

EPICYCLICAL ASTRONOMY: A CASE FOR GEOGEBRA

Amit Dhakulkar and Nagarjuna G.

Homi Bhabha Centre for Science Education, TIFR, Mumbai, India

amit@hbcse.tifr.res.in, nagarjuna@gnnowledge.org

Epicyles were historically used by the ancient Greeks to explain the retrograde motion of planets. This episode in history of science is used as a case to show how we can use computer simulations to visualize complex, abstract ideas and difficult to imagine constructions. We present here a method developed using the dynamic mathematics software GeoGebra, to teach the concept of epicyles.

Keywords: GeoGebra, Astronomy education, Epicyles, Visualization, Constructionism

INTRODUCTION

When students face complex and abstract ideas which need a lot of imagination, they find it difficult to visualize and concretize the ideas. For visualizing and understanding these abstract ideas some concrete ideas are needed. Concrete models here, would mean some physical objects or computer simulations which will try to render the nature of the phenomena or concept under question as closely as possible (Hesse, 1966). The physical objects or the computer simulations in this case would fall in the category of 'external' visualizations. External visualizations provide support for all perception, including that in science (Gilbert, 2005). While building actual physical models is desirable in many cases, it may not be always possible to actualize in practice. Instantiating all possible values of the model are difficult, often impossible. However it is important to work out the possible implications of the model to gain proper understanding of the model. In such cases computer simulations clearly win over the physical models. The computer simulations offer the flexibility and ease to change the parameters of the model, which is not always possible in the physical models. Due to this very fact students can be left to explore a particular simulation and this can possibly lead to the construction of knowledge by students themselves. We present here one such case and how using a computer could help gain understanding and construction of the required domain knowledge. We have prepared a generic model using the dynamic mathematics software GeoGebra for explaining the motion of the celestial bodies using the idea of *epicyles* and its variants in the context of elementary astronomy. This example also underlines the use of computer for scientific imagination, in the current case that of ancient Greek astronomers.

The present work was implemented in an undergraduate history of science course, which is taught by one of the authors. The course uses an approach to teach the ideas that the ancient scientists used to develop the subject matter. We used a computer simulation to explain how the Greeks attempted to solve the problem of retrograde motion of planets. This exercise helped the students realize and appreciate the rigorous mathematical calculations performed by the ancients and also the possible implications of using such a model.

To teach the concepts in elementary astronomy in this course we used two Free and Open Source Softwares (FOSS), GeoGebra¹ and Stellarium². Stellarium is a planetarium software which was used to show the movements of the celestial bodies across the sky. For presenting the geocentric world view we used the dynamic mathematics software GeoGebra. In what follows we briefly present an account of the ancient Greek's scheme of the heavens, and their construction using GeoGebra and its possible implications.

For this course we have mostly followed (Hogben, 1938; Hoskin, 2003; Rogers, 1960; Toulmin & Goodfield, 1961) in presenting and developing the ideas regarding astronomy of the ancient times.

EPICYCLES AND ECCENTRICS

The Alexandrian school of astronomy, begins from about 330 B.C. and continued for a few centuries. There are many remarkable discoveries in astronomy attributed to this school. For our purpose we will consider the schemes produced by them to explain the motion of the planets. The scheme of the Alexandrian school peaked with Ptolemy's model, which lasted till the time of Copernicus in 15th century. The idea that the Earth is stationary was firmly established in the later period, but the model with so many spheres was complicated. They instead thought about an idea that would simplify the complexity of system. The uneven motion of the Sun, faster in the winter and slower in the summer, can be predicted by just using one eccentric circle as shown in Figure 1. The Sun is moving on a circle, which rotates at a constant speed, the Earth is not at the centre of the circle, but is off-centre. A similar scheme was devised for the moon.

For explaining the motion of the planets each of the planet was made to steadily go around the centre once in its own “year”, and the center of the planets’ orbit was made to revolve around the Earth once in 365 days (Rogers, 1960). Thus the planet has a superimposition of two circular motions, producing observed epicycloid track. The eccentric scheme for the planets is shown in Figure 1.

