these aims,
- Allowing teachers to relate the content of the program to their experiences and also to find opportunities to reflect on their experiences, and
- Need to respect the professional identity and knowledge of a teacher and to work with and from it (NCTE, 2009, pp. 66-67).

Besides these, the principles advocate creating spaces for sharing of teachers' experiences, addressing teachers' needs and extended interaction with a group of teachers. They caution against compromising interactivity especially through the use of electronic media, aiming at quick fixes, over-training, and routinised and superficial training. The principles highlighted above are especially important for the focus of this article on in-service teacher professional development (TPD) workshops.

In-service TPD programs in India

As emphasized by the NCFTE, there is a need to develop clear vision of the goals that programs must achieve and the means by which they can be achieved. Most in-service TPD programs in India are designed in response to the need of curriculum reform and view teachers as agents of the state, who implement the reforms rather than as participants in the process of reconstruction of the curriculum. Underlying this is the assumption that teaching can be changed by directing changes in the content or structure of interactions in classrooms while not directly addressing the teacher's own conceptions of teaching, learning and mathematics. In-service TPD is seen as training for content or pedagogy, mostly revolving around the changed curriculum, but not necessarily as important for continuous teacher development. Content-focused interventions often consist of lectures delivered by “experts” and the mathematical content is typically divorced from the context of teaching and learning. Another common focus of TPD programs is “how to teach a particular topic”. This may appear to be close to the work of teaching and hence directly relevant to teachers. However, there is a large variation in the contexts and life experiences that students bring to the classroom and teachers need to be flexible and adaptive in addressing the needs of a student (NCTE, 2009). Instruction to teachers are guided by a *transmission model*, where recommendation on how to teach a topic tend to be recipe-like. The effectiveness of such an approach is limited and is not consistent with the vision articulated in the NCFTE 2009. Teachers need to develop their own vision of the changed goals of instruction and adapt their teaching in self-determined ways to meet these changed goals.

In India, workshops are an important component of TPD programs on which the greatest time, effort and resources of the state are spent. In our experience, and as reported

elsewhere, TPD workshops are often organized in an ad hoc manner on the basis of expediency, sometimes driven by the need to utilize funds (MHRD, 2009, pp. 2, 15-16). There is no clear consensus about what needs to be done in these workshops and how it is to be done. In structured large-scale programs, TPD is sought to be achieved through the “cascade model” of training (MHRD, 2009, p.15), where master resource teachers are trained first, who in turn train other teachers. The design and content of the modules, which are used repeatedly at each tier of the cascade training, is generally not research-based. The vision underlying most of these programs restrict teachers' agency to implementing a new textbook, a pre-designed pedagogy or a prescribed assessment technique. In our view, TPD programs however need to have a broader vision of the needs of a teacher as a developing professional, view the teacher as an 'active learner', and must address issues of knowledge, beliefs, attitudes and practices in a comprehensive manner, rather than in the narrow context of a particular reform.

The new curriculum, arguably expects from the teacher a deeper understanding of subject matter as well as the teaching learning process, rather than merely adopting new techniques. Teachers in elementary and middle grades are expected to not only make their students fluent in computational mathematics but to also address process goals in the learning of mathematics, such as reasoning, using multiple ways to solve problems, justifying their solution, making generalizations and conjectures, and analyzing the mathematical work of others (NCERT, 2006b). However, there have been few TPD programs in India, which have focused on the skills and knowledge required to facilitate this kind of teaching. Research studies of teachers' knowledge in other countries have pointed to the importance of knowledge that integrates subject matter and pedagogy. Although pedagogical content knowledge and subject matter knowledge have been considered as useful constructs to

describe essential knowledge for teaching (Shulman, 1986; Ma, 1999), it is rarely the central focus of any phase of teacher education in India (Naik, 2008; Kumar, Dewan & Subramaniam, 2012). Thus, we consider providing opportunities for deepening teachers' knowledge of mathematics and of pedagogy revolving around mathematical practices to be one of the central goals for TPD programs.

Bringing about change in teachers' knowledge of mathematics relevant to teaching is clearly a challenging task, but only partly addresses the TPD need. Studies have shown that teachers' beliefs also strongly influence teaching practice and determine what teachers notice in the classroom (Thompson, 1992; Phillip, 2007). In the Indian context, commonly held views include the belief that mathematics is a body of knowledge consisting of known solutions to a well defined set of problems and that not all children are capable of learning mathematics (Kumar & Subramaniam, 2013). A study by Dewan (2009) indicates that such beliefs, which stand in contrast to the ones envisioned in the National Curriculum Framework, are held by not only teachers but even administrators, faculty members and directors of teacher education institutions, thereby indicating the extent of challenge to implement the new framework. This points to the need to create spaces where teachers articulate and reflect on the beliefs that they hold while respecting the identity of the teacher. Teachers need to not only experience alternative ways of doing mathematics, but also to build an awareness of and sensitivity to students' mathematical thinking.

