Mathematics
Student Handbook

Geometric Reasoning
Proportional Reasoning

CLI

CONNECTED LEARNING INITIATIVE

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The **Connected Learning Initiative (CLIx)** is a technology enabled initiative at scale for high school students. The initiative was seeded by Tata Trusts, Mumbai and is led by Tata Institute of Social Sciences, Mumbai and Massachusetts Institute of Technology, Cambridge, MA USA. CLIx offers a scalable and sustainable model of open education, to meet the educational needs of students and teachers. The initiative has won UNESCO's prestigious 2017 King Hamad Bin Isa Al-Khalifa Prize, for the Use of Information and Communication Technology (ICT) in the field of Education.

CLIx incorporates thoughtful pedagogical design and leverages contemporary technology and online capabilities. Resources for students are in the areas of Mathematics, Sciences, Communicative English and Digital Literacy, designed to be interactive, foster collaboration and integrate values and 21st century skills. These are being offered to students of government secondary schools in Chhattisgarh, Mizoram, Rajasthan and Telangana in their regional languages and also released as Open Educational Resources (OERs).

Teacher Professional Development is available through professional communities of practice and the blended Post Graduate Certificate in Reflective Teaching with ICT. Through research and collaborations, CLIx seeks to nurture a vibrant ecosystem of partnerships and innovation to improve schooling for underserved communities.

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Any questions, suggestions or queries may be sent to us at:
contact@clix.tiss.edu

Centre for Education, Innovation and Action Research
Tata Institute of Social Sciences
V.N.Purav Marg, Deonar,
Mumbai – 400088, India
Phone: +91 – 22- 25525002/3/4
www.clix.tiss.edu

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Student Handbook

Geometric Reasoning

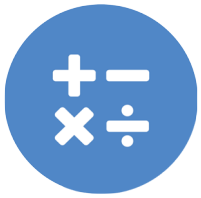
Proportional Reasoning

This handbook belongs to:

Name: .....

Class: .....

Section: .....



CLIx subject team
Amit Dhakulkar
Arati Bapat
Arindam Bose
Bindu Thirumalai
Jayashree Subramanian
Jeenath Rahaman
Ruchi S. Kumar
Saurabh Khanna
Saurabh Thakur
Sayali Chougale
Suchismita Srinivas
Sumegh Paltiwale
Vijay Wani
Shweta Naik (Consultant)

Academic mentor
Aaloka Kanhare
K. Subramaniam

Academic support
Archana Correa
Arnab Kumar Ray
Jaya Mahale
Jayashree Anand
Samir Dhurde
Shikha Takker
Tuba Khan

Editors
Arindam Bose
Bindu Thirumalai
Ruchi S. Kumar
Suchismita Srinivas

Copy editors
Aparna Tulpule
Venkatnarayanan Ganapathi

Translators
Amrit Upadhyay
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Jitender Kumar
Pramod Pathak
Praveen Allamsetti
Ravi Kant

Production team
Dhammaratna Jawale
Jaya Mahale
Jayashree Anand
Sheetal Suresh

Video development support
Gitanjali Somanathan
Manoj Bhandare
Shiva Thorat

Voice over
Arindam Bose
Ruchi S. Kumar
Saurabh Thakur
Suchismita Srinivas

Platform development
Brandon Muramatsu
Cole Shaw
Harshit Agarwal
Jeff Merriman
Kathleen McMahon
Kedar Aitawdekar
Keerthi K.R.D
Kirky DeLong
Mrunal Nachankar
Nagarjuna G.
Padmini Sampath
Prachi Bhatia
Rachana Katkam
Ramjee Swaminathan
Sadaqat Mulla
Satej Shende
Sumegh Paltiwale
Saurabh Bharswadkar

Tool development
Ashwin Nagappa
Kedar Aitawdekar
Mrunal Nachankar
Prachi Bhatia
Rachana Katkam
Sadaqat Mulla
Saurabh Bharswadkar
Tanvi Domadia
Tejas Shah

Platform design
Aditya Dipankar

Platform content authoring
Ashirwad Wakade
Rajiv Sambari
Roshan Gajbhiye
Saurabh Thakur
Sumegh Paltiwale
Vijay Wani

Publication team
Rachna Ramesh Kumar
Sunita Badrinarayan
Usha Iyengar

Cover design and formatting
Ramesh Khade

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To the students and teachers of all the schools where we piloted our modules.
To all the teachers, copyeditors and Rajasthan and Telangana team for their time and effort in the revalidation of content

Geometric Reasoning

Table of Contents

Note: * Lessons without hands-on activities are not included in this workbook

Module Overview

About Geometric Reasoning Module

Geometric Reasoning Module (Parts I and II) has been designed to develop the reasoning abilities of 8^{th} and 9^{th} class students. There are a total of 5 units in the Geometric Reasoning module which will help students develop an understanding of (2D) shapes, and the ability to analyze, discuss and argue confidently about shapes in terms of their attributes and properties. The module will also help students develop definitions of various special quadrilaterals on the basis of their properties. Moving forward from Geometric Reasoning Part I, the Part II will help students develop a relational understanding of the different types of special quadrilaterals, and also to understand the need for proofs in Mathematics. A digital game named PoliceQuad is designed to help students think deeper about the properties of Geometric shapes while playing the game. LOGO Turtle and Geogebra are free and open source software which give students visual experiences of verifying many concepts, properties and theorems in Geometry.

The modules have digital as well as hands-on activities and formative assessments form an integral part of this course. There are Pre and Post assessments at the start and end of each module on the digital platform. The course is prepared by conforming with the current State and NCERT syllabi and the activities designed in this course focus on developing thinking and reasoning abilities of the students. The Mathematics modules will be installed in the school computer labs using a server based model.

How to use this book ?

This book contains some of the activities of the module that are hands-on (involving classroom discussions) which are to be used in conjunction with other materials that are present on the CLIx platform. These hands-on activities and worksheets help to elicit and consolidate learning of Geometrical shapes and should be done in a proper sequence along with the digital activities. Students may do the problems in the space provided in this workbook or in their notebooks and discuss with their teachers and peers.

The CLIx platform is a digital platform that makes use of both the digital content and the workbook content. The platform has features like Notebook, Discussion and Gallery where students can give their responses, comments and upload their work respectively.

Geometric Reasoning
Part I

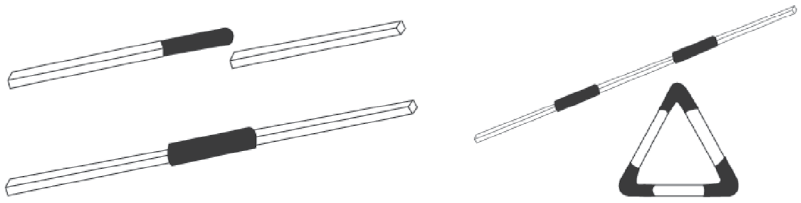
Unit 1: Concept of Shape

Lesson 1.1: What is shape?

Activity 1: Matchstick shapes

Work in your group. You will need a set if (used) matchsticks and cycle valve tubes.

Task 1: Make a triangle, a square and pentagon using the matchsticks and valve tubes.



Task 2: Perform these actions (mentioned in the table) on the square. Discuss in your group and note whether the shape changes or not.

	Action	Does the shape change?	Why do you think so ?
1	Sliding the shape on the floor/desk		
2	Rotating the shape on the floor/desk		
3	Flipping the shape on the floor/desk		
4	Pressing on the opposite vertices of the shape		

Task 3: Try pressing on the opposite vertices of the pentagon. Does it change shape ? Now try doing the same with the triangle. Does it change shape?

Extension Task 1: Try making as many different shapes as possible by deforming/twisting the pentagon (without breaking it or opening up the joints). In particular, try to make
a triangle that has exactly two sides equal

- a four sided polygon
- a star shape

Extension Task 2: Try making a triangle in which all three sides are of different length. What would be the minimum number of matchsticks needed for this?

Unit 2: Analysing and Describing Shapes

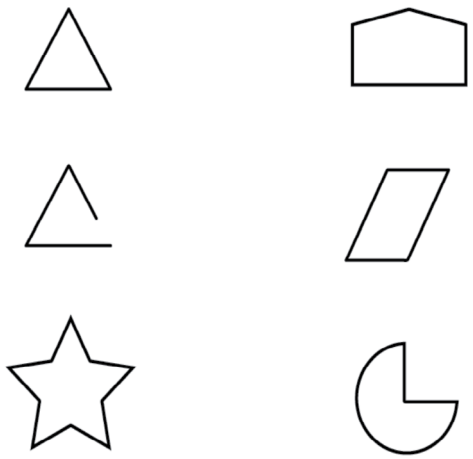
Lesson 2.1: Analysing shapes

In this lesson, students play the digital game **"Police Quad"** - Mission 1 on the platform. This Mission of the game helps students in developing understanding of shapes through property-based reasoning tasks.

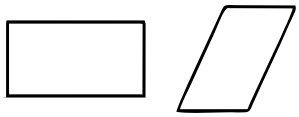
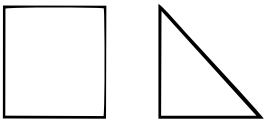
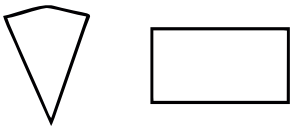
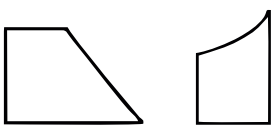
Activity 1: Sorting shapes

Work individually on the following tasks and then discuss with your group.

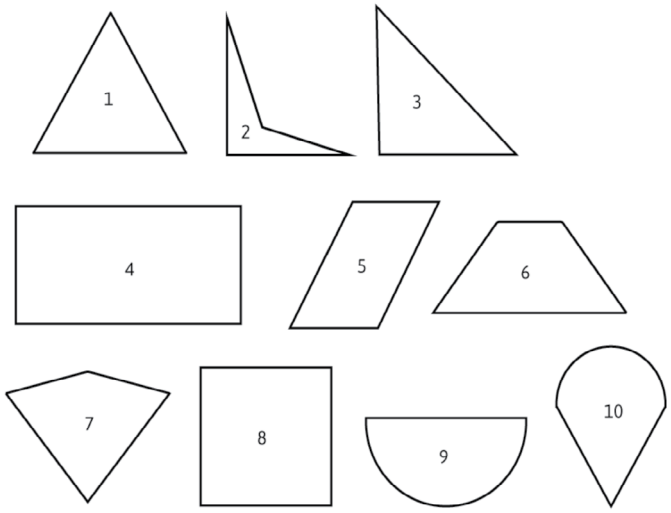
Task 1: Write 1-2 lines about (or properties of) each of the following shapes



Task 2: Observe the two shapes given in each of the following sets. List as many similarities and differences that you can between the two. One example is given for the first set.

Set	Similarities	Differences
	1) Both have exactly 4 sides 2) Both have 2 pairs of parallel sides	1) The first shape has right angles, the second does not
		
		
		

Task 3: Look at the collection of shapes below. Based on their properties, sort them into two groups in as many different ways as you can. An example is shown in the table.



Property	These shapes have it	These shapes don't have it
Has exactly 4 straight sides	2, 4, 5, 6, 7, 8	1, 3, 9, 10

Extension Task 1: Draw 3 different shapes that have the following property 'all sides equal'. 'All sides equal' is one way which the shapes that you got are similar. What is one difference between them?

Extension Task 2:
Draw a shape that has both these properties:
i. exactly 5 sides
ii. exactly 2 right angles

Extension Task 3:
Draw a shape that has all these properties:
i. exactly 4 sides
ii. exactly 2 right angles
iii. exactly 1 pair of sides parallel

Lesson 2.2: Describing shapes

*Please refer to this lesson on the **CLIx platform***

In this lesson, students play **"Police Quad"** - Mission 2, which helps students in strengthening property-based understanding of shapes through shape description tasks. The Mission also helps develop strategic thinking.