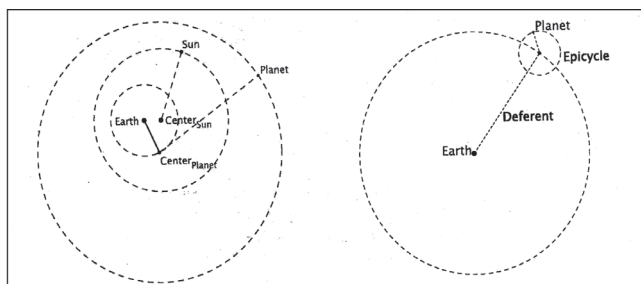


Figure 1: The idea of an eccentric circle for one planet and the Sun

Figure 2: The idea of an epicyclic model for a planet

These schemes though are operational with circles, a more grand description would be in terms of spheres. For many centuries the astronomers indeed thought in terms of “motions of the heavenly spheres” (Rogers, 1960).

Epicyclical model as shown in Figure 2 equivalent model to explain the same effect. Hipparchus (~140 B.C.) showed the equivalence of the eccentric and the epicyclical schemes to represent the heavenly motion. In this case there is a main circle whose radius arm is rotating at a constant rate. This is the deferent circle. The end of this arm carries a smaller circle the epicycle. The track thus produced by a point on the smaller circle is called epicycloid, when both are moving at constant speeds. The word epicycle literally means outer-circle. This curve can be produced mechanically by rolling a smaller circle on the circumference of the larger one. The mathematical details and the parametric equations of the curves generated in this way, are provided in chapter 6 of (Lawrence, 1976).

After Hipparchus the final model that we present is that of Ptolemy (~120 A.D.). In Ptolemy’s model the Earth is stationary, non-rotating at the center. The sphere of the stars is rotating once every 24 hours. The Sun has a simple epicyclic scheme as given by Hipparchus. The Moon has a bit more complex epicycle. For the other remaining five planets Ptolemy found that there were discrepancies, with simple epicyclical models. The theory and the data did not fit. Ptolemy came up with a scheme which could “save the phenomena”. In his scheme, shown in Figure 3, the Earth is not at the centre of the larger circle, as is in the case of epicyclical model. It is placed at some distance from the centre of the main circle. There is another point called as equant, which is at the same distance from the centre as the Earth. The equant is the centre for the epicycle of the planet.

Ptolemy’s scheme was remarkably successful in explaining the observed motion of the planets. By adjusting the speeds and the lengths of the orbits, one can get the desired results. But the discrepancies mounted with this scheme also. This scheme was set to rule the models for the next thousand years.

BUILDING THE GREEK COSMOLOGICAL MODELS

The physical model for depicting the epicyclical movements can be made with the arrangement given in Roger’s book (1960). The problem with such a physical model is if we want to change the length of the deferent or the radius of the epicycle, it is not easy to do so. Another problem would be actually tracing the actual orbit produced by the planet. These things can be done, but it will take more efforts to do so. In this case we preferred the computer simulation of the model using GeoGebra.

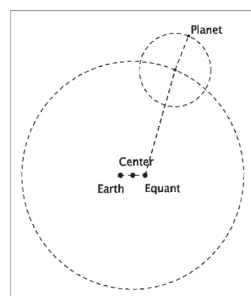


Figure 3: Ptolemy’s scheme to explain the motion of planets

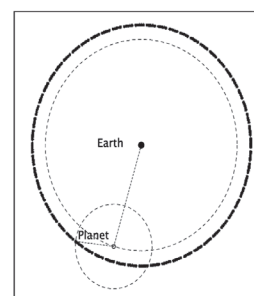


Figure 4: A screen shot to show the phenomena of being able to watch the same face of moon. The radius of the epicycle and length of the deferent are the same. Here $a = 1$, $h = 1$, $Radius = 4$, $Deferent = 10$