Research studies have illustrated how the development of professional learning communities contribute to teachers' professional growth, by providing a site for articulation and reflection on the beliefs, for sharing the knowledge held and practice adopted by the teachers. (Kazemi and Franke, 2004; Jaworski, 2007; Brodie, 2012).

Thus, in the Indian context as elsewhere, the goals that TPD programs need to focus on include

- Enabling teachers to develop a vision for the changed goals of instruction and become “active learners”
- Providing opportunities to make teachers' knowledge and beliefs explicit,
- Strengthening teachers' knowledge integrating content and pedagogy,
- Building on beliefs through reflection and engagement, and
- Fostering professional communities as spaces for developing shared understanding about teaching and learning of mathematics.

In this chapter, our focus is how the components and interaction in a teacher professional workshop can be shaped to address these goals. The design as well as the enactment of the workshop contribute towards meeting these goals. Hence we develop a framework that illuminates both these aspects. The framework is drawn from our own experience of TPD, from the literature on teacher development and from guiding policy documents in the Indian context such as the NCFTE. We analyze two interaction episodes from the workshop and illustrate how the framework illuminates the task design and the agency of the participating teachers and teacher educators in addressing the workshop goals. Our purpose is to illuminate critical aspects in a particular professional development workshop for mathematics teachers through an analysis of sessions enacted in it.

Framework for Workshop Design and Analysis

In designing TPD workshops to address the goals described above, we consider three guiding principles as essential. These are drawn mainly from our own practice, but are related to the theoretical perspectives of situated learning theory (Lave & Wenger, 1991) and communities of practice (Wenger, 1998). These principles are also related to those from the NCFTE highlighted previously. The three principles, which, in our view, must inform the design and conduct of TPD workshops through all its activities in a comprehensive manner are

- Situatedness in the work of teaching,
- Offering challenges to teachers' to revisit their knowledge and beliefs, and
- Developing a sense of belonging to a professional community.

The aspect of situatedness is addressed through the choice of tasks as well as the mode of presentation of the task. The use of artifacts like students' errors, examples from textbooks, or examples emerging from live or video records of classroom teaching with questions, prompts and examples used in the interaction can recall the context of teaching and learning. It is this aspect that allows teachers to make strong connections with their own practice thereby providing a stimulus for participation and reflection. Moreover, the use of artifacts from the daily activity of teaching has been emphasised by practice based professional development (Ball and Cohen, 1999).

The second principle of challenging teachers' beliefs and knowledge needs to be built into the tasks chosen for the sessions and reflected in the actions by teacher educators such as

re-voicing individual teachers' views for consideration by the participants, and providing counterarguments, explanations and questions to help teachers think about the tacit aspects of teaching and mathematical content. In the TPD workshops, such responses were made not only by teacher educators, but teachers on their own also reacted to their colleagues' articulations by making conjectures, arguments, assertions, counterarguments, explanations and reflective remarks.

The third principle of building a sense of a professional learning community acknowledges that teaching is a cultural activity, and the development of a teacher is not to be viewed in individual terms, but in the setting of a community. We adopt a broad view of community as encompassing teacher educators, researchers and teachers, all of whom are engaged in the enterprise of analyzing teaching. We provided opportunities for discussion, sharing and inter-animation of ideas to enable members of the community to share their histories as a resource for participation (Wenger 1998). In the workshops organized by the authors, this aspect was addressed by posing tasks and questions for the whole group rather than to individual teachers. The teacher educators attempted to situate themselves as members of the larger teaching community by using “we” in their language as well as drawing on their own teaching experiences with students in the course of their research work. Teacher educators adopted several words and categories commonly used by teachers and also elicited and acknowledged teachers' knowledge about students and teaching gained through years of experience.

The three principles described above of challenge, situatedness and community building are interrelated. Focusing on the work of teaching helped in fostering the solidarity among teachers, who were regarded as knowledgeable members of community as they are

engaged in the work of teaching and thus are entitled to have and voice their views. Belonging to a community entails the work of making claims and conjectures, making arguments or counterarguments to support one's claims drawing on the knowledge gained from experience, and supporting the growth of knowledge in a community. Thus challenging beliefs and knowledge was an integral aspect of community building as much as situatedness in the work of teaching.

Understanding the role that interventions such as workshops play in the professional development of teachers requires consideration of not only design aspects, but also of enactment aspects. The affordances of the task that participants work on, and the interaction among the participants determine whether the workshop addresses the goals adequately. The framework outlined thus far includes goals and principles for the design and conduct of the workshop. To facilitate the analysis of the enactment of the workshop, we add further elements to this framework relevant to key features of the interaction during a workshop. We draw these elements from the notion of the teacher education triangle adapted from the didactic triangle (Goodchild & Sriraman, 2012) as shown in Figure 1. The interaction during a TPD session can be conceptualized as an interaction between the three elements of the task, the teachers and the teacher educators. We focus on the affordances of the tasks and on the agency of both teachers and teacher educators. Rather than viewing agency as “associated with the individual subject as a self-standing entity,” we describe how this “arises out of engagement” (Wenger, 1998, p. 15). The engagement that we focus on is with colleagues and teacher educators who share the common enterprise of improving mathematics education in schools.