Unit 3: Classifying and Defining Shapes

Lesson 3.1: Classifying shapes

*Please refer to this lesson on the **CLIx platform***

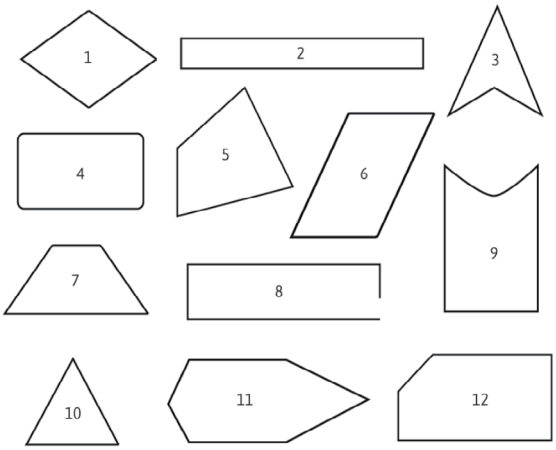
In this lesson, students play **"Police Quad"** - Mission 3. This Mission helps students engage in property-based classification, reasoning and informal deduction tasks.

Lesson 3.2: Defining shapes

Activity 1: What is a quadrilateral

Work individually on the following tasks and then discuss with your group.

Task 1: Look at the collection of shapes and sort them based on their properties into two groups- 'Quadrilaterals' and 'Not quadrilaterals'.



Now, fill in the table below.

These are qauadrilaterals	These are not quadrilaterals

For each shape, discuss why you think it is a quadrilateral, or not. Now complete the following: I think "a quadrilateral is

Lesson 3.3 : Defining special quadrilaterals

In this lesson, students use **"Turtle Logo"** to construct, identify and define special quadrilaterals.

Activity 1: Constructing rectangles

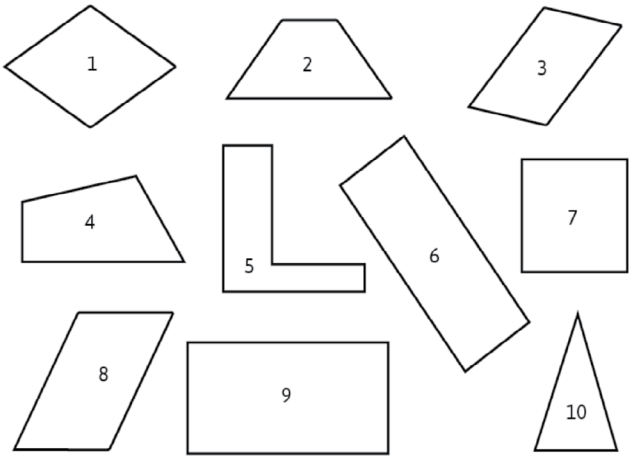
*Please refer to this activity on the **CLIx platform***

Activity 2: Exploring special quadrilaterals

Work individually on the following tasks and then discuss with your group.

Task 1: Look at the collection of shapes and sort them into those that are parallelograms and those that are not. Fill in the table.

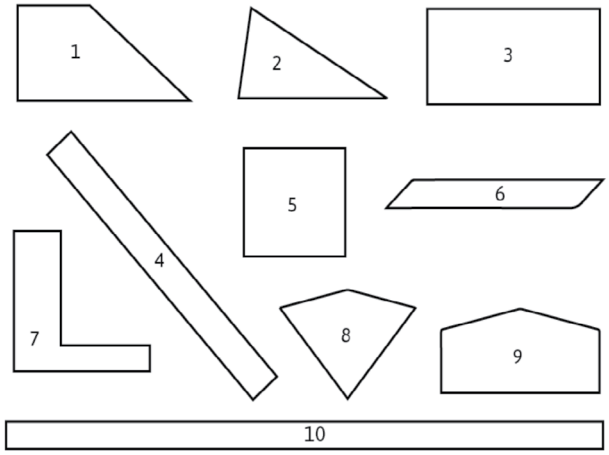
These are parallelograms	These are not parallelograms



For the shapes which are not parallelograms, explain why. Based on this, write your definition of a parallelogram. I think "a parallelogram is

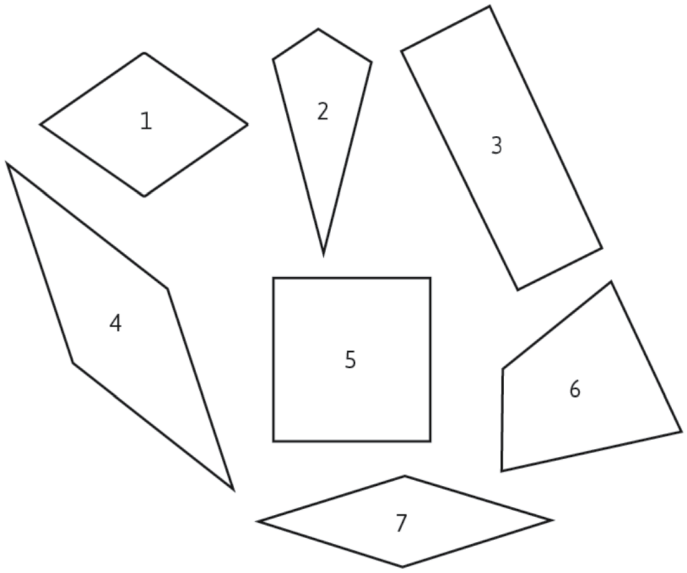
Task 2: Look at the collection of shapes and sort them into those that are rectangles and those that are not. Fill in the table

These are rectangles	These are not rectangles



Based on this, write your definition of a rectangle.
I think "a rectangle is

Task 3: Look at the collection of shapes and sort them into those that are rhombuses and those that are not.



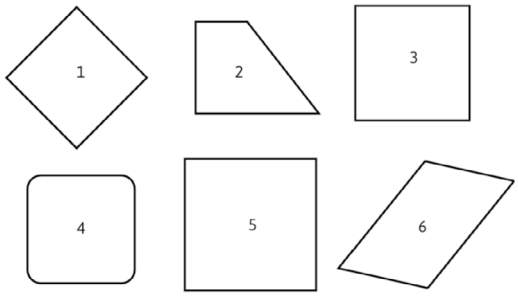
Now, fill in the table below.

These are rhombuses	These are not rhombuses

Based on this, write your definition of a rhombus
I think "a rhombus is

Task 4: Look at the collection of shapes and sort them into those that are squares and those that are not. Fill in the table

These are squares	These are not squares



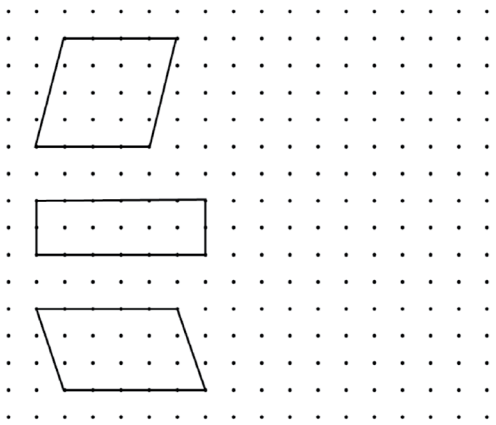
Based on this, write your definition of a square.

Lesson 3.4: Properties of special quadrilaterals

Activity 1: Making property lists

Work individually on the following tasks and then discuss with your group.

Task 1a: Shown here are some examples of parallelograms. Draw two more examples of parallelograms on the dot paper. (Make sure that your parallelograms are different from the ones already given)

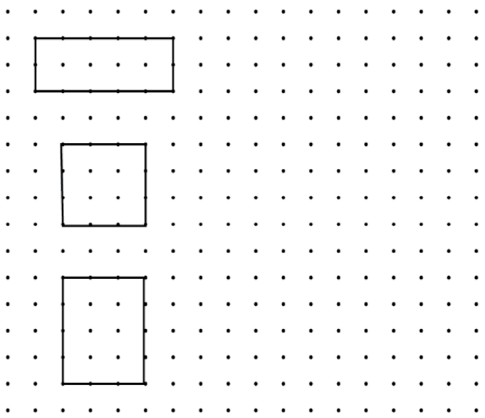


Task 1b: Write down the ways in which these parallelograms are different from each other.

Task 2: Observe the parallelograms in Task 1 and make a list of as many properties as you can. Remeber, the properties should be common to ALL the examples!

Properties of a Parallelogram		
Side Properties	**Angle Properties**	**Diagonal Properties**

Task 3a: Shown here are some examples of rectangles. Draw 2 more examples of rectangles on dot paper. (Make sure that your rectangles are different from the ones already given!)



Task 3b: Write down the ways in which these rectangles are different from each other.

_____

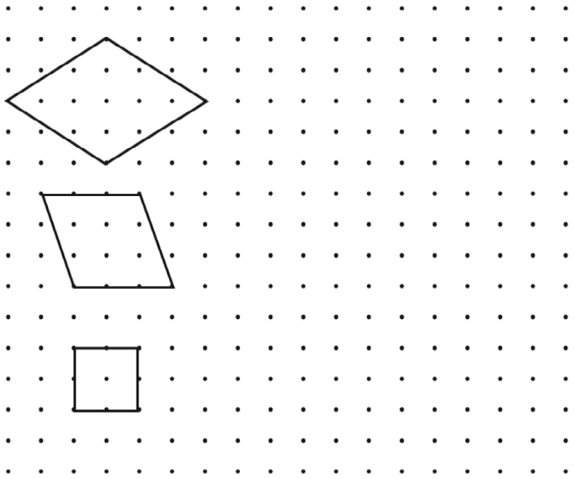
_____

_____

Task 4: Observe the rectangles in Task 3 and make a list of as many properties as you can. Remember, the properties should be common to ALL the examples.

Properties of a Rectangle		
Side Properties	**Angle Properties**	**Diagonal Properties**

Task 5a: Shown here are some examples of rhombuses. Draw 2 more examples of rhombuses on dot paper. (Make sure that your rhombuses are different from the ones already given!)



Task 5b: Write down the ways in which these rhombuses are different from each other.

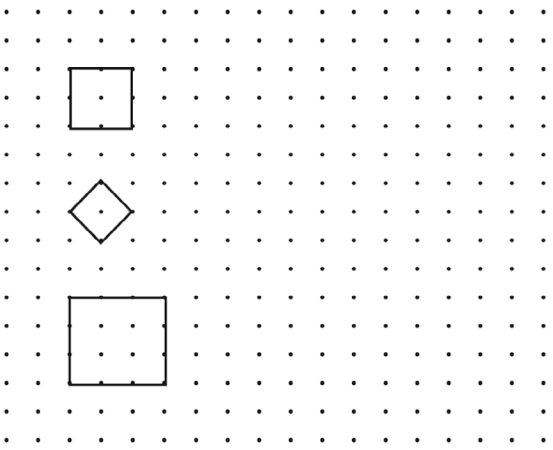
_____

_____

Task 6: Observe the rhombuses in Task 5 and make a list of as many properties as you can. Remember, the properties should be common to ALL the examples!

Properties of a Rhombus		
Side Properties	**Angle Properties**	**Diagonal Properties**

Task 7a: Shown here are some examples of squares. Draw 2 more examples of squares on dot paper. (Make sure that your squares are different from the ones already given!)



Task 7b: Write down the ways in which these squares are different from each other.

_____

_____

Task 8: Observe the squares in Task 7 and make a list of as many properties as you can. Remember, the properties should be common to ALL the examples!

Properties of a Square		
Side Properties	**Angle Properties**	**Diagonal Properties**

Geometric Reasoning
Part II

Unit 1: Property-Based Reasoning

Lesson 1.1: Relationships among special quadrilaterals

Activity 1: Representing relationships 1

*Please refer to this activity on the **CLIx platform***

In this activity, students play **"Police Quad"** - Mission 4. This Mission of the game is meant to initiate discussion on hierarchical class-relationships among parallelograms, rhombuses, rectangles and squares.

Activity 2: Creating property stacks

Work individually on the following tasks and then discuss with your group.

Task 1: Study the table of properties below. Put a tick (√) mark in a cell if the corresponding shape has the given property. Leave the cell blank if it doesn't.