To build the cosmological models of the ancients we used the already existing tools in GeoGebra. In the sections that follow the radius of the deferent circle is denoted by *Deferent* and radius of the epicycle as *Epicycle*. In the model we also have options for changing these radii using the *Slider* function in GeoGebra. In the sliders we have an option to *Animate*. When this option is selected the variable in the slider, changes its values automatically, with a given step size, from minimum to maximum values. The rate at which the slider changes its values can also be changed. In our model of the epicycles we need to change the rate of the angular speeds for the deferent and the epicycle quite often, so we have provided, sliders for them too. For changing the speed of the point on the main circle we use the slider named a , and for the epicycle slider named h . Finally we can enable the option of *Trace On* on any of the points in the construction. We have put it *On* for the planet on the epicycle. The *Trace On* option generates the trajectories in red color which we have shown in the screen shots. In the next section on exploring the models, the values of the four parameters which generated, the screen shots will be given.

GeoGebra offers an inbuilt tool which enables one to see how a particular file was constructed. In the *View* option on the main Menu-bar in the GeoGebra window, there is an Option of *Construction Protocol*, which lists the order in which the objects were created. Also just below this option is the option for *Navigation Bar for Construction Steps*, which shows a screen video for the construction steps. The GeoGebra files for the different models are available at: <http://hos.gnowledge.org/geogebra>.

EXPLORING THE MODELS

We would like to mention that in 1960 Norwood Hanson wrote a paper on epicyclical astronomy, from which we borrow our title, states that “When confronted with this triangular “orbit” and the square one which follows, several historians and philosophers of my acquaintance have registered startled, incredulous reactions” (Hanson, 1960). We couldn’t agree more. We were not aware of the work of Hanson when the models were prepared in GeoGebra. But we did register the “startled, incredulous reactions” indeed. Also when these models were presented to different groups of students and academics they also were “startled”.

In our construction of the epicycles all the 12 figures which appear in Hanson’s paper can be produced, with proper adjustments of speeds and the radii of the orbits. Apart from these, there is almost an infinite number of figures which can be produced using the constructed model. In this article we show some screen-shots for the same.

The static screen shots which appear in this article do not do justice to the *dynamic* movement of the planets as seen in GeoGebra. We present some of the most startling ones.

The circle, and the face of the moon

If we keep the radius of the epicycle smaller than the length of the equant and keep the speeds of the two same then we see a simulation of a natural phenomena concerning the moon. The period of revolution of moon and its period of rotation around the earth is same. This is the reason why we see the same face of moon. With these settings we see that the same face of the epicycle, even when it is rotating about its own axis and revolving around the Earth. The Planet revolves around the Earth in a circular orbit. The screen shot of Figure 4 shows when the planet moves in such an orbit, in the resultant

animation one can clearly see that the same face is directed towards the Earth.

The ellipse

Using the same setting for radius and the length of the deferent, but changing one of the speeds to -1 instead of 1, we get the resultant orbit of an ellipse, Figure 5. We can vary the shape of the ellipse by changing either the radius of epicycle or the length of the deferent. Figures 2, 3 and 9 in Hanson’s article (1960) have elliptical orbits. When the radius is taken to lower limit of zero, the resulting motion is circular with radius being equal to that of the deferent. If we make the radius of the epicycle larger than the length of the deferent, the resultant orbit is an ellipse. But the intermediate case when both of them are equal presents an interesting orbit. If we keep the length of the radius of epicycle equal to that of the deferent, resulting motion is a perfect rectilinear motion, the screen shot is shown in Figure 6. This appears as Figure 5 in Hanson’s article (Hanson, 1960, p. 153). In this case the angular speed for the two was kept at 1 and -1.

The triangle

By changing the radius of the epicycle to a smaller value than the length of the deferent, and keeping the speeds of the two to 1 and -3 one can approximate a triangular orbit. The triangular orbits appear as Figure 6 and 7 of in Hanson’s paper (1960).

The square

Just a little change in the settings gives us a square, Figure 8. If we change h to -3, the resultant orbit that we get is that of a square. The square orbit appears in Figure 8 of Hanson’s paper (1960).