Figure 1: The teacher education triangle

In the next section, we briefly describe the components of the workshop and how they relate to the goals and design principles in the framework. In the subsequent section, using the framework consisting of goals, design principles and interaction elements (see Table 1), we provide an analysis of the interaction during two episodes from the TPD workshop under sections dealing with *the nature of the tasks, the agency of the teachers and the agency of the teacher educators*. The choice of the task, the communicative devices used by the teacher educators and the efforts to shape the interaction, all reveal the importance and inseparability of the aspects of situatedness and challenge. We describe the efforts made to situate the discussion of teaching and learning both in the context of the work of classroom teaching and within the community of teachers. The questions that we specifically address are the following:

- How is the teachers' repertoire of knowledge and beliefs gained from experience brought into play in their engagement in the workshop?
- How are learning opportunities or opportunities for reflection on beliefs created in the course of the teachers engagement?
- What aspects of teachers educators' enactment of the task and interaction with the teachers facilitated the engagement of teachers?

Table 1

Framework for analysing design and enactment of a TPD workshop

Workshop goals	Principles for designing components and tasks	Interaction aspects
Strengthen teachers' knowledge relevant	Situatedness	Task structure

to mathematics teaching	Challenge	Teachers' agency
Provide opportunities to articulate and reflect on beliefs relevant to teaching mathematics	Community building	Teacher educators' agency
Fostering the development of professional communities of learning		

Table 1

Components of the TPD Workshop

The design of the TPD workshops held at the HBCSE is informed by the goals of providing opportunities to teachers to strengthen their knowledge for teaching mathematics, to reflect on their beliefs, and to foster the building of professional learning communities. The principles of situatedness, challenge and community building guide the design of the workshop components, which include sessions involving the study of classroom teaching to learn about content rooted in pedagogy, learning about students' thinking from students' responses, working on mathematical problems and understanding relevant research on teacher learning and the work of teaching. Every experience with teachers has helped us modify our thinking about what form of the content is most relevant and functional for teachers' reflection and development. Our aim here is not to present ideal designs for workshops, but to highlight the opportunities for the development of teachers' knowledge and reflection on beliefs that emerged in the workshops. To provide concrete details of the design, we focus on a residential workshop that was held for ten days, describing the various components and their functions. Following this description, we discuss in detail two episodes from this workshop, focusing on the enactment aspects.

The majority of teachers who participated in this workshop belonged to a nationwide government school system catering to children from a range of socio-economic backgrounds.

The school system had taken various steps to implement reform based on National Curriculum Framework mainly through teacher training and issuing notices and circulars. Of the 20 teacher participants, 12 were from this school system: four middle school teachers from outside Mumbai, four primary and four middle school teachers from Mumbai. All the eight local teachers were engaged in follow up activity after the workshop. Among these, four teachers also received support in their classroom teaching by one of the authors. The follow-up phase of the project is not reported in this chapter.

The following is a description of the components in the workshop, with an elaboration of their role in the development of mathematical knowledge required for teaching and in reflecting on beliefs and attitudes related to teaching and learning of mathematics.

Observing classroom teaching. Teachers viewed and participated in three consecutive days of live teaching during the workshop. The activity involved teachers in contributing to the plan for the class, observing the lesson and then reflecting on it. The teaching observed was atypical, in its focus on eliciting students' ideas and building on them and thus was intended to be a source of reflection and challenge for teachers. These sessions provided teachers a context for making their situated knowledge about pedagogical approaches and students' capability explicit. Prompts to elicit reflection on the lesson included inviting teachers to make conjectures about the intentions of the teacher in making specific moves, what children were thinking and what alternative pathways could have been taken at critical points in the lesson.

Learning through problems. Teachers worked on mathematical problems during these sessions, which were posed in contexts close to either their teaching practice or daily life. The problems were content specific and therefore separate sessions were conducted for

different content topics such as number sense, fractions and ratios, and algebra. The main objective was to create distractions in a mathematics problem based on familiar alternative conceptions that teachers or students have, leading to cognitive conflict and eventually to reflective learning. The problem presented in Table 2 is an example.

Table 2: Example of a workshop problem task

A student in the class had added fractions like this: 3/7 + 2/3 = 5/10. Why do you think students add in this way?