The first row is done for you.

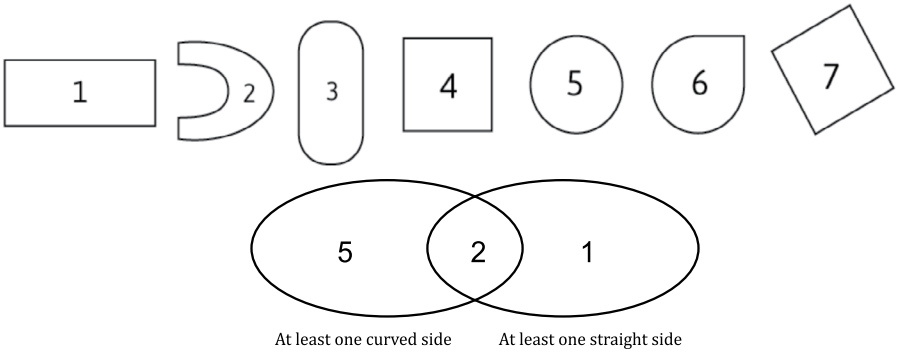
	Properties	Square	Rectangle	Parallelogram	Rhombus
1	Closed figure made of 4 line segments only	✓	✓	✓	✓
2	Pairs of opposite sides equal in length				
3	Pairs of opposite sides are parallel				
4	Opposite angles are congruent				
5	Diagonals bisect each other				
6	All angles are right angles / are equal				
7	Adjacent angles are equal				
8	Diagonals are equal in length				
9	Adjacent sides are equal in length				
10	All 4 sides are equal in length				
11	Diagonals are perpendicular to each other				

Lesson 1.2: Representing relationships

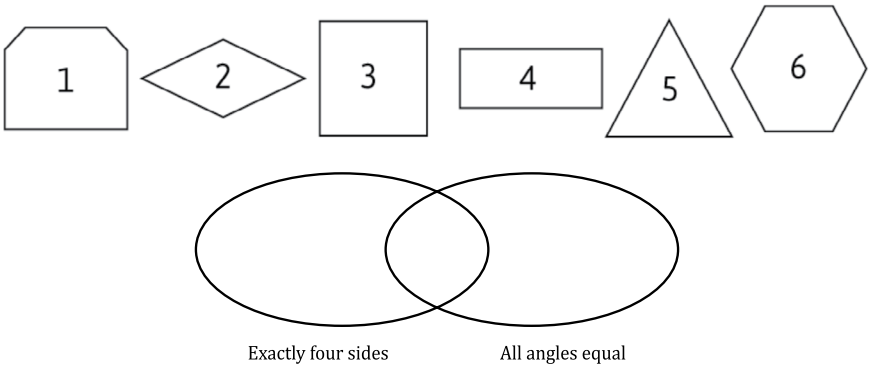
Activity 1: Representing relationships 1

Work individually on the following tasks and then discuss with your group.

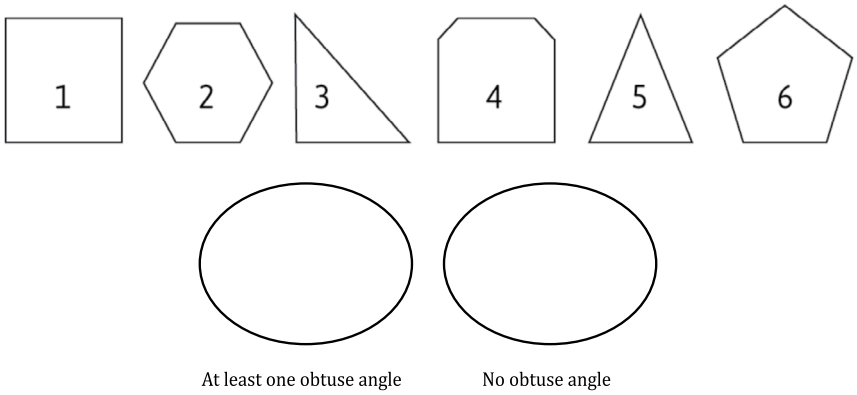
Task 1: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram. Some are done for you.



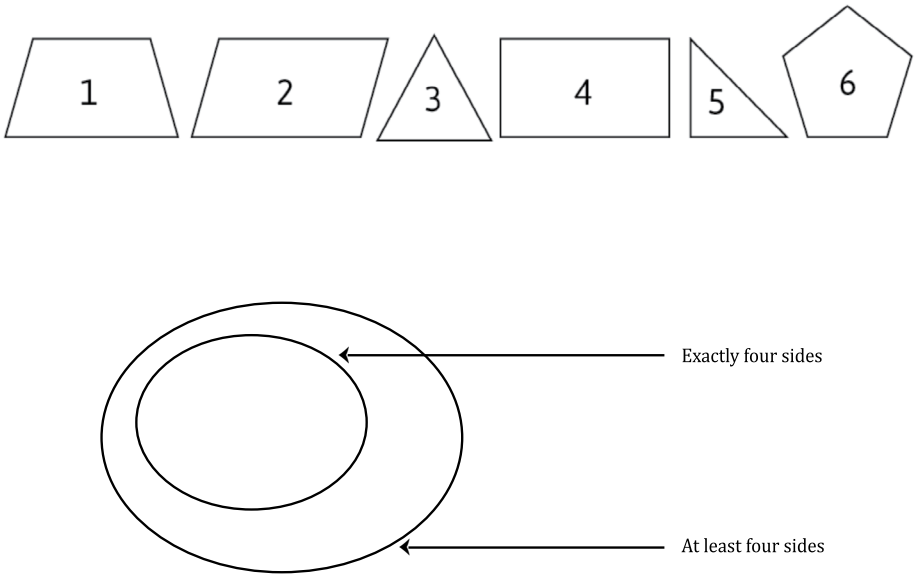
Task 2: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram.



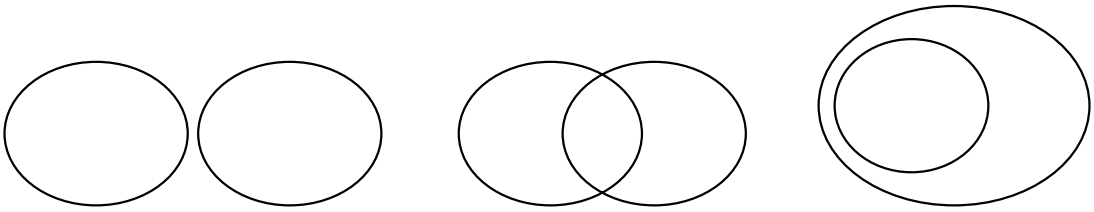
Task 3: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram.



Task 4: Write the numbers corresponding to each of the shapes below in the appropriate place in the given Venn diagram.



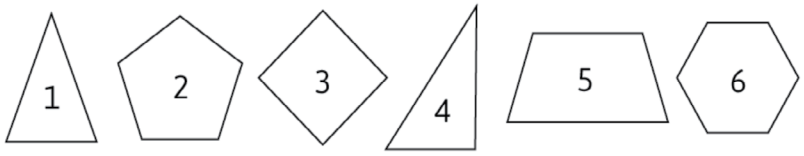
For Tasks 5 to 8, consider the three types of Venn diagrams Type 1, Type 2 or Type 3 shown below.



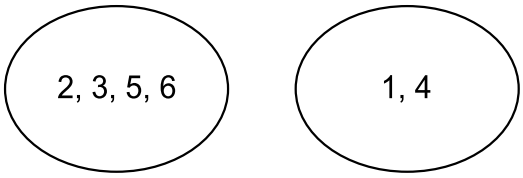
Type 1 Type 2 Type 3

Task 5: Consider two groups of shapes:
- Group 1: has at least 4 straight sides
- Group 2: has less than 4 straight sides

a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
b. Now classify these shapes into the Venn diagram you selected. (Write the numbers in the appropriate place. Some are done for you)



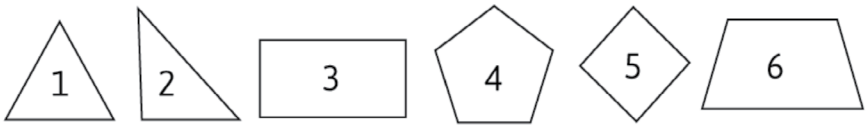
(This task is done as an example for you.)

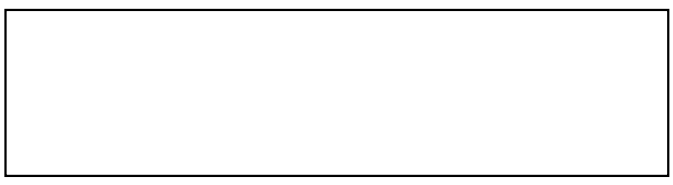


Atleast four straight sides Less than four straight sides

Task 6: Consider two groups of shapes:
- Group 1: has all sides equal
- Group 2: has at least 1 right angle

a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
b. Now classify these shapes into the Venn diagram you selected. (Write the numbers in the appropriate place.)

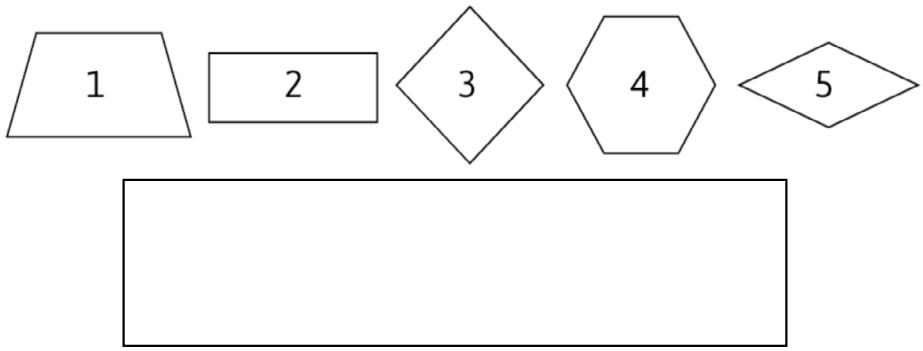




Task 7: Consider two groups of shapes:

- Group 1: has at least one pair of opposite sides parallel
- Group 2: has no sides parallel

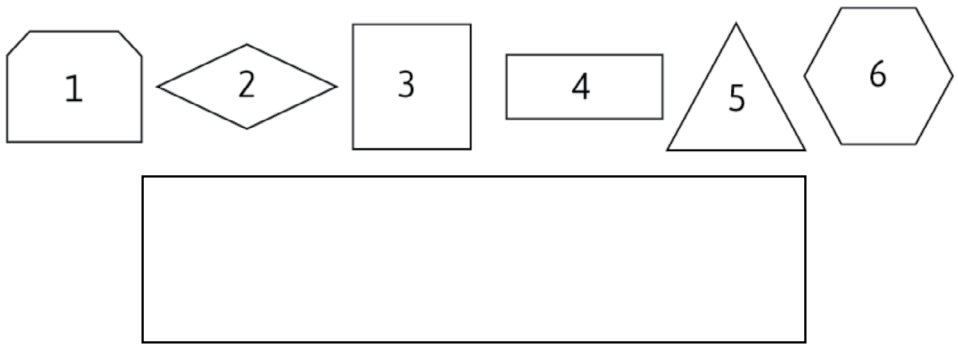
a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
b. Now classify these shapes into the Venn diagram you selected. (Write the numbers in the appropriate place.)