Non-integer speeds

Experimenting with the speeds and the lengths in the model generates very beautiful patterns. If we keep one of the speeds as a non-integer, we generate beautiful spirographs. For example in case of Figure 5, if we change $a = 1.1$ instead of 1, we generate the following beautiful spirograph Figure 9. The planet comes back to its starting point, since the orbits resulting from such a scheme are closed. Depending on the angular speed of this may happen in one revolution as in Figure 5 or it can take many cycles as in Figure 9.

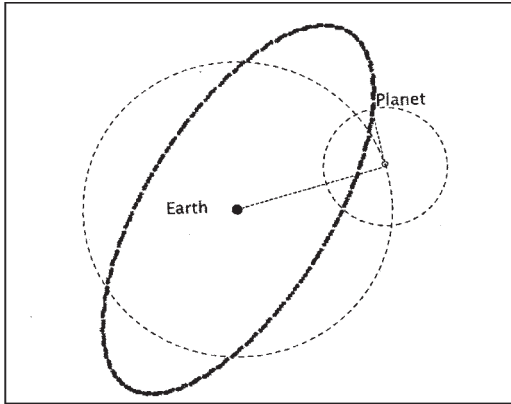


Figure 5: The resultant orbit of an ellipse. The settings are $a = 1$, $h = -1$, *Epicycle* = 4, *Deferent* = 10

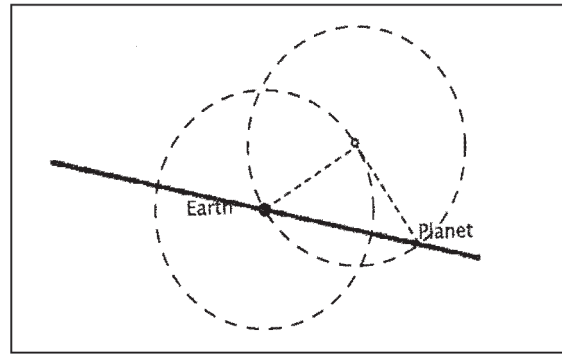


Figure 6: A 'rectilinear' orbit. The radius of the epicycle and length of the deferent are the same. Here $a = 1$, $h = -1$, *Epicycle* = 4, *Deferent* = 4

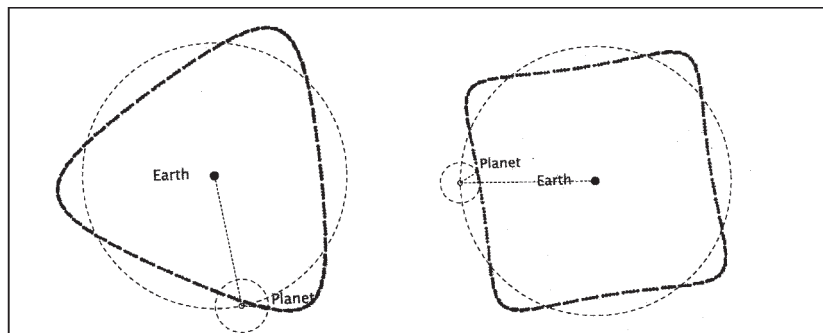


Figure 7: A 'triangular' orbit. The radius of the epicycle and length of the deferent are the same. Here $a = 1$, $h = -2$, *Epicycle* = 2, *Deferent* = 10

Figure 8: A 'square' orbit. The radius of the epicycle and length of the deferent are the same. Here $a = 1$, $h = -3$, *Epicycle* = 1.5, *Deferent* = 10