When the teacher asked the student why she had done it in this way, the student said that her father had taught her. The teacher explained that this method was wrong. On the following day there was a complaint from the father. He pointed out that the teacher had added exactly like his method. This was his example, Marks in history: 35/50. Marks in geography: 24/50. Total marks in social studies: 35/50 + 24/50 = 59/100.

How would you respond to the parent's criticism?

These sessions occasionally led to deep exploration of mathematical concepts and making connections between various mathematical constructs, and providing a space for teachers to reflect and build upon their mathematical knowledge for teaching.

Working on students' thinking. These sessions included working on students' errors, uncovering students' thinking by analysing strategies, and inferring potential misunderstanding underlying these errors. These led to discussions on issues such as – which questions are efficient in evaluating understanding of a specific concept, what do these errors tell us about students' and teachers' own conceptions including their beliefs about the nature

of mathematics, and what do students' errors imply in terms of shaping the instruction. In the next section, we discuss in detail the interaction in two sessions of the workshop that involved working on students' errors.

Reading literature based on research. In these sessions, teachers in groups of three to four studied a research article from the field of mathematics education and made presentations to colleagues. The sessions were found to be valuable in fostering the sense that a teacher is a part of a community that systematically studies content and pedagogy with the goal of improving teaching and learning. The readings stretched the boundaries of the participant teacher community from the immediate peer group to the professional community of mathematics educators including researchers.

Analysing curriculum material. These sessions were included to add connection, coherence and depth to teachers' comprehension of textbooks so as to use them efficiently. Teachers in groups of two to three analysed textbooks from grade 3 to 6 for a specific mathematical topic for the following prompts – what is the hierarchical development of the topic, how are context and real life connections brought about, what is the role of the various examples provided, and how are representations used in the textbook. Although teachers use these textbooks on an everyday basis, these sessions provided an opportunity to distance themselves from the sole purpose of teaching and look at the textbook critically.

Expressing beliefs about teaching, students and mathematics. In this session held at the beginning of the workshop teachers completed a 6-part questionnaire based on the Likert-type scale, which provided them an opportunity to reflect about their own beliefs about teaching, students, self and mathematics. Teachers worked individually on these questionnaires. The questionnaire items at times framed the discussions in subsequent

sessions. Teachers also reported that the statements mentioned in the questionnaire made them think about issues that they had never thought of. At the end of the workshop, teachers were given parts of the belief questionnaire on mathematics, mathematics teaching and preferred practices and were asked to record the changes in their views. Table 3 presents a sample of the items used in the questionnaire.

Table 3: Sample items used in the questionnaire for teachers

Domain	Sample item	5 point Likert-type scale
Practice	I ask students to practice problems very similar to one done in the class as homework.	Almost never to Almost always
Belief about mathematics	Being good at mathematics means being able to perform calculation quickly and accurately.	Strongly agree to strongly disagree
Belief about teaching mathematics	Listening carefully to the teacher explain the mathematics lesson is the most effective way to learn mathematics.	Strongly agree to strongly disagree
Belief about self	If something is not clear in the textbook I am confident that I can work it out on my own.	Strongly agree to strongly disagree
Belief about students	Student from poor homes tend to struggle in mathematics.	Strongly agree to strongly disagree

Workshop Enactment

The sessions of the workshop were generally characterized by high levels of interaction and participation by teachers. The two episodes of the workshop that have been

selected for analysis here, deal with errors made by students and the learning for teachers from these errors. The two sessions were led by different teacher educators, and were structured differently. Transcripts of the two episodes were prepared from the video records of the sessions. The coding process was adapted from Miles and Huberman (1994). The transcripts were coded broadly into three categories: the task design features, the interaction features and teachers' explorations and reflections. Dimensions within these categories were identified and consensus about the coding was established by discussion among the coders (authors). After the initial coding, the themes explicated in this chapter were arrived at keeping in view the framework presented in Table 1.

The Task as a Resource for Teacher Education

In Episode 1 (Day 2, about 30 min), the task was to describe and explain student errors from looking at their responses to seven test items on the topics of number, place value and fractions (see Table 4 for sample items). At the beginning of the session, the teacher educator provided a set of prompts to guide the discussion for each question: identify competencies being tested, find all possible correct answers and understand what caused the student errors.

Table 4: Example of test items and student errors shown to teachers in Episode 1

Questions	Student error 1	Student error 2
Write next three numbers: 3097, 3098, ------, ------, -------	3097, 3098, 3099, 30910, 30911	3097, 3098, 3099, 30100,30101
14 tens + 23 ones	1423	14023
Draw 7/4	Drawing representing 4/7	Drawing representing 7/11

In Episode 2 (Day 1, about 30 min) teachers were given a handout that described a

student Mohsin's difficulty in writing numerals despite his familiarity with numbers (see Table 5). The teachers discussed in groups and presented their suggestions about how Mohsin could be helped.