Task 8: Consider the property set:

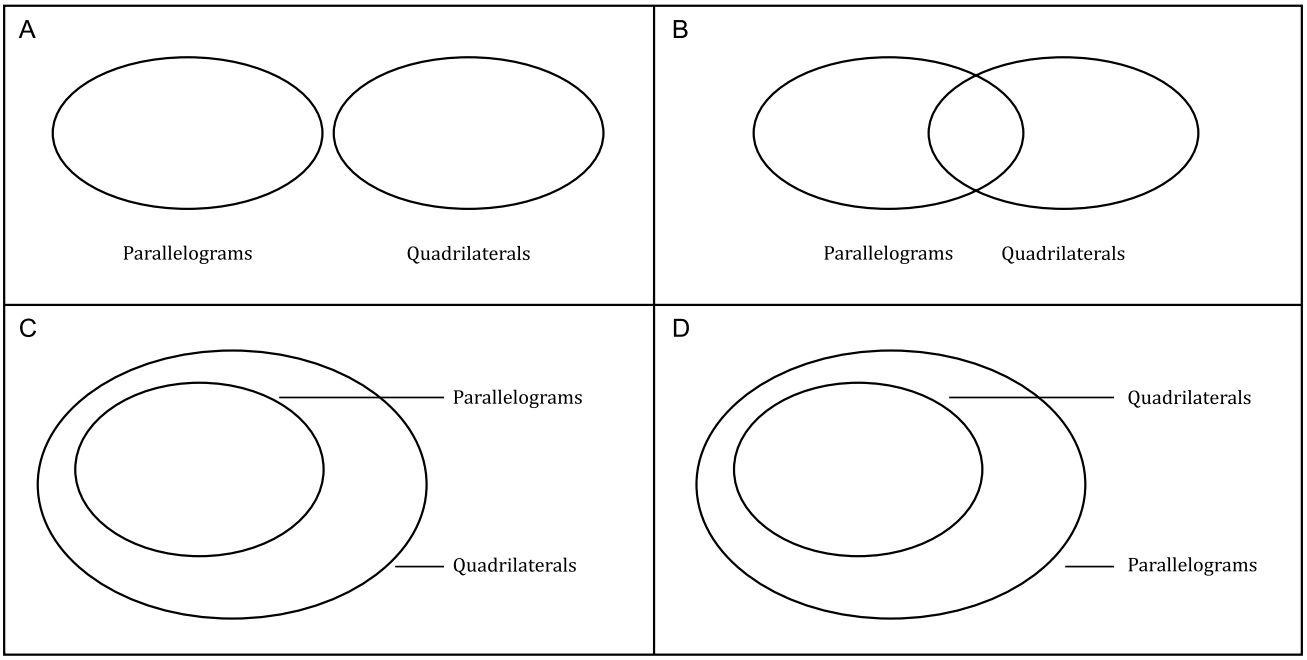
- at least 3 straight sides
- exactly 4 straight sides

a. Which type of Venn (Type 1, 2 or 3) would be the most appropriate to classify a group of shapes by these two properties? Justify your choice. Draw it in the space provided below.
b. Now classify these shapes into the Venn diagram you selected, by the given properties.

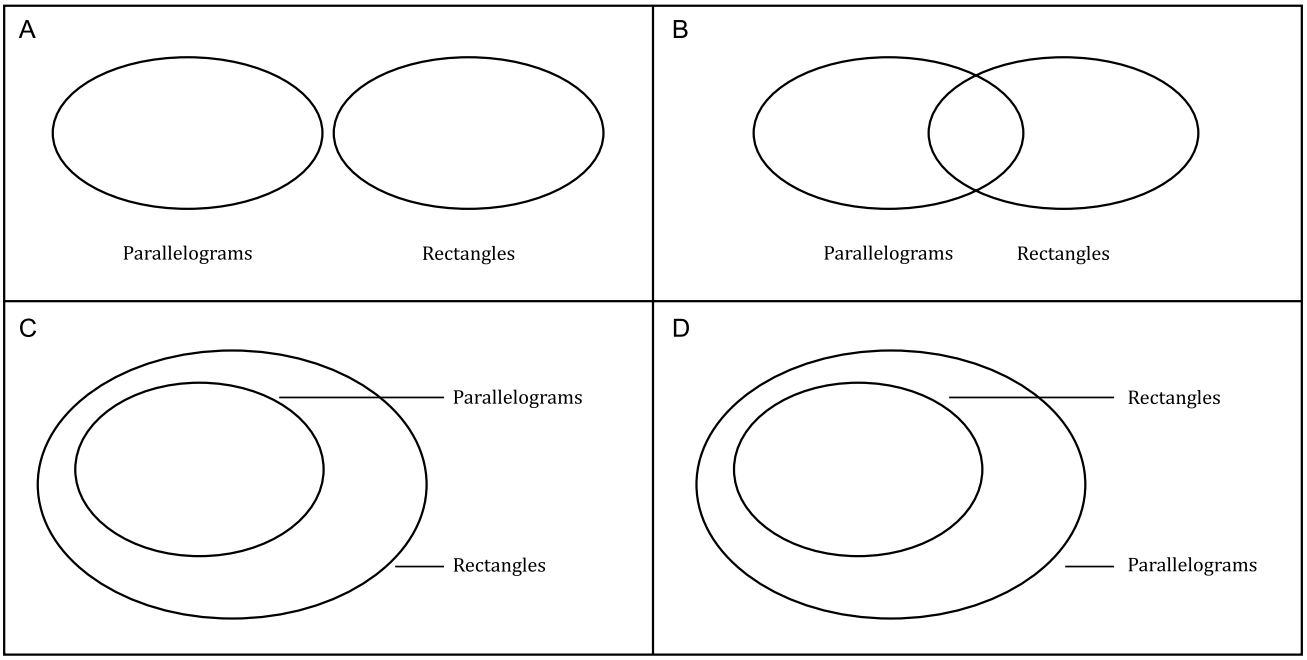


Activity 2: Representing relationships 2

Task 1: Which of these is the correct representation of the set of parallelograms and quadrilaterals, and why? (You may want to use the definition of quadrilaterals and parallelograms and their properties to see how they are related.)



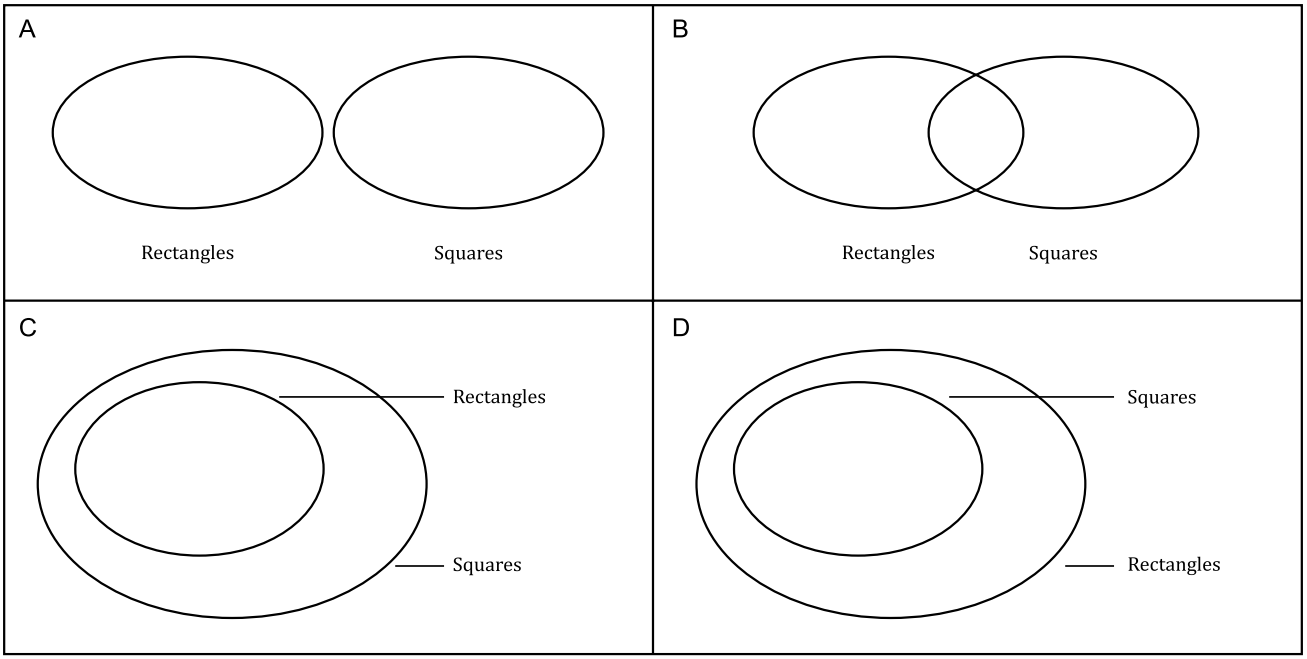
Task 2: Which of these is the correct representation of the set of parallelograms and rectangles, and why? (You may want to use the definition of parallelograms and rectangles and their properties to see how they are related.)



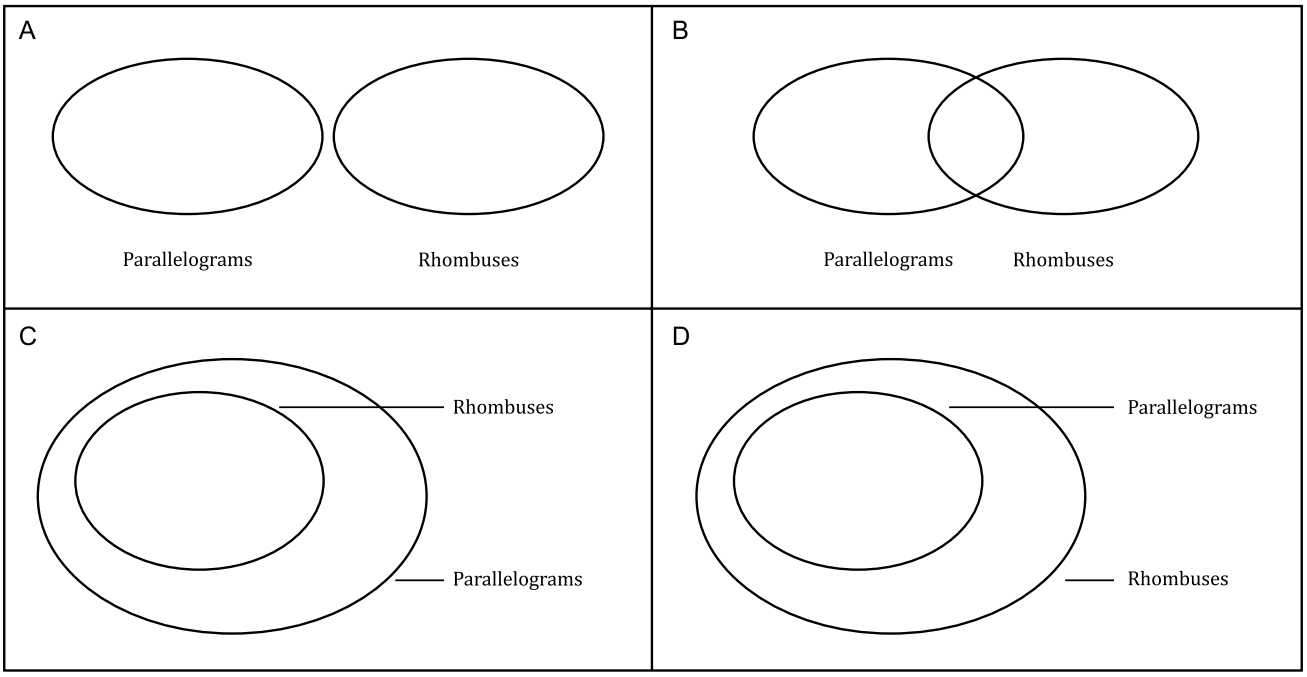
Task 3: Based on task 2, which of the following statements is correct? And why?

a. All parallelograms are rectangles
b. All rectangles are parallelograms
c. Some rectangles are parallelograms (and some are not)
d. No parallelogram is a rectangle

Task 4: Which of these is the correct representation of the set of rectangles and squares, and why? (You may want to use the definition of quadrilaterals and parallelograms and their properties to see how they are related)