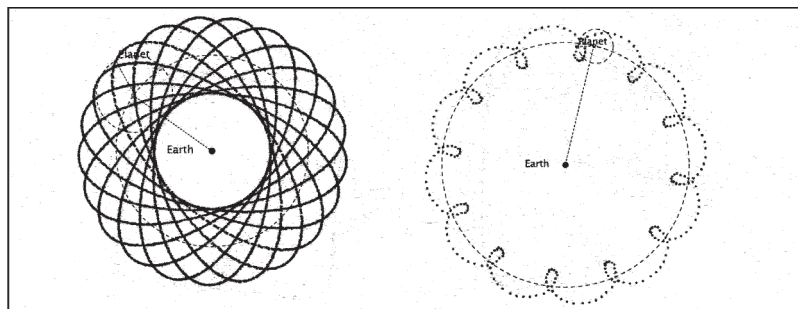


Figure 9: A modified version of Figure 5. The radius of the epicycle and length of the deferent are the same. Here $a = 1$, $h = 1$, *Epicycle* = 4, *Deferent* = 10

Figure 10: Explaining the retrograde motion of planets. Here $a = 1$, $h = 13$, *Epicycle* = 1.4, *Deferent* = 10

EXPLAINING THE RETROGRADE MOTION

The epicycles and the eccentrics were introduced to explain the retrograde motion of the planets. We now present some of the screen-shots for explaining the retrograde motion of the planets. Figure 10 shows the motion of a planet, which can be seen as a retrograde motion by an observer on Earth.

The perspective of the entire system which our model offers is one in which the observer is very far away from the system, and is seeing from the 'top'. For an observer on Earth only the projection of the epicycloidal path will be visible, which explains the retrograde motion.

SOME IMPLICATIONS

The idea of a body performing a circular motion at an uniform speed is common in elementary physics. When two such uniform circular motions are combined one gets some counter-intuitive results. Even then, expecting curves like the ellipse and the epicycloid from the combination of two uniform circular motion, is intuitive. What is further intriguing and counter-intuitive in case of epicycles is that apart from the figures which are of the curved type like the ellipse, epicycloids, some regular polygons like the triangle and the square are also generated in the process. We think this is a very important result. Apart from the elementary astronomy course where we used this model, it can also form good material in an advanced mechanics course.

FURTHER

One of the constraints of using the current version of GeoGebra [we have used version 3.2.0.0 for developing the epicycles] is that it is 2-D. Although the work for 3-D version is under progress. Once the 3-D version is available we can probably work out the details of having the planet move on a cube, pyramid or an ovoid as mentioned by Hanson in his paper (1960). In the 3-D version the actual observed paths of the planets as seen in the night sky can be replicated.

CONCLUSION

The exploration of theoretical models using dynamic software like GeoGebra help in visualizing highly abstract and difficult

to imagine constructions. This kind of use of software tools for education has been widely researched and developed by constructionist studio based education.

In my vision, the child programmes the computer and, in doing so, both acquires a sense of mastery over a piece of most modern and powerful technology and establishes an intimate contact with some of the deepest ideas from science, mathematics, and from the art of intellectual model building (Papert, 1980, p. 5, emphasis in original).

We presented an elaborate case from history of science to demonstrate the efficacy of dynamic visualization software. Such software, unlike computer based tutorials, help the learners construct and explore the domain on their own to achieve greater understanding of difficult subjects.

NOTES

¹ <http://www.geogebra.org>

² <http://www.stellarium.org>

REFERENCES

- Gilbert, J.K. (Ed.). (2005). *Visualization in science education*, Vol. 1. Netherlands: Springer.
- Hanson, N.R. (1960, June). The mathematical power of epicyclical astronomy. *Isis*, 51(2), 150-158.
- Hesse, M. (1966). *Models and analogies in science*. University of Notre Dame Press.
- Hogben, L. (1938). *Science for the citizen*. New York, NY: Alfred Knopf.
- Hoskin, M. (2003). *The history of astronomy - a very short introduction*. Oxford: Oxford University Press.
- Lawrence, J.D. (1976). *A catalog of plane curves*. New York, NY: Dover.
- Papert, S. (1980). *Mindstorms*. New York, NY: Basic Books.
- Rogers, E.M. (1960). *Physics for the inquiring mind*. Princeton, NJ: Princeton University Press.
- Toulmin, S., & Goodfield, J. (1961). *The fabric of the heavens*. New York, NY: Harper and Brothers.