Table 5: Task given to teachers in Episode 2

Mohsin is in class 5. He helps his father, who is a vegetable seller, with home deliveries. He can find the total amount a customer has to pay and often does the addition mentally. He also knows a lot about how much things cost: Televisions, cycles, two wheelers, washing machines, etc. But when his teacher asked him to write 'rupees two thousand twenty five' in numerals, he wrote 'Rs 200025'. Think about what Mohsin's problem is and how his teacher can help him. How can the teacher make use of what he already knows so that he can learn something he doesn't know.

Both the tasks are invoked together with supports drawn from teaching practice itself to give authenticity to the task and to invite deeper engagement on the part of the teachers. In the first episode, the student errors were drawn from a pre-test of students participating in the vacation course, the same group of students whom the teachers knew they would be observing later. In the second episode, the teacher educator informs teachers that Mohsin is a real boy whom he teaches, and furnishes details about Mohsin's responses to other tasks in the course of the discussion. Teachers identified the students' responses from both the episodes as “common” among their students. However it required further probing on the part of teacher educators to make teachers think about students' thinking underlying the errors and the sources of the errors, beyond identifying them just as “common” errors.

The tasks chosen for the two episodes are situated in the context of teaching while challenge is introduced by requiring the teachers to think beyond the normal requirements of everyday teaching. In the first episode, this is achieved through the three prompts inviting teachers to uncover a deeper layer of students' thinking that can explain their responses. Teachers at first thought that the errors surfaced because of the non-typical questions asked in the test. For example, to explain why the student incorrectly showed 4/7 instead of 7/4, teachers said “because numerator is greater than the denominator in the given fraction.” Several teachers thought that it was not possible to represent improper fractions. The teachers interpreted fractions in terms of the “part-whole” meaning as number of parts out of total parts, which made representation of improper fraction impossible. The task thus challenged them to re-consider meaning attributed to fractions. We will return to the discussion of this example later.

In the second episode, the teachers were asked to consider both what the child does and does not know so as to induce a sense of conflict by juxtaposing these together. For e.g., teachers identified that Mohsin knows numbers in the thousands, can add mentally and can read the price of a bicycle but cannot write the amount 2025 correctly. What accounts for the child's capability in the context of everyday calculations, and his profound lack of understanding of a related part of school mathematics? This tension sets a dialectic in motion allowing the teachers to revisit the relatively hidden and unspoken aspects of their everyday teaching.

Both the tasks worked as a vehicle for reflection and engagement on the part of teachers by articulating their beliefs and knowledge. The discussion moved beyond the immediate demands of the tasks to broader concerns like connecting teaching of mathematics

with out-of-school experiences or considering students' thinking underlying their mistakes as a resource for teaching mathematics. Thus the discussion of tasks need to recede from focus, opening up a space for deeper engagement, where teachers can share and critically reflect on what they know, understand, believe and practice. The role of teachers' and teacher educators' agency in engaging with task and the emergent issues will be discussed in the sections below.

Teachers' Agency in Engaging with Knowledge and Beliefs

An important part of understanding the work of teaching as a profession is a shared agreement about the specialized knowledge and expertise that informs the work of teaching. Professional development programs need to elicit and build on such knowledge, much as teachers elicit and build on students' knowledge in the classroom. This process also allows the community of teachers and teacher educators to develop a shared view of the contours of such knowledge. Providing opportunities for the process of eliciting teachers' knowledge and beliefs lead to a understanding of teachers' agency. We define teachers' agency as initiatives and autonomy expressed by teachers during the course of interaction to assert and justify their beliefs. In this section, we explore how teachers' agency was expressed in the course of the interaction during the selected episodes.

Anticipating students' responses. Requests to teachers to anticipate and predict student responses were either built into the task itself or were made by the teacher educator in the course of discussion. This aspect is embedded in teachers' everyday work of teaching. Over the years teachers develop an implicit knowledge about typical and atypical student responses. In the TPD context, making this knowledge explicit works as a resource in building the shared knowledge between teachers and teacher educators and providing ways to discuss students' thinking. In episode 1, teachers were able to anticipate some student errors

for the questions (see Table 4) which paved the way for discussing student thinking. In episode 2, for example, the teacher educator asked teachers to predict Mohsin's strategy to find the cost of 10 kg potato given the cost of 1 kg. The teachers anticipated that Mohsin would repeatedly add the unit cost to arrive at the cost of 10 kg, which the teacher educator confirmed was what Mohsin actually did. These questions were significant as they directed teachers' attention towards what the student did know at a point when they were focusing only on his incapability. Sharing the anticipations paved the way to a discussion about the differences between mathematics students learn outside school and in school and the need to bridge the gap between the two.