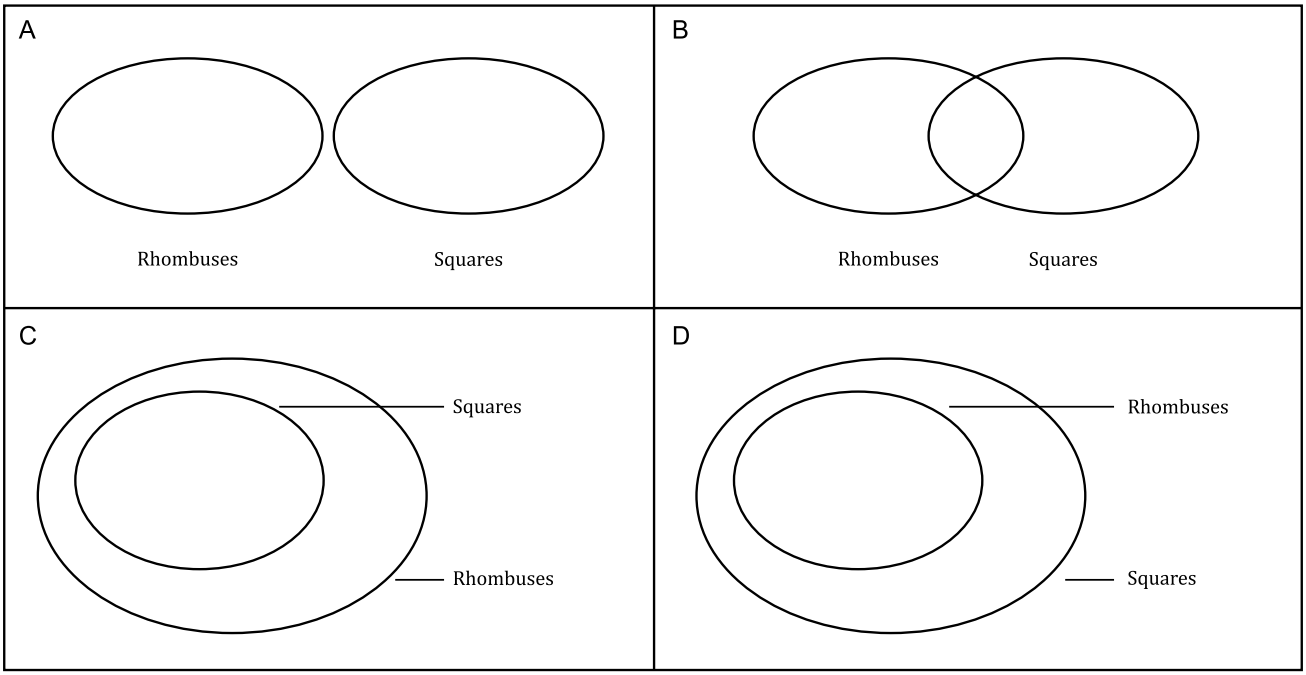
Task 5: Which of these is the correct representation of the set of parallelograms and rhombuses, and why? (You may want to use the definition of parallelograms and rhombuses and their properties to see how they are related.)



Task 6: Based on task 4, which of the following statements is correct? And why?

a. All parallelograms are rhombuses
b. All rhombuses are parallelograms
c. Some rhombuses are parallelograms (and some are not)
d. No parallelogram is a rhombus

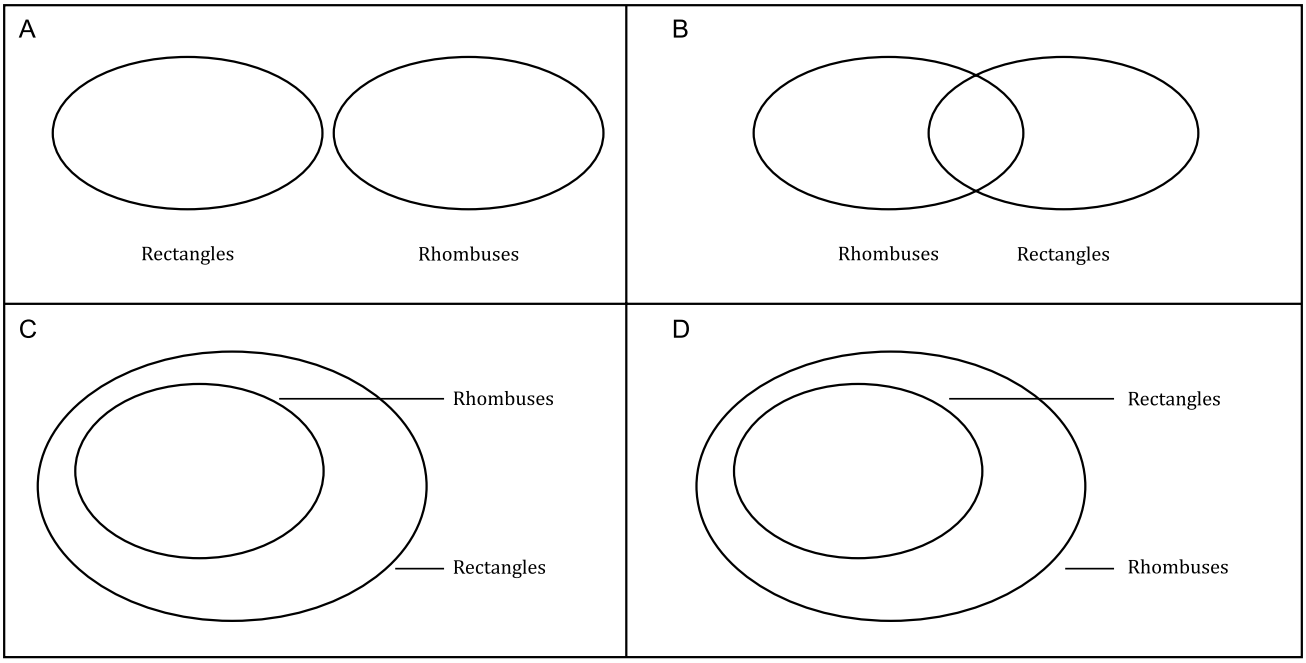
Task 7: Which of these is the correct representation of the set of rhombuses and squares, and why? (You may want to use the definition of rhombuses and squares and their properties to see how they are related.)



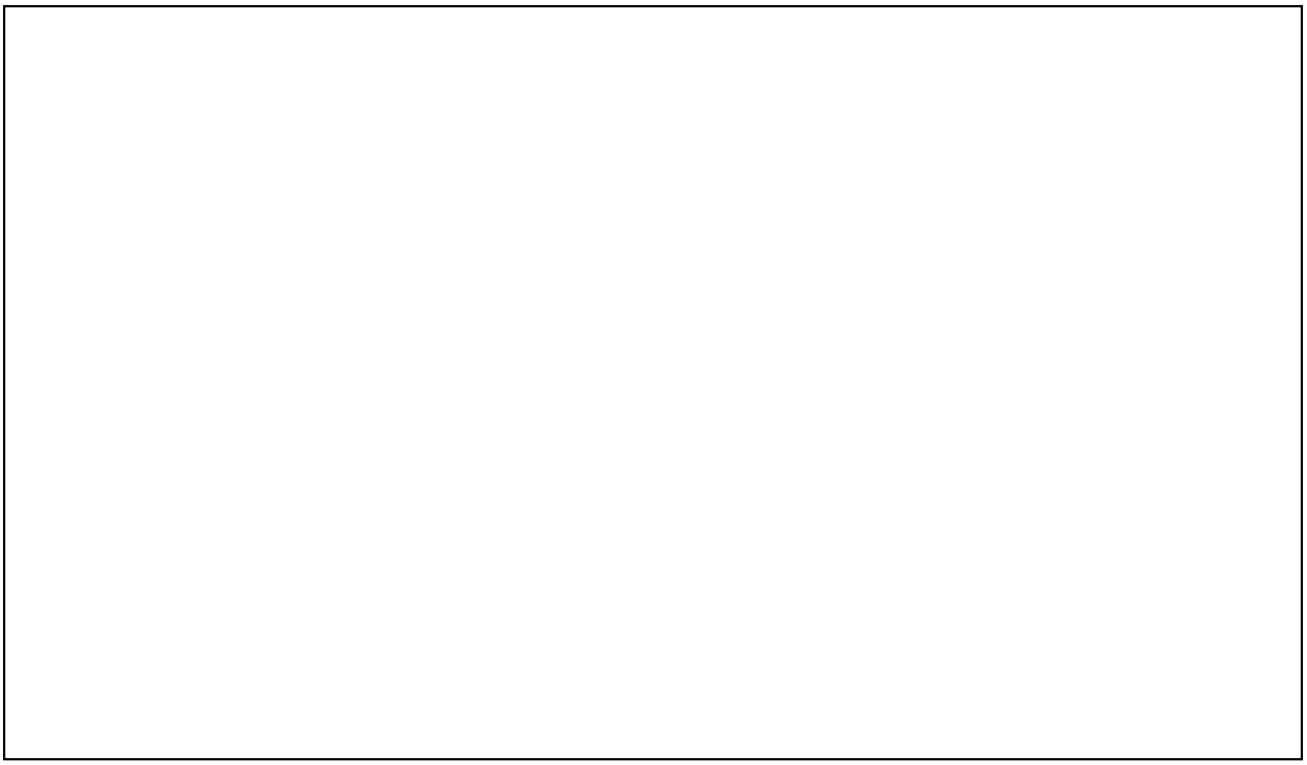
Based on this choose the correct word that completes the sentence.

i. _____ rhombuses are squares. (All/Some/No)
ii. _____ squares are rhombuses. (All/Some/No)

Task 8: Which of these is the correct representation of the set of parallelograms and rhombuses, and why? (You may want to use the definition of parallelograms and rhombuses and their properties to see how they are related.)



Task 9: Using the representation chosen in the above tasks, represent quadrilaterals, parallelograms, rhombuses , rectangles and squares in one diagram. Represent quadrilaterals using a rectangle and use appropriate circles to represent the remaining quadrilaterals.



Lesson 1.3: Discussing definitions

*Please refer to this lesson on the **CLIx platform***

In this lesson, students will be discussing definitions of different quadrilaterals and will represent the relationship between quadrilaterals using Venn diagrams.

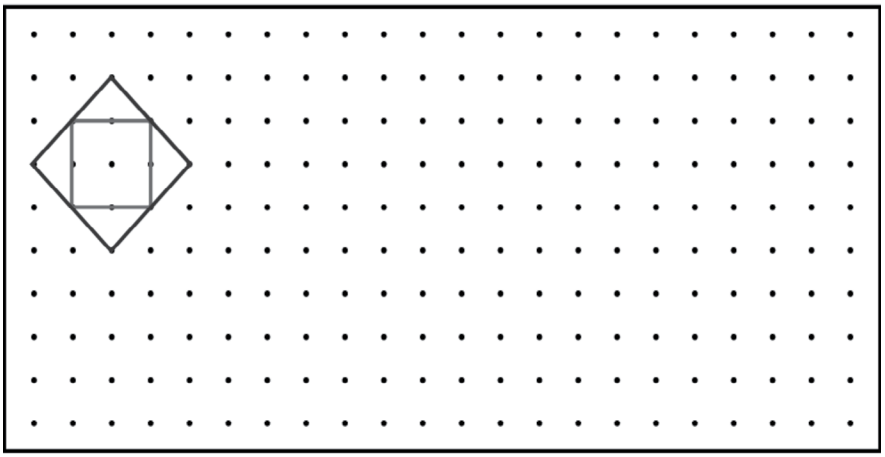
Unit 2: Understanding the Need for Proof

Lesson 2.1: Midpoint explorations

Activity 1: Midpoint explorations

Work individually on the following tasks and then discuss with your group.

Task 1: On the dot paper below, draw different squares. Join the midpoints of the sides of each of these squares (in order) to create a new quadrilateral. The first one is shown as an example.

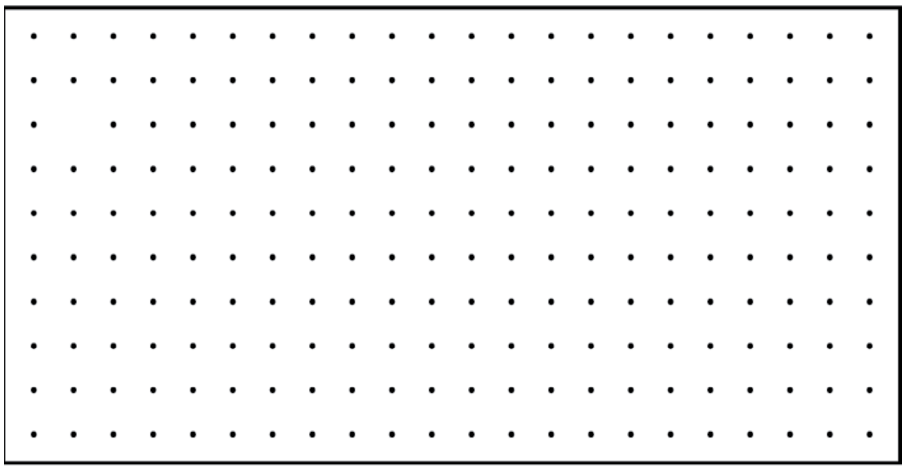


Observe each of the new quadrilaterals formed, and complete the following: The quadrilateral formed by joining the midpoints of sides of a square is a _____

Task 2: Suppose you were to join the midpoints of sides of a rectangle in a similar fashion. What shape do you think you might get? Think about it, and write your conjecture here:

The quadrilateral formed by joining the midpoints of sides of a _____is a_____

Task 3: Now verify your conjecture by drawing different rectangles on the dot paper below and joining the midpoints of the sides.

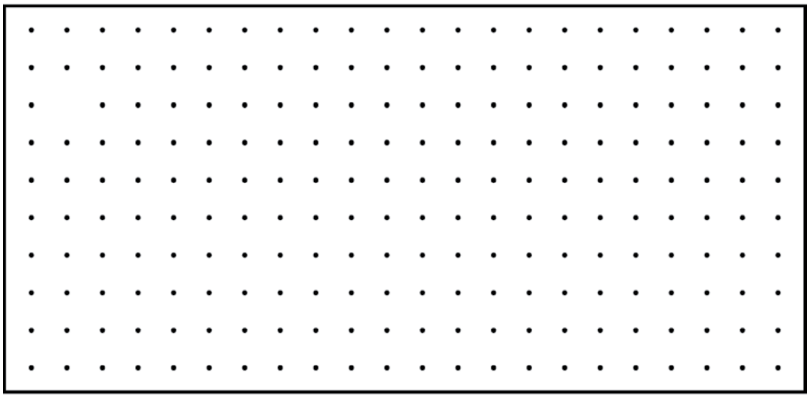


Task 4: Based on Task 3 does your conjecture hold? If not, how would you modify it?

Task 5: Now make similar conjectures about other special quadrilaterals, rhombus and parallelogram and verify them. Write your conjectures in the space provided, and use the dot grid for verifying.

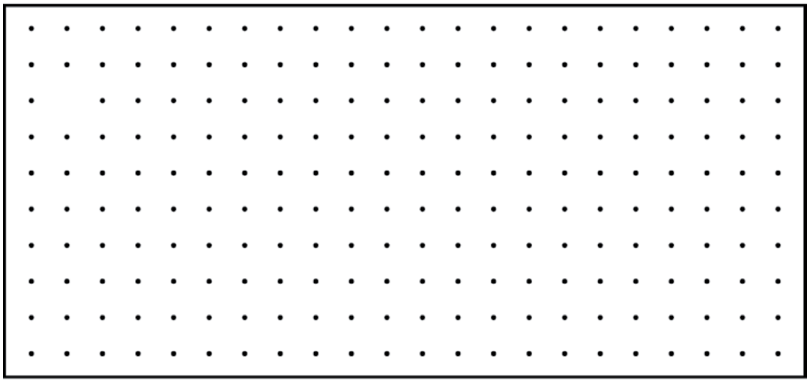
Conjecture 1

Conjecture 2



Task 6: Drawing on your observations in the 5 previous tasks, make a conjecture about the shape formed by joining the midpoints of sides of any quadrilateral.

Points to think about:
Would this be true for ALL quadrilaterals? How do you know? Explain your reasoning here. Use the dot grid below if necessary.



Extension Task 1: if possible, draw a quadrilateral, joining whose midpoints of sides in order gives a figure that is NOT a parallelogram. If not possible, explain why?

Lesson 2.2: Angle sum property

In this lesson, the **"Geogebra"** application is used to help students explore and verify angle sum property of quadrilaterals and also of other polygons.

Activity 1: Angle sum property of quadrilaterals

Work individually on the following tasks and then discuss with your group.

Task 1: Draw a quadrilateral ABCD and measure its (interior) angles. Record in the table below.

∠**A**	∠**B**	∠**C**	∠**D**	**Sum of all (interior) angles**

Task 2: Compare your quadrilateral with those of others in your class.

a. Do they appear the same?
b. Do you observe a pattern/regularity across all quadrilaterals? Write observation in the form of a conjecture:

Task 3**:** Refer to the pattern observed in Task 2.

a. Do you think this pattern will hold true for ALL quadrilaterals ?
b. Why or why not?

Task 4: Draw a quadrilateral and join any one of its diagonals. Without measuring, can you say what the sum of the interior angles of this quadrilateral will be? Write your reasons.

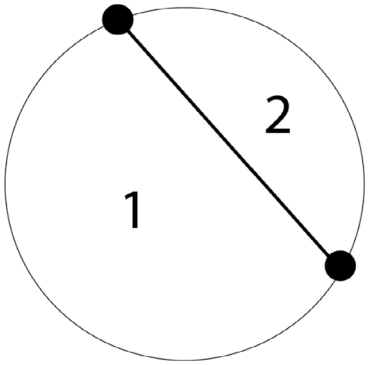
Extension Task 1: Do you think this property (sum of interior angles) will hold true for all parallelograms? Why or why not?

Lesson 2.3: Need for proof

Activity 1: Need for proof

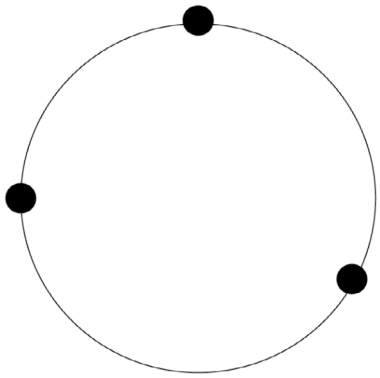
Work individually on the following tasks and then discuss with your group.

Task 1: Mark 2 distinct points on a circle and join them. Note how many separate regions the circle is divided into.



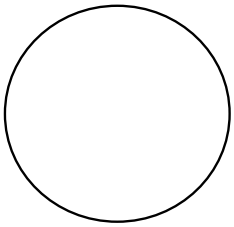
- Number of points on circle : _____
- Number of separate regions : _____

Task 2: Now draw another circle. Mark 3 distinct points on it. Join all possible pairs of points. How many separate regions is the circle divided into?



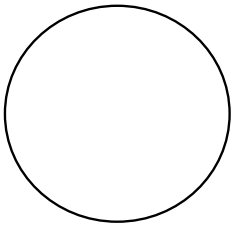
- Number of points on circle : _____
- Number of separate regions : _____

Task 3: Mark 4 distinct points on one circle, join all possible pairs of points. Note the number of (separate) regions the circle is divided into. In the other circle, do the same with 5 distinct points.



For four points on the circle

- Number of points on circle : _____
- Number of separate regions : _____



For five points on the circle

- Number of points on circle : _____
- Number of separate regions : _____

Task 4: Now record your observations from Task 3 in the table below.

Number of points on circle:	**2**	**3**	**4**	**5**
Number of separate regions:				

Task 5: What is the pattern you observe? Write it down. (You could write it as a 'rule' about the relationship between the number of points taken on the circle, and the number of separate regions the circle is divided into.)

_____

_____

Task 6: Do you think your 'rule' will hold true for ANY number of points taken on the circle ? Why or why not?

_____

_____

Task 7: Verify your 'rule' by taking:

i. 1 point on circle
   Number of separate regions: _____
ii. 6 points on circle
   Number of separate regions: _____

Task 8: Does your rule hold true? Based on this, would you like to change your response to Task 7? If yes, put the new response here.

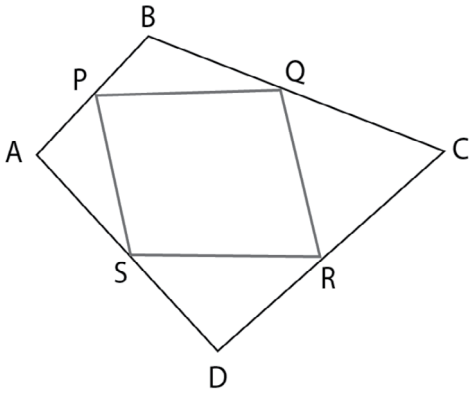
Points to think about:
How many examples do you think are 'enough' to prove a conjecture ?

How many examples do you think are 'enough' to disprove a conjecture ?

Lesson 2.4: Writing a proof

Activity 1: Proving midpoint result for quadrilaterals

Task 1: In the figure, PQRS is formed by joining the midpoints of a quadrilateral ABCD. Prove that PQRS is a parallelogram.



(Hint: **Use the result of the Midpoint Theorem The line joining the midpoints of two sides of a triangle is parallel to the third side and half of it.*)***

Lesson 2.5 : Proving and disproving

Activity 1: True and false statements

Work these out in your group, and then present your solution to the class. Use the space provided for working out your solution.

1. If you double a whole number, you get an even number.

☐ True ☐ False

2. If you add two odd numbers you will get an even number.

☐ True ☐ False

3. If you multiply two odd numbers you get an even number.

☐ True ☐ False

4. If you add 1 to a whole number you get a number less than 1,000,000,000,000,000,000,000.

☐ True ☐ False

5. If a parallelogram has one pair of adjacent sides equal, it is a rectangle.

☐ True ☐ False

6. If a parallelogram has at least one right angle, it is a rectangle.

☐ True ☐ False

7. If a quadrilateral has one pair of opposite sides equal, and the other pair parallel, then it is a parallelogram.

☐ True ☐ False

8. If a quadrilateral has one pair of opposite sides equal, and parallel, then it is a parallelogram.

☐ True ☐ False

Proportional Reasoning

Table of Contents

Note: * Lesson without hands-on activities are not included in this workbook

Module Overview

About Proportional Reasoning Module

The proportional reasoning module is developed to facilitate students' shift from additive to multiplicative reasoning and to be able to appreciate the idea of scaling and recognizing scaling in situations involving the relationship between two or more quantities. The module is made for students of 8^{th} and 9^{th} classes. There are a total of 4 units in the Proportional Reasoning module, unit-1 is about the move from additive to multiplicative reasoning, unit-2 is about the multiplicative thinking, unit-3 is about Ratio and Proportion and unit 4 is about the application of proportional reasoning in different contexts. There are three major digital tools used in this module, food sharing tool, pattern task and ice-cube activity, each with several variations involved in it.

The modules have digital as well as hands-on activities and formative assessments form an integral part of this course. There are Pre and Post assessments at the start and end of each module on the digital platform. The course is prepared by conforming with the current State and NCERT syllabi and the activities designed in this course focus on developing thinking and reasoning abilities of the students. The Mathematics modules will be installed in the school computer labs using a server based model.

How to use this book ?

This book contains some of the activities of the module that are hands-on (involving classroom discussions) which are to be used in conjunction with other materials that are present on the CLIx platform. These hands-on activities and worksheets help to elicit and consolidate learning of Geometrical shapes and should be done in a proper sequence along with the digital activities. Students may do the problems in the space provided in this workbook or in their notebooks and discuss with their teachers and peers.

The CLIx platform is a digital platform that makes use of both the digital content and the workbook content. The platform has features like Notebook, Discussion and Gallery where students can give their responses, comments and upload their work respectively.

Unit 1: Additive to Multiplicative Thinking

Lesson 1.1 : Jamuni learns to share

*Please refer to this lesson on the **CLIx platform***

The five digital activities help in understanding equal share and fair distribution using fractions.