Identifying "key knowledge pieces". At times, teachers contributed centrally to the goal of building mathematical knowledge for teaching. In episode 2, while trying to elaborate why Mohsin made the error of writing two thousand twenty five as 200025 a teacher explained that understanding the "meaning of zero" involves understanding how it changes value with position – it has no value when written in the leftmost position of the numeral and in other positions it determines the place value of the other digits in the numeral. Her explanation of the concept of the position of zero can be characterized as a "key knowledge piece" (Ma, 1999) that is important in understanding place value of a number. Her intervention led other teachers to also identify the conceptual gap in the student's thinking and a teacher asserted, "he knows 2000 and he knows 25 but how to write [2025] he doesn't know".

Conjecturing underlying causes. In episode 1, the discussion on understanding student errors led to discussing underlying causes of the errors. For the question of drawing a representation of the fraction 7/4, some students had drawn part-whole representations of 4/7

or 7/11. Teachers tried to explain the thinking that might underlie these responses. As discussed earlier, initially teachers identified the cause of the error as unfamiliarity with question. In the course of the discussion, a teacher put forth an alternative explanation of why students drew 7/11 to show the fraction 7/4. The student, he argued, may have interpreted 7/4 to mean “7 shaded and 4 unshaded parts” thus making a total of 11 parts of which 7 were shaded. Thus teachers began to engage with the reasoning that the student must have applied to create such representations. The discussion moved to how counting of shaded parts (using whole numbers) was generally over-emphasized while teaching fractions. Thus, teachers also reflected on their own teaching as a possible cause for student difficulties in learning. To cite another example, when a question was raised in episode 2 about why students are not able to learn mathematics even after five years of schooling while they learn quickly outside the school, a teacher observed that “we do not correlate mathematics taught in school with everyday life”.

Articulating and contesting beliefs. We consider the occasions when teachers' beliefs were explicitly articulated as important moments in the workshop. At times this took the form of reflection suggesting a revisiting of beliefs. For e.g., in episode 1, after discussing student errors and students' thinking underlying these, a teacher reflected “in fact we know their mistakes but we don't really see into their thinking”. Teachers contested and challenged views articulated not only by their colleagues, but also by teacher educators. For e.g., in episode 1, the question, “Add 337 + 33700” was discussed, where students had made an error in vertically aligning the digits. A teacher reacted to the example and said “Addition questions should not be given in horizontal manner as it will lead to error” thus indicating her belief that errors should be avoided during instruction. Similarly, during a discussion of teaching aids for teaching place value, a teacher voiced his opinion that using teaching aids will cause

lot of confusion and it would be better if students are told the rules and asked to practice. Another teacher contested this by saying that they themselves (teachers) had learnt mathematics by rote when they studied in school but it is important now to emphasize understanding concepts. At a point in the discussion, a teacher asserted that the abacus was easier for students to learn place value. Another teacher responded by explaining how stick bundles representing different units (tens and hundreds) can build understanding of place value better. Thus voicing of assertions led to sharing of alternative viewpoints, which created a need for justification, thus deepening the engagement in the workshop. In this instance, the difference between the stick bundles and the abacus led to exploring the difference between the grouping principle and the positional value principle discussed later.

These articulations were important points in the sessions, which provided a window into teachers' thinking as well as created a space for revisiting and reflecting on beliefs relevant to teaching and learning. Teachers also assessed their own learning in the workshop as reflected in an appreciative comment by a teacher at the end of episode 1. The active interventions by the teachers described above were indicative of teachers' agency as they were not merely involved in affirming or contesting what the teacher educator or other teachers were saying but were engaging in their own sense making about the aspects discussed related to the task.

Agency of the Teacher Educator: Inter-animation, Knowledge and Beliefs

In a TPD context to what extent teachers' knowledge and understanding are elicited, what aspects of knowledge are negotiated and in what direction the discussion moves during a session depends critically on the interventions made by the teacher educator. Not only is the participants’ engagement crucial, but also the degree of inter-animation of ideas. Scott and

Mortimer (2003) have used Bakhtin's idea of inter-animation to illustrate how an interaction in the classroom is “functionally dialogic” when more than one point of view about an issue is represented as well as explored. Scott and Mortimer define low inter-animation as just listing of the varied responses shared in the group while high inter-animation means that there is an engagement with the different views expressed by the participants as a group. In the context of TPD too, we found that the aspect of inter-animation of teachers’ responses was crucial in how teacher perceived their roles in the session. In episode 1, inter-animation was low partly because the interaction was structured in such a manner that there was one correct answer for the questions posed and even when different opinions were voiced, the teacher educator responded to the individual teacher by elaborating and emphasising the interpretation that she thought was correct. On the other hand in episode 2, the teacher educator considered different conjectures voiced and posed the differences in opinions as questions to be considered by the whole group. Not only did teachers contest the views of other participants but they also contested views expressed by teacher educator leading to discussion. The moves that led to high inter-animation in episode 2 included inviting teachers to respond to each other, problematizing teachers' responses to be discussed by the group and the use of open-ended questions.