Lesson 1.2 : Let's compare and distribute

*Please refer to this lesson on the **CLIx platform***

The five digital activities help in understanding the concepts of proportional and additive reasoning.

Lesson 1.3 : Make the share equal across groups

*Please refer to this lesson on the **CLIx platform***

The four digital activities further clarify the concepts of proportional and additive reasoning.

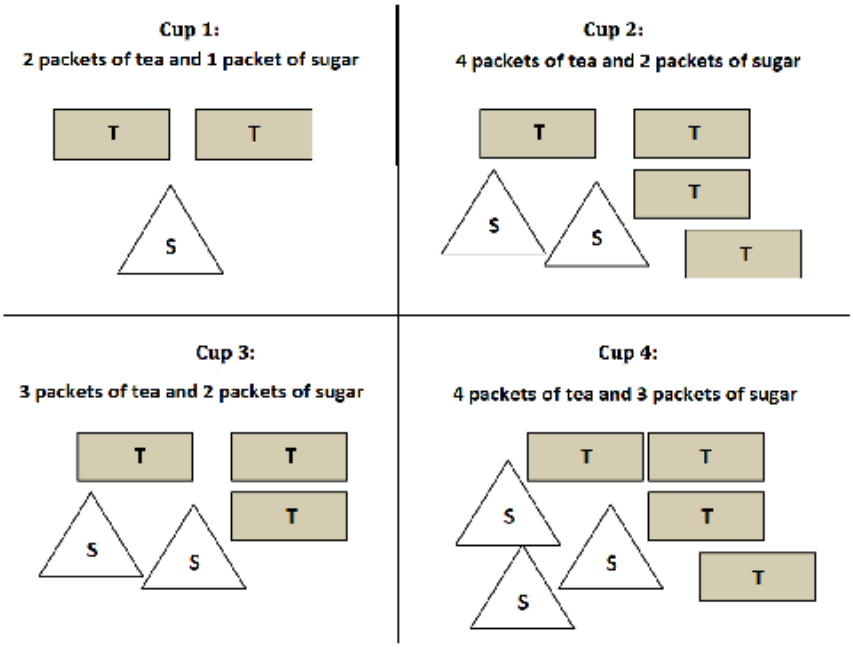
Unit 2: Multiplicative Thinking

Lesson 2.1 : Sweeter tea

Activity 1

In the image, each group of shapes represents a cup of tea. The rectangular shapes represent tea packets and the triangular shapes represent sugar packets.

Study each cup of tea carefully and find out which of the 4 cups of tea is most sweet.

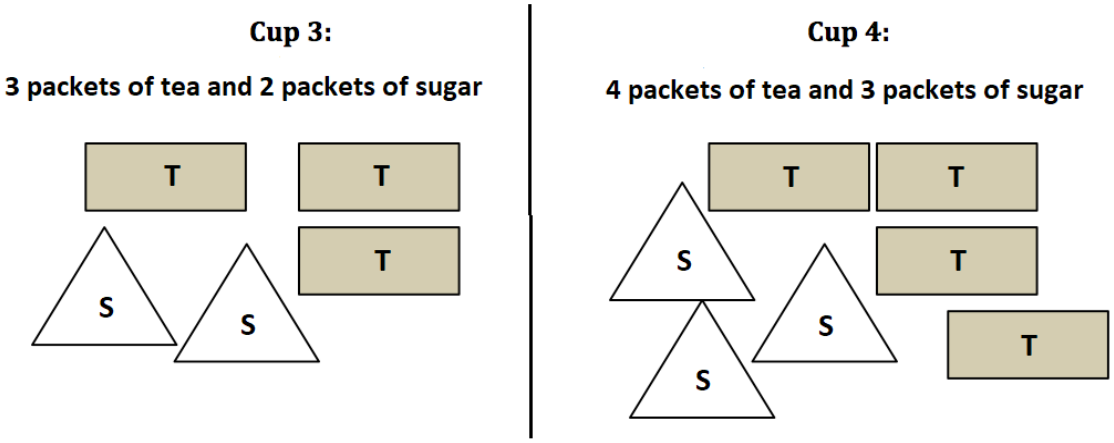


Activity 2

In the image, each group of shapes represents a cup of tea. The triangles represent sugar packets and the rectangles represent tea packets.

You are given 6 packets of tea. How many packets of sugar would you need to make a cup of tea that is exactly the same as:

- The one shown in Cup 3
- The one shown in Cup 4



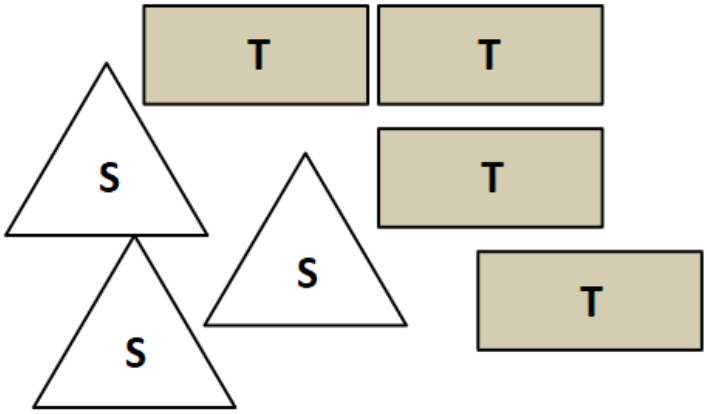
Activity 3 - Part 1

In the image, each group of shapes represents a cup of tea. The triangles represent sugar packets and the rectangles represent tea packets.

You have 15 packets of tea and 11 packets of sugar. How many packets of tea and sugar will you need to make 4 cups of tea that are as sweet as the one shown in Cup 4?

Cup 4:

4 packets of tea and 3 packets of sugar

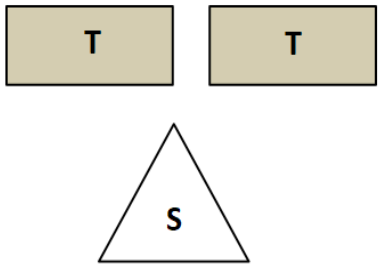


Activity 3 - Part 2

Study the following pictures of cups of tea. Each rectangle represents a packet of tea and each triangle represents a packet of sugar. Which cup has less sweeter tea?

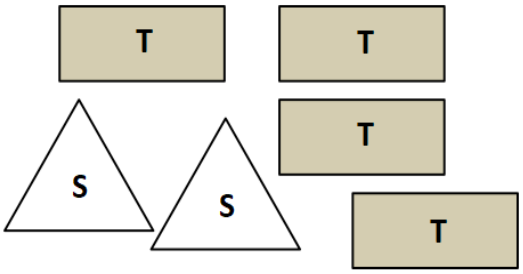
Cup 1:

2 packets of tea and 1 packet of sugar



Cup 2:

4 packets of tea and 2 packets of sugar



Lesson 2.2 : Jamuni solves puzzles

*Please refer to this lesson on the **CLIx platform***

The six digital activities help in building multiplicative thinking through scaling up and scaling down of patterns.

Lesson 2.3 : Jamuni goes to the bazaar

Activity 1

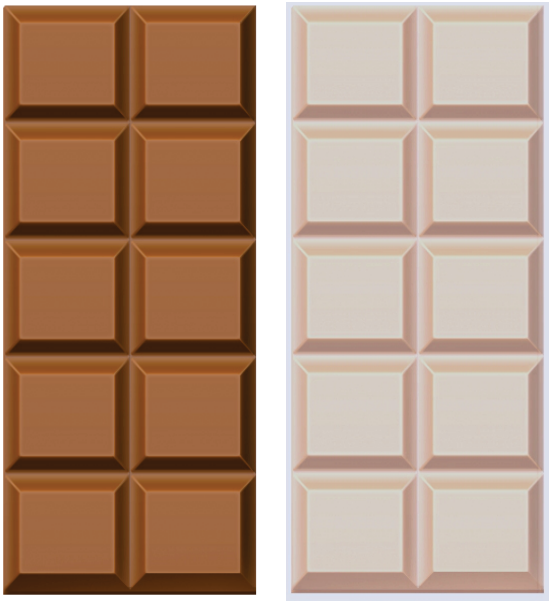
Jamuni and her friends are at an egg shop in the mela.

- They see a tray of eggs. The tray contains 12 eggs and costs Rs.36. Now if they want an egg tray that has one and half times more eggs than this tray, how much will they need to pay ?
- Shabhana is Jamuni's friend and she wants to buy two egg trays. She finds that there is a mix of both white and brown eggs in each tray. The first tray holds 12 eggs of which 4 are brown and 8 are white. The second tray holds 18 eggs. If the proportion of brown and white eggs is the same in both trays, how many eggs of each colour does the second tray have ?

	Tray 1	**Tray 2**
Total number of eggs	12	18
Number of white eggs	4	
Number of brown eggs	8	

Activity 2

Aman, Jamuni's friend, loves chocolate! He decides to buy a bar of chocolate to share with his friends. Help him solve some problems he faced when he went to a chocolate shop.



1. A white chocolate bar contains 10- small pieces. If Aman decides to give each of his friends 2 such small pieces, how many children can share the bar ?

2. The shopkeeper sells 3 small pieces of white chocolate for Rs.4. If Aman spends Rs.40, how many such pieces of chocolate can he buy?

3. The shopkeeper charges Rs.4 for a small piece of brown chocolate. If Aman decides to buy 10 such pieces, how much money he would need?

Activity 3

Jamuni and her friends were thirsty and went to a juice shop. The juice shop had two options for orange juice: 6-litre cartons for Rs. 200 and 4-litre cartons for Rs. 150.

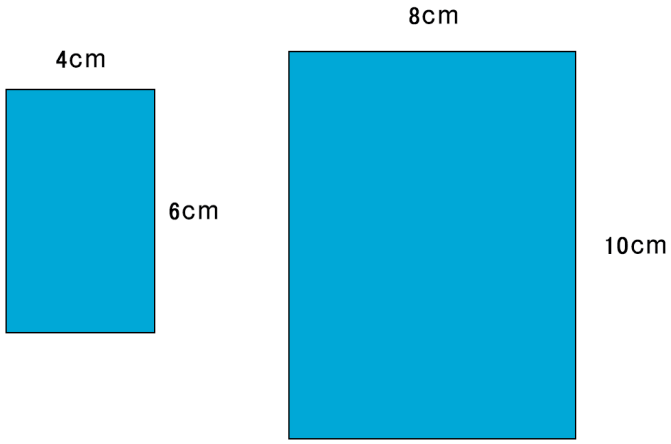
Which of the two cartons is cheaper?

- 6-litre carton
- 4-litre carton

Tell your partner how you found the answer. Find out what method your partner used.

Activity 4

Jamuni wants to buy a square paper napkin but the shopkeeper only has rectangular ones. Look at the two rectangular paper napkins shown here. Which of them is more squarish? Why?



Lesson 2.4 : Sahir makes a poster

*Please refer to this lesson on the **CLIx platform***

The two digital activities are based on pattern scaling and examining relationship between original and scaled shapes.

Unit 3: Ratios and Direct/Inverse Variations

Lesson 3.1 : Understanding ratio notation

Activity 1

Jamuni is sitting on the giant wheel ride. She is able to scan the entire mela scene whenever she goes to the top. She makes many observations. Can you write out Jamuni's observations in the form of a ratio?

a. There is 1 boy for every 2 girls in the fair. _____
b. Leena's mother is thrice as tall as her. _____
c. A farmer is standing with 4 cows and 8 pigs. _____
d. Geo is 2½ times shorter than Inspector Kaata. _____

Activity 2

27 children are sitting inside a video game parlour at the mela. The ratio of girls to boys is 3:6. Which of the following statements is/are true?

a. The ratio of boys to girls is 6:3. _____
b. Half the children in the parlour are female. _____
c. We know exactly how many boys are in the parlour. _____
d. We know exactly how many girls are in the parlour. _____
e. If we randomly choose 9 children in the parlour, we can expect that 3 will be girls. _____
f. We can figure out how many boys there would be if the parlour was visited by 36 children. _____

Activity 3

A circus hall in the mela has 100 seats. It is divided into two zones. Zone 1 has 30 seats and zone 2 has 70 seats. A total of 80 tickets was sold for a show. All the seats in zone 1 were filled.

a. What is the ratio of seats in zone 1 to seats in zone 2? _____
b. What is the ratio of empty seats to occupied seats? _____
c. What is the ratio of empty seats to occupied seats in zone 2? _____

Lesson 3.2 : Map reading with Jamuni

Jamuni has an interest in map reading and a curiosity to find the distance between two places using different possible routes. She likes reading her world atlas and uses the scaling factor given in the map to calculate the exact distance between two places.

Activity 1

Carefully look at the map-scale (scale given in the map.) What do you see? Compare 1 unit of the map-scale with 1 unit of the ruler (scale) you have. Now fill the following table and put correct unit-name

	Map-scale	Real distance
1 Unit		

The ratio between map-scale and real distance can be written as _:_. This ratio is called the "scale factor" for a given map. It is a matter of convenience that we choose different scaling factors for showing or calculating the distance between places. Now use the above scale factor to find the distance between any two cities of your choice on the map. Think about different ways of doing this task. Fill in the following table:

(Hint: Use a thread to measure the circuitous route and using a ruler and the given map-scale, find the actual distance between these two cities)

Map-scale	Length of the thread used	Real distance
1 cm = 12 km		
1 cm = 12 km		
1 cm = 12 km		

The other way in which a map-scale is shown on a map is by expressing a unit distance and a real distance, for example, 1 cm = 15 km which indicates that 1 cm in the map distance is equivalent to 15 km in actual distance.

Map-scale (1 cm = 15 km)	Distance on the map	Real distance
	6 cm	
	10.5 cm	
	50 cm	

Activity 2

In one map, Jamuni saw the map-scale was given in ratio form. Fill in the table by calculating the real distance.

	Thread Length	Real distance
Map-scale (1:25000)	10 cm	
	18 cm	
Map-scale (1 cm = 2.5 km)	12 cm	
	21 cm	

Activity 3

In this task, thread length or the real distance between two places are given below in the table. Fill in the missing value.

Map-scale	Distance on the map	Real distance
1:1500	50 cm	
10:2000	25 cm	
1 cm = 12 km		1800 km
1:250		500 km
	10 cm	75 km
	32 cm	960 km

Activity 4

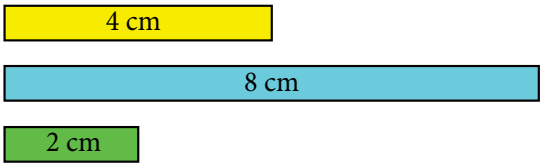
Jamuni found that the scale for the distance is represented differently in four different maps. Help her locate the map in which the distance between the cities A and B is different from all the other maps.
[Hint: Use a method other than cross-multiplication.]

Map	Map Distance between location A and B	Map-scale	Scaled distance between location A and B
Map 1	25 cm	1:600	
Map 2	12 cm	1:1250	
Map 3	24 cm	3:1800	
Map 4	30 cm	5:2500	

Lesson 3.3 : Finding length using strips

Activity 1

Aman, Sahir, and Leena are trying to measure the length of a sheet using strips. One of them uses only green strips; another uses only yellow strips; and the third uses blue strips.



Aman found the length as 8 units.
Leena found the length as 16 units.
Sahir found the length as 4 units.

Given these conditions, predict:
Who used the green strips ?
Who used the yellow strips ?
Who used the blue strips ?

Activity 2

Place the strips in line along the sheet's length to verify your prediction.

Now complete the following table:

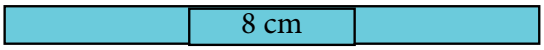
Length of strip (1)	Number of strips used (n)	1xn	1/n

If the strip is bigger, the number of strips used is _____ (smaller/bigger).
If the strip is smaller, the number of strips used is _____ (smaller/bigger).
Do you see any pattern in the 1st and 2nd columns of the above table ?
Do you see any pattern in the 3rd and 4th columns of the above table ?

We see that the _____ column contains all equal values. What does this value signify ?

Activity 3

Aman, Sahir, and Leena bought 3 sheets of different lengths from the stationery shop. They are now using the pink strip (size: 8 units) to measure its length.



Aman found the length as 24 units.
Leena found the length as 16 units.
Sahir found the length as 48 units.

Given these conditions, predict:
Who had the longest sheet ?
Who had the shortest sheet ?

Activity 4

Place the strips in line along the sheet's length to verify your prediction.

Now complete the following table:

Length of strip (1)	**Number of strips used (n)**	**1xn**	**1/n**

If the sheet is bigger, the number of strips used is _____ (smaller/bigger).
If the sheet is smaller, the number of strips used is _____ (smaller/bigger).
Do you see any pattern in the 1st and 2nd columns of the above table ?
Do you see any pattern in the 3rd and 4th columns of the above table ?

We see the _____ column contains all equal values. What does this value signify ?

Lesson 3.4 : Ice cubes in lemonade

*Please refer to this lesson on the **CLIx platform***

The two digital activities help in understanding inverse proportion.

Unit 4 : Applications

Lesson 4.1 : Proportions in linear equations and probability

Activity 1

Jamuni and her friends are planning to return home from the mela via train. They are checking the train time-table and found the following train:

	Station Name	Arrival time	Departure time	Distance (in km)
Train 12345	A		08:00	0
	B	12:30	13:00	225
	C	14:30	14:40	300
	D	16:40	17:00	400
	E	19:00		500

1. Plot a curve by putting distance travelled on the x-axis and time taken on the y-axis. Plot different stations A, B, C, D, E on the curve.
2. Find the distance that the train covers between station A and B and the time it takes. Similarly, find these values for other stations in the following table.

Stations	Distance Travelled (x)	Time taken (y)	x/y
A to B			
B to C			
C to D			
D to E			

3. Find the value of x/y in each case. What are the different values that you get? Do you see a pattern? Can you give a name to x/y ? Think and discuss with your friends.
4. Now write an equation that satisfies the above data table in terms of x and y.

Activity 2

Jamuni visits a stall where 3 buckets are kept. Bucket A has 2 red balls and 4 yellow balls. Bucket B has 4 red balls and 8 yellow balls. Bucket C has 7 red balls and 14 yellow balls. In order to win the prize, Jamuni has to answer the stall owner's questions correctly.

The stall owner asks: "If you pick out one ball fro each bucket what is the probability that the ball will be red?" Enter your answers in the table below:

	Red balls	Yellow balls	Probability of finding a red ball
Bucket A	2	4	
Bucket B	4	8	
Bucket C	7	14	

Do you see any pattern in the last column of the table ? Can you explain why you see this pattern ?

Lesson 4.2 : Compound ratio and proportion

Activity 1 : Compound Proportion

1. Jamuni's parents are construction workers. She observed that a team of construction workers can construct a wall of 400 metres in 12 days by working 8 hours everyday. How long will it take if the wall size is 600 metre and the workers put in 9 hours everyday ?

2. Jamuni's mother deposited Rs. 4500 in a bank and received Rs. 360 after two years. How much interest amount will she get in 5 years if she deposited Rs. 6000?

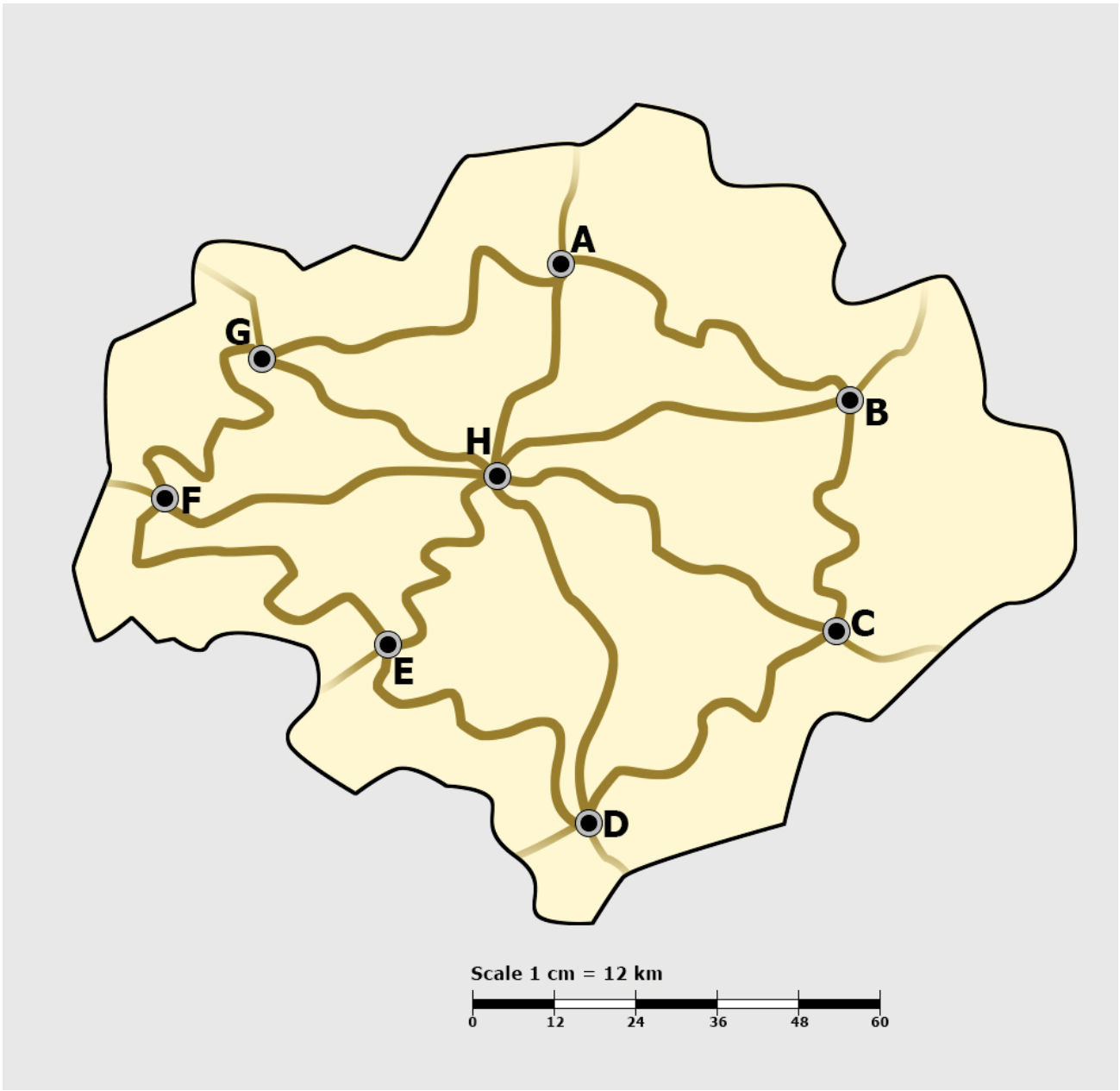
Activity 2: Mixture problem - Mini recipe problem

Jamuni, Aman, Leena, Sahir are sitting at a chai ki dukan. The recipe used for making tea for 4 persons is provided here:

- Tea powder - 2 teaspoons
- Sugar - 4 teaspoons
- Milk - 12 teaspoons
- Water - 20 teaspoons

After half an hour, Jamuni's parents also join the group, and they all decide to have a cup of tea. List the ingredients and their amounts to make tea for 6 persons now, which would taste exactly the same as the tea made earlier.

- Tea powder - ____ teaspoons
- Sugar - ____ teaspoons
- Milk - ____ teaspoons
- Water - ____ teaspoons


A
G
B
H
F
C
E
D
Scale 1 cm = 12 km
0
12
24
36
48
60

Centre for Education, Innovation and Action Research
Tata Institute of Social Sciences
V.N.Purav Marg, Deonar,
Mumbai – 400088, India
Phone: +91 – 22- 25525002/3/4
www.clix.tiss.edu