The teacher educators' interventions during the course of the interaction in turn are guided by their own beliefs and the knowledge that they bring to bear on the discussions. In this section, we briefly discuss first the moves made by the teacher educators that reflect the goals and the principles outlined in the workshop design framework and second, the interaction between beliefs and knowledge as reflected in the teacher educators' interventions.

Teacher educators’ moves in alignment with the goals of TPD. The teacher

educators frequently invited teachers to respond to the views expressed by their colleagues communicating that the teacher educator is not solely responsible for evaluating teachers' responses but that it has to be decided by the deliberation of the whole group. This led to situations where the teacher educators had to handle disagreement and conflict of views. Teacher educators usually welcomed disagreement and considered it a healthy sign of engagement, which allowed teachers to articulate their beliefs, which could then be taken up for discussion. The teacher educator sometimes restated a view in more general terms by placing it in the broader educational context. In episode 2, when there was disagreement over whether it is better to teach students rules for algorithms, the teacher educator contrasted learning inside and outside school (translated and summarised: “if students hold strongly what they learn from outside the school, but they are not able to hold on to what is learned at school, why it is not held we must think and talk about it”). This can be interpreted as an attempt to build a sense of community by inviting teachers to reflect on their beliefs in the context of larger educational debates.

The use of open-ended or probing questions by the teacher educator elicited more and more varied responses from the teachers leading to richer discussion strands. There were several examples of this in episode 2. The teacher educator asked teachers to suggest a variety of learning aids that could be useful for Mohsin. He then invited them to think about which teaching aids are better and why. Teachers' responses to such questions were often elaborate with some teachers recounting their own teaching experiences. Another significant move by the teacher educator was asking for clarification of the meaning of terms used by teachers and moving towards shared meaning and vocabulary. For e.g., in episode 2, the teacher educator asked for a deeper probing of the meaning of “place value”, a term that occurred frequently in the discussion.

Interaction between teacher educators' beliefs and knowledge. The teacher educators held and acted on beliefs that were at times different from those held by teachers. Since the teacher educators' actions were guided by an expectation that teachers accept these beliefs, they could be considered to be belief goals for the TPD program. Some of the beliefs had to do with the emphasis or value ascribed to elements in the teaching-learning context. For example, the teacher educators believed that what a student knows is more important or at least as important as what he or she does not know. The emphasis placed on this was prominent in episode 2, when the facilitator repeatedly brought teachers' focus back to what the student (Mohsin) knew when the discussion turned to what he did not know. In episode 1, the following student error was discussed. In response to the question "show the number made of 14 tens and 23 ones," a student wrote "14023". Teachers thought that this student did not have a concept of place value. A teacher educator present in the audience pointed out that in fact it did show a partial understanding of place value since the student knew that 14 tens can be written as 140 and 23 ones as 23.

Another visible belief of teacher educator was – the emphasis on what the student knows in contrast to what she does not know is consistent with ascribing value to students' thinking as a resource for teaching. This belief interacts with knowledge about students' ways of thinking in enhancing teachers' awareness and sensitivity. The teacher educators attempted at times to lead the discussion into understanding students' responses more deeply. In the example discussed earlier of why some students incorrectly represented the fraction 7/4 by drawing a picture for 7/11, a teacher educator conjectured that it could be due to excessive emphasis on the part whole representation of fractions by making as many parts as the denominator and shading as many parts as the numerator. Thus instruction treats numerator and denominator as separate whole numbers. Taking the discussion further, a teacher

educator in the audience proposed an analysis of the fraction concept, by distinguishing between counting and measuring contexts. He suggested that measuring rather than counting is a better context to understand a fraction as a single number indicating a particular quantity. Counting contexts tend to reinforce the idea that a fraction is made up of two numbers. These interventions not only led to a deeper probing of students' thinking, but also to understanding how the choice of a teaching approach may play an important role in students developing certain conceptions.

Another belief held by the teacher educators, which is related to valuing students' thinking, is the belief in the efficacy of using students' previous knowledge (especially knowledge acquired from the everyday life/culture) as a resource for teaching. This was foregrounded in episode 2 by asking teachers to anticipate Mohsin's responses while using information about the daily life activities in which he engages and raising the question about the need to bridge the gap between the mathematics learned outside and inside the school.

The teacher educators attempted to communicate that umbrella concepts like “place value” need to be understood in detail, in terms of how they play a role in specific contexts of learning related concepts, of problem solving, or of understanding an algorithm. To become more useful, they need to be decomposed into sub-concepts like “grouping principle” and “positional value principle”. The grouping principle determines that in the decimal number system, 10 units form the next higher unit in the sequence of units, tens, hundreds and so on. Number words encode the grouping principle by naming the different powers of ten: “four thousand”, “six hundred”, etc. The positional principle in contrast determines that in the written numeral the value of a “digit” depends on its position. This principle is essential to understand written numerals and includes understanding that when a zero appears at a certain

position, it indicates that there are no units corresponding to that position. Both the grouping and the positional value principles help to reconstruct the number from the written numeral. This distinction was important in episode 2 in understanding what Mohsin knew (prices of articles and composing money in terms of currency units) and what he did not know (writing a number). The distinction between the grouping and positional value principles was new to many teachers, at least in an explicit sense.

The teacher educators believed that the usefulness of a “teaching aid” depends on the context and specific needs/difficulties faced by students. In episode 2, to help address Mohsin's difficulty, teachers had selected aids embodying both grouping principle as well as the positional principle. This did not take account of the fact that while Mohsin had a weak understanding of the positional principle, he had a strong grasp of grouping of multi units because of his familiarity with money. The teacher educator was able to use teachers' responses to explicate how understanding of place value embodies both principles and how the abacus specifically caters to develop understanding of the positional principle.

Discussion and Conclusion

The effectiveness of TPD workshops depend both on design and enactment aspects. We have attempted here to present a framework that can aid in the understanding of both aspects and to illustrate how the framework may be applied through an analysis of the components of a workshop and two interaction episodes. The framework assumes that the central goals are to address teachers' knowledge and beliefs relevant to mathematics teaching. The framework does not describe what constitutes knowledge for teaching mathematics, nor does it elaborate on the nature of beliefs conducive to teaching for understanding. A framework that elaborates on the specifics of knowledge and beliefs relevant to teaching

mathematics will need to be contextualized with regard to topics and to teacher communities. The framework presented here, in contrast, identifies certain principles that are important for the design of tasks and their enactment in workshop sessions. The study reported here does not aim to provide evidence for the effectiveness of a TPD intervention. The framework proposed here, we believe, is useful in identifying and providing rich descriptions of elements that are important in a TPD intervention.

The principles we consider important are situatedness, challenge and community building. The components of the workshop and the tasks worked on were chosen and designed to embody these principles. As the analysis of interactions in the two episodes shows, not only is the design of tasks important, but also how interactions between teachers and teacher educators are shaped to support teacher learning. We used the teacher education triangle having the three corners as task, teachers' agency and teacher educators' agency, as a framework to analyse the interaction aspects. The task incorporated contexts and artifacts that are situated in the work of teaching thus facilitating the involvement of teachers by identifying elements common with their teaching practice and engaging in deeper exploration of the contexts and artifacts. In the episodes discussed above, a prominent artifact was students' errors or responses. Discussion centered on these led teachers to analyse conceptual gaps, alternative explanation of errors and develop a perspective of explaining student errors by thinking about students' sense making efforts.

The evidences of the types of teacher engagement that occurred during the episodes throw light on the kind of opportunities that arise for teacher learning. Teachers' engagement took the form of anticipating and predicting students' responses, identifying key knowledge pieces, conjecturing underlying causes, articulating and contesting beliefs and assessing a

teaching resource or a teaching approach. Such engagement was crucial in building shared understanding not only among teachers, who rarely get opportunities to reflect collectively about teaching in their schools and professional development contexts, but also for teacher educators by providing windows into teacher thinking. Teachers' assertions, counterarguments, alternative explanations and assessments were also a resource, which deepened fellow teachers' and teacher educators' understanding about mathematics teaching as it takes place in classrooms. The key knowledge piece of the meaning of zero, identified by an expert teacher, was important in deepening the participants' understanding of the conceptual gap that needs to be addressed to help Mohsin in writing numbers correctly.

We have elaborated on the principles of situatedness of tasks, challenges and development of community as guiding the design and enactment of the sessions. These aspects inform the decisions of the teacher educators about how interventions are to be made in sessions to facilitate active learning of the teacher. The belief goals of the teacher educators helped in guiding what interventions are to be made in terms of prompts presented to the teachers and in identifying aspects of teachers responses that could be problematized. The agency exercised by the teacher educator is important to not only actualize the opportunities afforded by tasks, but also in guiding discussions beyond the resolution of the tasks in order to relate to the broader goals of teaching mathematics. The actions of the teacher educator like inviting teachers to respond to each other, handling disagreement and conflicts by posing issues as more general questions, use of open ended questions and building shared vocabulary paved the way for building a sense of community while challenging teachers to explain and justify their thinking.

We claim that while designing workshops for teachers it is essential to not only focus

on the aspects that need to be discussed with teachers but also how the session needs to be enacted to allow teachers to exercise their agency and take ownership of their own learning rather than looking for answers from outside. This is important because we need to provide ways through which teachers can build on the knowledge of students and mathematics teaching that they already have rather than merely providing knowledge, which teachers may or may not find useful in their own classroom contexts. This point is important for designing workshops for in-service teachers as they have already developed identity as well as situated knowledge of students and teaching which must be respected and built upon.

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